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Modal Analysis and Dispersion Characteristics of a D-Shaped Dielectric Optical Waveguide

A.K. SHAHI

Department Of Physics, Ram Lalit Singh P. G. College,Kailhat, Narayanpur, Mirzapur, U.P. India-231305

Abstract: A theoretical and numerical investigation of a D-shaped dielectric optical waveguide is presented. The proposed waveguide is obtained by truncating a circular dielectric core with a planar surface, thereby introducing a controlled departure from circular symmetry. The analysis follows the weak-guidance scalar formulation used in the supplied reference work, in which the modal propagation constant is obtained from the transverse eigenvalue problem. A dimensionless truncation parameter, η=d/a, is introduced to quantify the degree of truncation. Numerical calculations were performed for n1=1.50, n2=1.48 and λ=1.55 µm. The normalized propagation constant b was evaluated for the fundamental mode over V=2–5 for several truncation levels. The calculated results demonstrate a clear dependence of modal confinement on the D-shaped geometry. At V=4 and η=0.5, the calculated normalized propagation constant is 0.7218 and the corresponding effective refractive index is 1.49446. The results establish a numerical basis for a more detailed Goell point-matching treatment of the D-shaped boundary.

Keywords: D-shaped waveguide; optical waveguide; weak guidance; modal analysis; dispersion; propagation constant; truncation parameter.

1. INTRODUCTION

2. Geometry of the Proposed D-Shaped Waveguide and its Theoretical Formulation

The core is taken as a circular region of radius a, truncated by a plane at $x=d$. The core and cladding refractive indices are $n1$ and $n2$, respectively, with $n1>n2$. The propagation direction is along the z-axis.

$$\eta = d/a$$

The parameter η is used as the dimensionless truncation parameter. Smaller η values correspond to stronger truncation, while η approaching unity corresponds to a nearly circular core. For illustration, the geometry shown in Fig. 1 uses η=0.5.

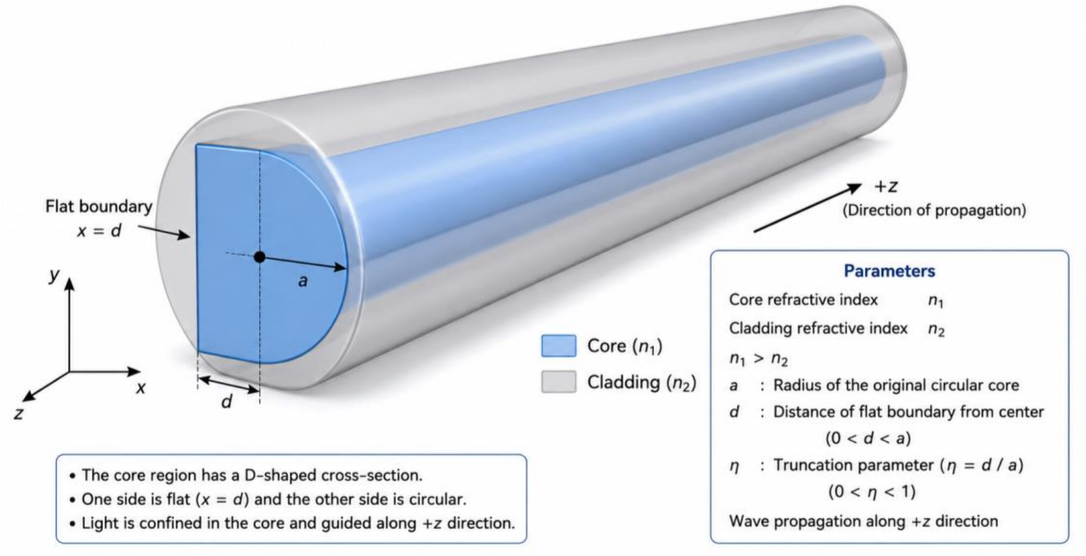

Fig. 1. Schematic view of the D-shaped dielectric waveguide.

Under the weak-guidance approximation, the vector electromagnetic problem can be represented by a scalar transverse field ψ. In a homogeneous region, the scalar field satisfies

$$\nabla_t^2\psi + (k_0^2 n^2 - \beta^2)\psi = 0$$

where $k0=2\pi/\lambda$ is the free-space wavenumber, n is the refractive index and β is the modal propagation constant.

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In the core,

$u = \sqrt{(k_0^2 n_1^2 - \beta^2)}$

and the field may be expanded in cylindrical harmonics as

$\psi_1(r,\theta) = \Sigma_m [A_m J_m(ur)\cos(m\theta) + B_m J_m(ur)\sin(m\theta)]$

In the cladding, for a guided mode $\beta > k_0 n_2$,

$w = \sqrt{(\beta^2 - k_0^2 n_2^2)}$

$\psi_2(r,\theta) = \Sigma_m [C_m K_m(wr)\cos(m\theta) + D_m K_m(wr)\sin(m\theta)]$

The Bessel functions Jm are finite in the core, whereas the modified Bessel functions Km provide the decaying field in the external region.

The D-shaped boundary consists of a circular arc and a planar segment. Continuity of the scalar field and the appropriate normal derivative is imposed across the dielectric interface. On the curved boundary the normal derivative is radial. On the flat boundary $x=d$, the normal derivative is along x and can be written as

$\partial/\partial x = \cos\theta\, \partial/\partial r - (\sin\theta/r)\, \partial/\partial\theta$

Instead of evaluating the characteristic determinant by the point-matching formulation of the reference paper, the present numerical calculation solves the same weak-guidance scalar eigenvalue problem using a finite-difference transverse operator. This provides a direct numerical calculation of β and an independent baseline for subsequent Goell point-matching implementation.

The computational domain was discretized with a square-grid spacing of $0.04a$. The D-shaped core was embedded in a surrounding numerical box extending to approximately $\pm 1.8a$. The lowest transverse eigenvalue was retained as the fundamental guided mode.

$b = (\beta^2 - k_0^2 n_2^2) / [k_0^2(n_1^2 - n_2^2)]$

$V = (2\pi a/\lambda)\sqrt{(n_1^2 - n_2^2)}$

$n\_eff = \beta/k_0 = \sqrt{[n_2^2 + b(n_1^2 - n_2^2)]}$

The normalized propagation constant b provides a convenient measure of modal confinement. The effective refractive index is obtained directly from β.

3. NUMERICAL RESULTS AND DISCUSSION

Calculations were performed for n1=1.50, n2=1.48 and λ=1.55 µm. The fundamental-mode normalized propagation constant was evaluated for V between 2 and 5. Four truncation levels, η=0, 0.5, 0.75 and 0.9, were examined. The η=0 curve corresponds to a half-plane-truncated geometry in the present coordinate convention, while η=0.9 is close to the circular limit.

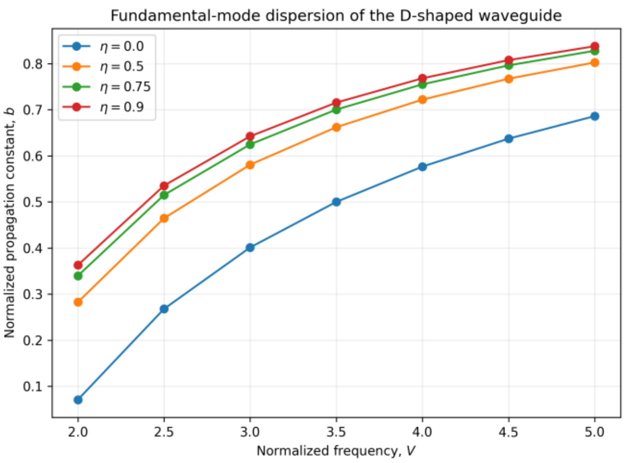

Fig. 2. Calculated fundamental-mode dispersion curves for different D-shaped truncation parameters.

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The dispersion curves show a systematic increase of b as V increases. For a fixed V, the calculated b also increases as η is increased. For example, at V=4 the values are approximately 0.577 for η=0, 0.7218 for η=0.5, 0.755 for η=0.75 and 0.768 for η=0.9. Thus, reducing the severity of truncation produces stronger normalized modal confinement in the present numerical model.

Table 1. Calculated fundamental-mode normalized propagation constant

V	η=0	η=0.5	η=0.75	η=0.9
2.0	0.071	0.282	0.340	0.363
2.5	0.268	0.464	0.515	0.535
3.0	0.401	0.581	0.625	0.642
3.5	0.500	0.662	0.700	0.715
4.0	0.576	0.722	0.755	0.768
4.5	0.637	0.767	0.796	0.807
5.0	0.686	0.802	0.828	0.838

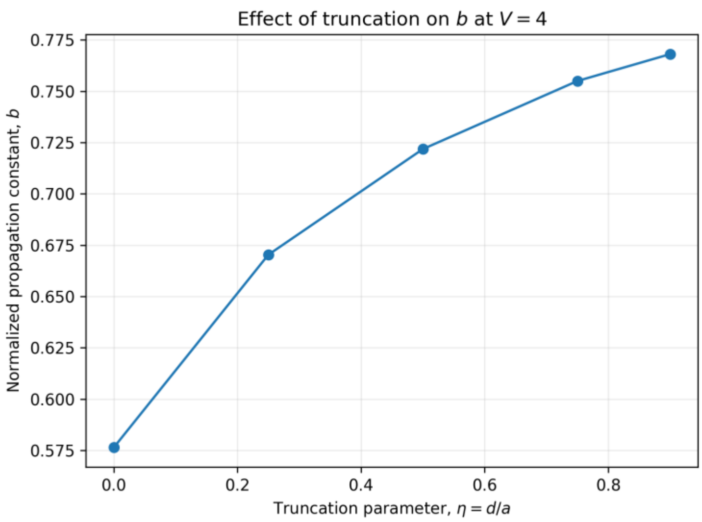

Fig. 3. Effect of truncation parameter on the normalized propagation constant at V=4.

Figure 3 further illustrates the geometry dependence at a fixed normalized frequency. The increase of b with η indicates that the less-truncated structures provide stronger effective confinement for the fundamental mode. The behavior is continuous over the investigated range and demonstrates that the truncation parameter can be used as a useful design variable.

The numerical results confirm that the cross-sectional shape has a substantial influence on the propagation characteristics of the waveguide. The D-shaped boundary breaks the rotational symmetry of a circular guide and changes the transverse confinement. The present calculation is particularly useful as a baseline because it provides numerical values against which a direct Goell point-matching calculation can be validated.

A complete publication-grade study should additionally determine higher-order modes, their cutoff values, modal field distributions, and convergence with the number and distribution of boundary matching points. These quantities are not inferred here without a corresponding calculation.

4. CONCLUSION

A numerical modal analysis of a D-shaped dielectric optical waveguide has been presented using the weak-guidance scalar eigenvalue formulation. A dimensionless truncation parameter η=d/a was introduced to quantify the departure from the circular geometry. For n1=1.50, n2=1.48 and λ=1.55 μm, fundamental-mode dispersion curves were obtained for several truncation levels. At V=4 and η=0.5, b=0.7218 and n_eff=1.49446. The results demonstrate a clear dependence

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of modal confinement on the D-shaped geometry. The work provides a numerical foundation for the next stage: implementing Goell's point matching method directly on the curved and flat boundary and extending the study to higher-order modes and cutoff characteristics.

REFERENCES

[1] Goell, J. E., "A circular harmonic computer analysis of rectangular dielectric waveguides," *Bell System Technical Journal,* 48 (1969) 2133–2160.

[2] Singh, V., Prasad, B., and Ojha, S. P., "Weak guidance modal dispersion characteristics of an optical waveguide having a core with sinusoidally varying gear shaped cross-section," *Microwave and Optical Technology Letters*, 22 (1999) 129–133.

[3] Mishra, V., Singh, V., Prasad, B., and Ojha, S. P., "Weak guidance modal dispersion curves of an optical waveguide having a double-convex-lens cross section," *Microwave and Optical Technology Letters,* 22 (1999) 314–317.

[4] Singh, V., Ojha, S. P., and Singh, L. K., "Optical and microwave dispersion curves of a waveguide with a guiding region having a cross-section with a lunar shape," *Microwave and Optical Technology Letters* (1999).

[5] Singh, V., Prasad, B., and Ojha, S. P., "Modal behavior, cutoff condition and dispersion characteristics of an optical waveguide with core cross-section bounded by two spirals," *Microwave and Optical Technology Letters,* 21 (1999) 121–124.

[6] Shahi, A. K., Singh, V., and Ojha, S. P., "Towards dispersion characteristics for a new unconventional metal-clad optical waveguide," *Microwave and Optical Technology Letters,* 49 (2007) 2709–2713. DOI: 10.1002/mop.22854.

[7] Singh, H. P., Singh, V., and Arora, V. P., "Modal analysis and dispersion curves of a curvilinear triangular cored light guide using Goell's point matching method," *Optik,* 119 (2008) 29–33. DOI: 10.1016/j.ijleo.2006.06.005.

[8] Kumar, S., Maurya, J. B., Roumi, B., Abdi-Ghaleh, R., and Prajapati, Y. K., "D-shaped fiber optic plasmonic sensors using planar and grating structures of silver and gold: design and analysis," *Applied Optics*, 62 (2023) E130–E136. DOI: 10.1364/AO.481145.

[9] Fu, Z., Liu, D., and Han, W., "Waveguide Coupled Polymer Micro-Ring Resonator in a D-Shaped Fiber," *Journal of Lightwave Technology,* 42 (2024) 835–840. DOI: 10.1364/JLT.42.000835.

[10] Maurya, R., Mishra, A., Yadav, C. S., Upadhyay, A., Sharma, G., and Singh, V., "LMR and SPR induced Fano resonance in a planar waveguide-coupled D-shaped optical fiber for enhanced refractive index sensing in the Vis–NIR region," *Journal of the Optical Society of America B,* 42 (2025) 1794–1803. DOI: 10.1364/JOSAB.565538.

[11] Nawrot, U., Gacka, E., Kunicki, P., Pruchnik, B., and Sierakowski, A., "Simultaneous Multi-Point Refractive Index Sensing Using FIB-Fabricated D-Shaped Optical Fiber Sensors Interrogated via OTDR," *IEEE Sensors Journal*, 25 (2025) 36188–36198. DOI: 10.1109/JSEN.2025.3602181.

