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A Spherical Fuzzy Set Based Approach to the Job Sequencing Problem

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Abstract: Job sequencing problems aim to find an optimal sequence of jobs, are fundamental in production and operations management. The optimal sequence helps to minimize objectives such as makespan, total tardiness, or flow time. Fuzzy set theory is considered better over classical theory as it handles vagueness more appropriately. These days, Spherical Fuzzy Numbers (SFNs) are gaining attention of researchers due to their ability to represent membership, non-membership and hesitancy simultaneously subject to a spherical constraint. In this paper sequencing problems are introduced in which processing times are taken as SFNs and a ranking based on score function is proposed for job prioritization. The proposed method is explained with the help of illustrative example.

Keywords: Job sequencing problem, Spherical fuzzy set, Spherical fuzzy numbers, membership function, non-membership function, hesitancy, score function.

I. INTRODUCTION

Job sequencing is an important application of operational research that determines optimal sequence of jobs to minimize total elapsed time. The main objective of job sequencing is that order of routing the jobs should be done with at most care that minimizes the total time. In classical job sequencing problem processing times of all jobs through different machines are taken exact figures However in real this is not possible. In measurement some errors are committed. Fuzzy logic helps to solve the problem of vagueness. The measurement of truth in this is done on the scale lying in [0,1]. Fuzzy set theory was first introduced by Zadeh (1965) in which a fuzzy set A is represented by a set of ordered pairs, where each element x in X is associated with a degree of membership $\mu_A(x)$ that ranges from 0 to 1. In fuzzy sets only membership functions are considered and non-membership is fully ignored. To solve this problem, Atanassov developed intuitionistic fuzzy sets (1986) in which both membership and non-membership degrees are included. After some time, T. Atanassov & G. Gargov (1989) gave the extension of intuitionistic fuzzy sets called interval-valued intuitionistic fuzzy sets. Intuitionistic fuzzy numbers were developed on the basis of the constraint that sum of membership grades and non-membership grades should be less than one. However, some situations may arise where it gets greater than one. To overcome, this problem, Yager (2013) developed Pythagorean fuzzy sets with constraint that sum of squares of membership and non-membership function should be less than or equal to one. After that, Cuong & Kreinovich (2013) developed fuzzy sets using membership, non-membership and neutral membership whose sum does not exceed one. However, there are situations where this sum exceed one. To handle these situations, Gundogdu & Kahraman (2019) introduced the idea of spherical fuzzy sets, where the sum of squares of three membership grades does not exceed one. Spherical fuzzy sets have been applied in various fields such as multi-criteria decision making by Ashraf et al. (2019), Balin (2020), Zeng et al. (2019), medical diagnosis by Mahmood et al. (2019), pattern recognition by Ullah et al. (2018). Mathew et al. (2020) applied spherical fuzzy sets on selection problems. Dhanalakshmi used spherical fuzzy sets to solve a fuzzy risk analysis problem whose parameters are presented as trapezoidal spherical fuzzy numbers. Due to realistic and versatile property of spherical fuzzy numbers in this paper sequencing problems are solved by taking processing times as spherical fuzzy numbers.

Preliminaries

Definition 1: Spherical Fuzzy Set (SFS)

Let X be a universe of discourse. A spherical fuzzy set A on X is defined as

$A = \{\langle x, \mu_A(x), \theta_A(x), \pi_A(x)\rangle : x \in X\}$ with the condition

$$\left(\mu_A(x)\right)^2 + \left(\theta_A(x)\right)^2 + \left(\pi_A(x)\right)^2 \leq 1$$

Definition 2: Spherical Fuzzy Number (SFN)

A spherical fuzzy number is a special case of an SFS representing numerical information as:

$$N = (\mu, \theta, \pi), \mu, \theta, \pi \in [0,1], \mu^2 + \theta^2 + \pi^2 \leq 1$$

Definition 3: Score Functions of SFNs

3.1: Existing Score Function:

Gundagdu and Kahraman (2019) proposed a simple score function $S(N) = \mu - \theta$ which includes the balance between membership and non-membership but ignores hesitancy.

3.2: Enhanced Score Function:

It is defined as $S(N) = \mu - \theta - \pi$ and in normalized form it is defined as

$$S(N) = \mu - \theta + (1 - \pi)$$

3.3: Generalized Score Function:

A weighted score function can better reflect preferences as

$$S(N) = \alpha\mu - \beta\theta - \gamma\pi \text{ where } \alpha, \beta, \gamma \geq 0 \text{ and } \alpha + \beta + \gamma = 1$$

Definition 4: Properties of score functions

A score function for SFNs should satisfy:

1. Monotonicity: Larger membership yields higher score.
2. Consistency: If two SFNs are identical, then they yield the same score.
3. Boundedness: $S(N) \in [-1,1]$.
4. Comparability: Enables ranking of alternatives in MCDM.

Algorithm:

The sequencing problem is defined as:

Jobs→ Machines↓	J_1	J_2	....	J_n
M_1	t_{11}	t_{12}	....	t_{1n}
M_2	t_{21}	t_{22}	....	t_{2n}

Here t_{ij} is spherical fuzzy number which denotes time duration taken by i^{th} job on j^{th} machine.

The following algorithm is given to solve the sequencing problem.

Step 1:

By using defined score function processing times taken as spherical fuzzy number are defuzzified.

Step 2:

Using Johnson's algorithm find optimal sequence for given sequencing problem.

Step 3:

Prepare In-Out tables for both machines and calculate total elapsed time.

Step 4:

From above table find idle time of machines.

Numerical Illustration:

Example:

Five jobs are to be processed through two machines and processing time of each job which is taken as spherical fuzzy number is presented in following table. Find optimal sequence of jobs and total elapsed time.

Jobs→ Machines↓	A	B	C	D	E
M_1	(0.7,0.7,0.2)	(0.9,0.4,0.2)	(0.7,0.2,0.4)	(0.6,0.3,0.1)	(0.6,0.4,0.4)
M_2	(0.2,0.3,0.5)	(0.8,0.5,0.7)	(0.7,0.4,0.5)	(0.8,0.2,0.3)	(0.7,0.2,0.1)

Solution:

Step 1: Using defined score function defuzzify processing time of each job .

Jobs→ Machines↓	A	B	C	D	E
M_1	0.8	0.7	1.1	1.2	0.8
M_2	0.4	0.6	0.8	1.3	1.4

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Step 2: using Johnson's algorithm optimal sequence is calculated.
Among all processing times minimum time is 0.4 which is corresponding to first job on second machine, so this job is placed first from last.

-	-	-	-	A

Leaving job A, for remaining jobs the shortest time is 0.6 which is for second job on second machine, so second job will be taken at second last position.

-	-	-	B	A

Leaving both jobs A and B, for remaining jobs the shortest time is 0.8 which is for third job on second machine, so put that in third last position.

-	-	C	B	A

Leaving these jobs from remaining jobs minimum time is 1.2 which is for fourth job on first machine, so put that job in first position.

D	-	C	B	A

Final sequence is

D	E	C	B	A

Step 3:

Job Sequence	M_1		M_2	
	Time in	Time out	Time in	Time out
D	0	(0.6,0.3,0.1)	(0.6,0.3,0.1)	(0.92,0.06,0.15)
E	(0.6,0.3,0.1)	(0.84,0.12,0.28)	(0.92,0.06,0.15)	(0.976,0.012,0.047)
C	(0.84,0.12,0.28)	(0.952,0.024,0.09312)	(0.976,0.012,0.047)	(0.993,0.004,0.014)
B	(0.952,0.024,0.09312)	(0.9952,0.0096,0.0095)	(0.993,0.004,0.014)	(0.999,0.002,0.002)
A	(0.9952,0.0096,0.0095)	(0.9986,0.0067,0.0028)	(0.999,0.002,0.002)	(0.999,0.0007,0.002)

Above table is prepared by using addition formula for spherical fuzzy numbers.
Step 4:
Total Elapsed time: (0.999,0.0007,0.002) hours

II. CONCLUSION

This paper explained how spherical fuzzy numbers can be effectively integrated into job sequencing problem to handle uncertainty in processing times. Processing times which are taken as SFNs can be converted into comparable crisp numbers by using score functions. In future research we can extend this to multi-machine flow shops and parallel machines.

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