IJIREEICE ISSN (O) 2321-2004, ISSN (P) 2321-5526

International Journal of Innovative Research in Electrical, Electronics, Instrumentation and Control Engineering

ISO 3297:2007 Certified ⋇ Impact Factor 8.021 ⋇ Peer-reviewed / Refereed journal ⋇ Vol. 11, Issue 8, August 2023

DOI: 10.17148/IJIREEICE.2023.11803

# Designing and implementing a Proportional Derivative-based Controllers for Ball and Beam Systems

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**Abstract**: The Ball and Beam System (BBS) is considered as one of the most common used approaches on benchmark for evaluating control algorithms in control engineering laboratories. This system demonstrates significant instability and nonlinearity, finding practical relevance in scenarios like maintaining balance of a ball on a beam, akin to the horizontal stabilization required during airplane landings. The actual system contains a ball, a beam, and a motor, with the core principle involving motor-generated torque to govern ball position on the beam. This facilitates smooth ball movement along the beam. Several theoretical approaches are used to controller design exist for stabilizing the ball and beam system. This paper proposed a Proportional Derivative (PD) controllers design to regulate ball position on the beam. MATLAB environment is utilised for simulating the mathematical model of this system. The study showcases the efficacy of the control algorithm in effectively managing the inherent nonlinearity and instability of the system. The achieved results from the conducted simulation, approved the ability of the proposed PD controller for stabilising ball position tracking on ball and beam system.

**Keywords:** PD controller, Ball and Beam System, DC motor, Root locus.

## I.INTRODUCTION

The management of unstable systems (BBS) holds significant importance for various control issues. Due to the potential risks associated with testing these systems in the context of aerospace and airplanes, their study is limited to laboratory environments where system modelling is employed. It is commonly associated with actual control difficulties, such as the horizontal stabilization of an airplane. Several authors have analyzed the stability of BBS.

According to Hrishikesh, the parameters of the PD controller are determined using the Particle Swarm Optimization (PSO) method [1]. Anitha has also developed a PID controller for BBS utilizing the POS method [6]. In his paper, Ibtihal presents a proposal for the design of a quantitative PID controller for the BBS. The PID controller parameters are determined using the Black Hole Optimization (BHO) method [2]. Moreover, the authors in [3] employed PID and an adaptive neuro-fuzzy controller to ascertain the desired location of the ball, while in the study conducted by Shahad, the parameters of the PD controller were achieved by employing a robust "H loop shaping controller" [4]. Furthermore, The objective of Radhi's research is to develop a controller utilizing fuzzy logic control [5]. The papers in [6, 7] utilized a PID controller to effectively regulate the position of the ball. In addition, Rahmat examined the stability of BBS through various types of controllers, including conventional, intelligent, and modern controllers [8].

The objective of this study is to regulate the proper position of the ball on the beam. The sensor is utilized to measure the position of the ball, while the motor is employed to adjust the angle of the beam, thereby altering the ball's location. The system employs multiple control algorithms in order to accurately guide the ball to its intended position. The designed PD controllers aim to optimize the rising time, overshoot, and offset to achieve the desired levels.

## II. BAL AND BEAM SYSTEM

The system is made up of two separate parts. The first part is called the DC servo motor. It is an electromechanical system that receives an electrical signal from a controller and produces a rotational displacement (angle) as its output. The second model is known as the ball and beam system( BBS). It's actually a mechanical system that takes the rotational displacement, or angle, from a motor and then converts it into linear displacement.
Firstly, the DC motor is capable of producing rotary motion directly and, when combined with wheels or cables, can also generate translational motion. In references [9] and [10], the electric equivalent circuit of the armature and the free-body diagram of the DC motor were derived. These diagrams are represented in Figures 1 and 2, while the corresponding transfer function is expressed in Equation 1.

  

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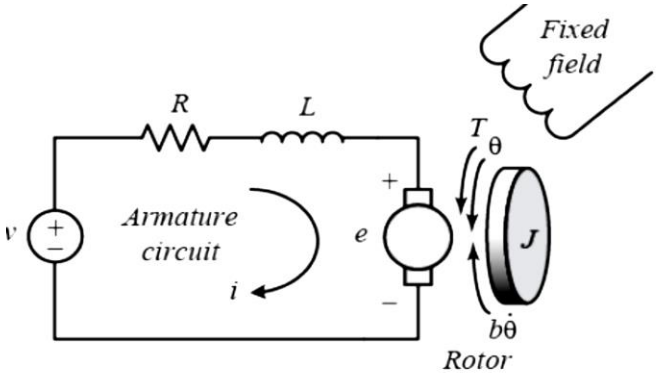


Figure1: Equivalent circuit of a DC motor[9]

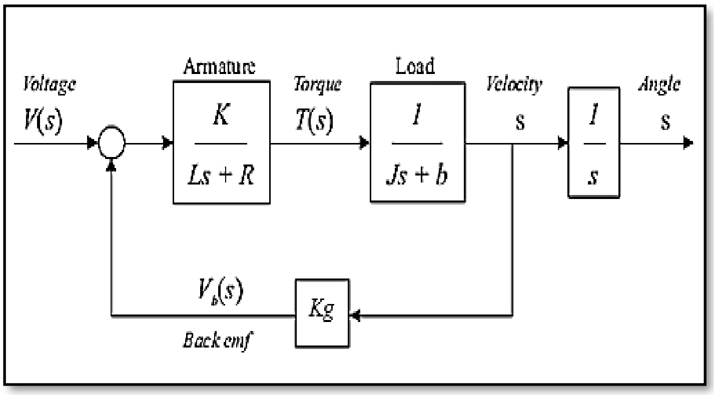


Figure 2: A block diagram of the DC motor[10]

The transfer function of the DC motor describes the relationship between the motor's position θ (s) and the applied voltage supply V (s) in the following equation:

$$G(s) = \frac{\theta(s)}{v(s)} = \frac{K * K_g}{s[(J\, s+b)(L_a+R_a)+K^2]} \qquad rad/v \qquad (1)$$

Table 1 DC motor parameters

| Parameter | Value |
|---|---|
| Gears ratio(Kg) | |
| Motor torque constant and the back emf constant (K) | K* Kg=K=0.01 |
| Moment of inertia of the rotor (J) | J=0.01 |
| Motor viscous friction constant (b) | b=0.1 |
| Motor electric inductance($L_a$) | $L_a$=0.5 |
| Motor electric inductance (La) | $R_a$=1 |

Secondly, a ball is placed on a beam, as shown in Figure 3, and it is allowed to roll along the length of the beam. A lever arm is linked to one end of the beam, while the opposite end is affixed to a servo gear. As the servo gear rotates at an angle( θ), the lever adjusts the angle of the beam(α). When the angle deviates from the horizontal position, the force of gravity prompts the ball to roll along the beam. A controller will be created for this system to control the position of the ball[12].

  

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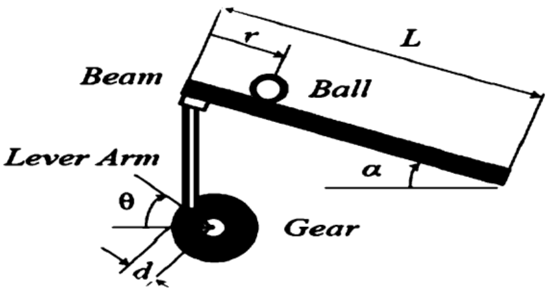


Figure 3: Ball and beam model [11,12]

Table 2: Ball and beam system parameters

| Symbol | Definition | Value |
|---|---|---|
| m | Mass of the ball | 0.111Kg |
| g | Gravity acceleration | 9.8 (m/s$^2$) |
| d | The distance between the center of the gear and the terminal point of the moving beam | 0.03m |
| L | The beam length | 1.0 m |
| R | Ball radius | 0.015m |
| J | Inertia moment of ball | 2*m*R$^2$/5 |

As mentioned in Eq. 2, various studies have obtained the transfer function of BBS according to [9–11]

$$P(s) = \frac{R(s)}{\theta(s)} = \frac{mgd}{L\left(\frac{J}{R^2}+m\right)s^2} \quad \left[\frac{m}{rad}\right] \qquad (2)$$

Finally, the overall transfer function has been calculated by cascading the BBS and DC motor transfer functions as presented in Equation (2).

$$\frac{R(s)}{V(s)} = \frac{K*m*g*d}{s^3\, L\left(\frac{J}{R^2}+m\right)(Js+b)(L_a\, s+R_a)+K^2} \left[\frac{m}{v}\right] \qquad (3)$$

## III. CONTROLLER DESIGN AND RESULT

The design method based on the root locus criterion and the algorithm mentioned in Figure 4

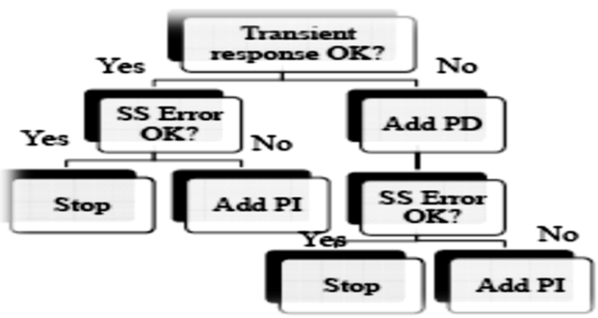


Figure 4: Schematic of the Design Algorithm

### 1. DC Motor

The open-loop transfer function of the motor is defined as follows:

$$G_{Motor} = \frac{\theta(s)}{V(S)} = \frac{2}{s\,(s+2)(s+10)} \qquad (4)$$

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### 1.1 DC motor without controller:

The following figures (figures 5 and 6) show the open loop step response and the location of poles and zeros, respectively.

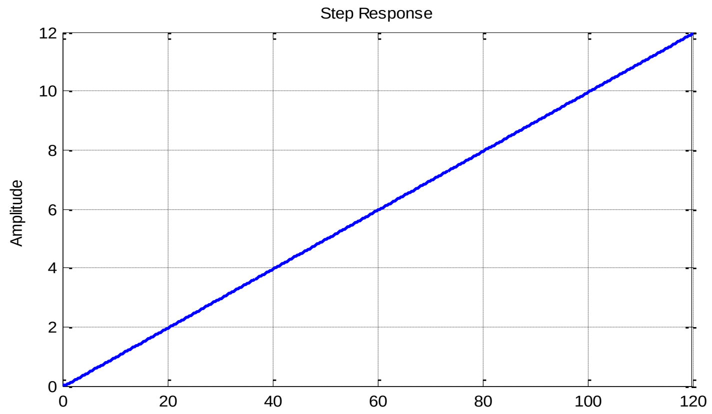


Figure 5: Open-loop response of the uncontrolled DC motor

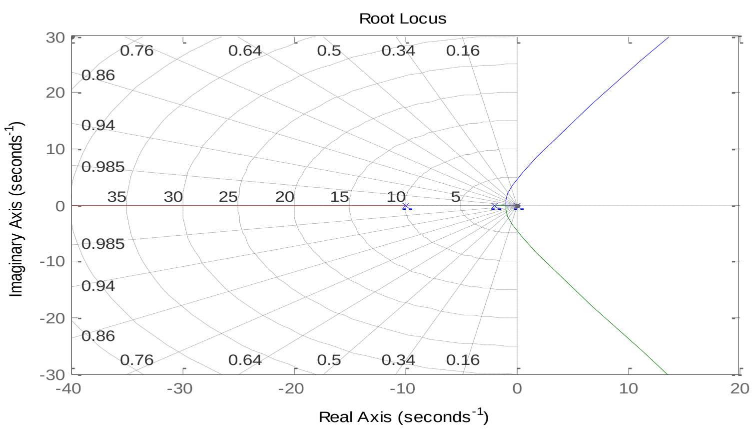


Figure 6: Root locus of the uncontrolled DC motor

### 1.2 DC Motor with controller:

In the forward path, the PD controller [k (s+a)] is added to the motor as follows:

$$G_{Motor} = \frac{2\ K(s+a)}{s\ (2+2)(s+10)} \qquad (5)$$

The closed loop of the controlled motor is as follows:

$$G\ _{CL\_Motor} = \frac{\overline{K}\,(s+a)}{s\ (s+2)(s+10)+\overline{K}\,(s+a)} \qquad (6)$$

Where $\overline{K}\ = 2\ K$

The mathematical derivation of the controller parameters is presented below:

Characteristic equation in standard form of a system 3rd order is$[(s+b)[s^2+2\,\omega_n\,\zeta\ \ s+\ \omega_n\ ^2\,]$. We compare the DC motor's characteristic equation with3rd order Characteristic equation as follows:

$$s^3+12\,s^2+20\,s+\ \overline{K}\,s+\overline{K}\ \ a=(s+b)[s^2+2\,\omega_n\,\zeta\ \ s+\ \omega_n\ ^2\,]$$

$$s^3+12\,s^2+(20\ +\ \overline{K}\,)s+\overline{K}\ \ a=\ s^3+(b+2\,\omega_n\,\zeta)\ s^2+(2\,\omega_n\,\xi\,b+\ \ \omega_n{}^2\,)s+b\ \ \omega_n{}^2$$

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The coefficients have identical power of s are compared to determine the PD controller's parameters as following:

$12 = b \ + 2\,\omega_n\,\zeta \qquad \therefore\ b \cong 788$

For our design we select $\omega_n = 404$, and $\zeta = 0.99$

$20 + \bar{K} = 2\,\omega_n\,\xi\, b + \omega_n^{\,2} \qquad \therefore\ \bar{K} \cong 4 \times 10^{05}$

$b\ \omega_n^{\,2} = a\,\bar{K} \qquad \therefore a = 328$

Secondly, controller's parameters are calculated via MATLAB program as follows

$$a = 328\,, K = 4.6708 * 10^{05}\,, and\ b = 787.9200$$

Finally, the designed parameters are selected as follows:

$$a = 275.33\,, K = 4.6708e + 05\,, and\ b = 788$$

The transfer function of the first PD controller is as follows:

$$G_{C1}(s) = K\,\frac{(s+275.33)}{(s+788.9)} \qquad (7)$$

The transfer function $G_{C1}(s)$ of the first PD controller is connected in series with the open-loop transfer function G(s) of the system. As a result, the transfer function of the system can be represented as follows:

$$\boldsymbol{G(s)} = \mathbf{K}\,\frac{(\mathbf{s} + \mathbf{275.3})}{\mathbf{s^2}\ (\mathbf{s} + \mathbf{787.9})\,(\mathbf{s^2} + \mathbf{799.9}\ \mathbf{s} + \mathbf{1.632} * \mathbf{10^{05}})} \qquad (8)$$

The step response of a controlled DC motor is shown in Figures 7 and 8.

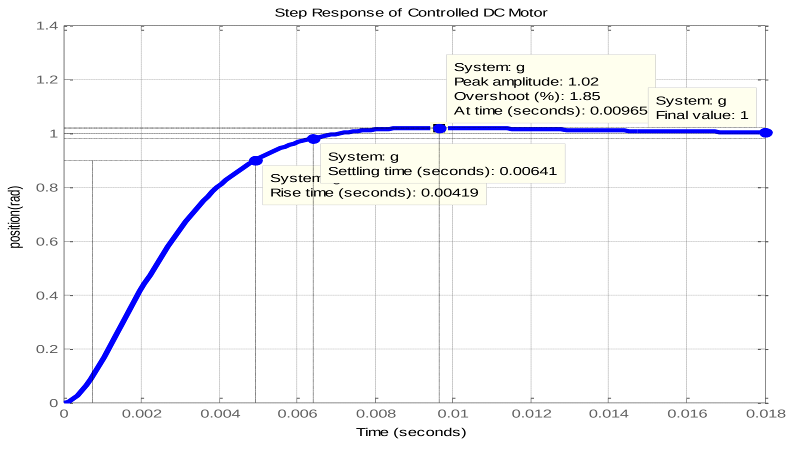


Figure 7 Step response of controlled DC motor.

It can be seen from figure 7 that the settling time $T_s$ is less than 0.007, the rise time $T_r$ is less than 0.005, the overshoot is less than 2%, and the steady state error is 0%.

## 2 ball beam system and DC Motor

Firstly, the DC motor and ball system are connected without a controller, as shown in figure 8.

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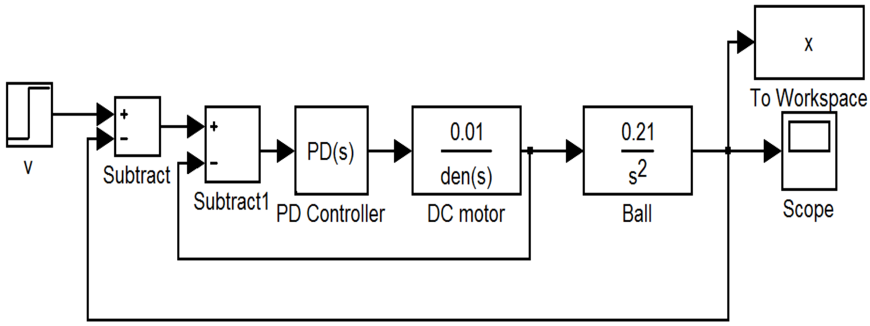


Figure 8: Overall systems

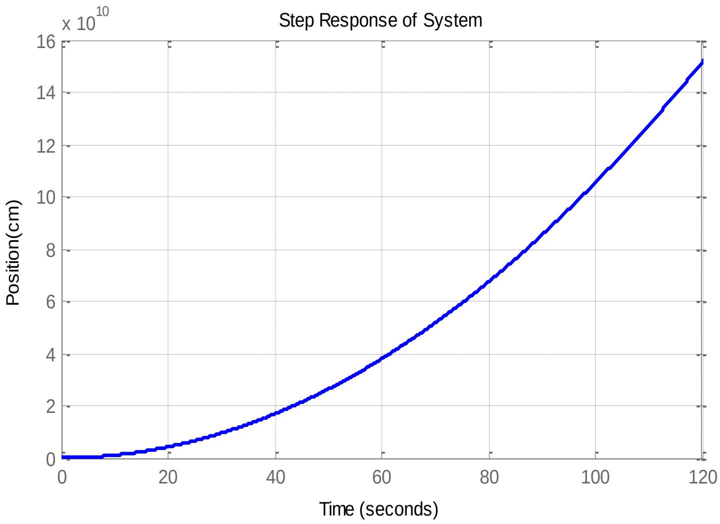


Figure 9: Step response of an uncontrolled ball and motor system

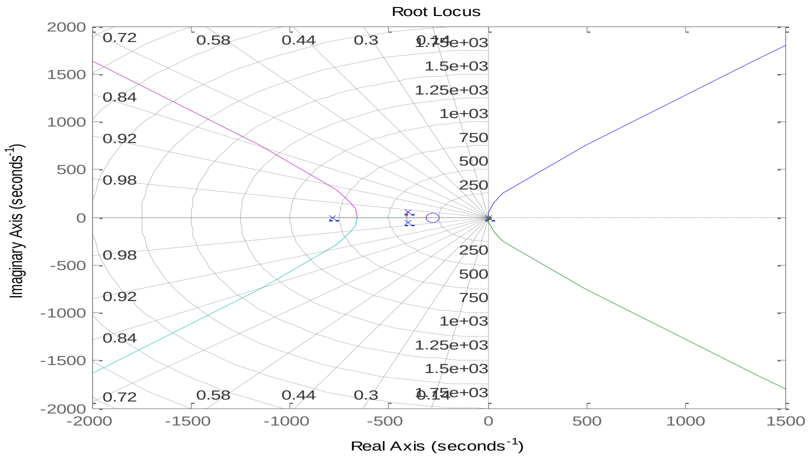


Figure 10: Root locus of the uncontrolled ball and motor system

The data presented in the above figure demonstrates that the system is exhibiting instability, with two poles located in the right half plane (RHP). This instability has led to the root locus being shifted to the left half plane (LHP) through the cancelation of the complex conjugate. Additionally, the second PD controller has been designed using the same approach as the first PD controller, outlined as follows:

Then $b_2 = 2.35 * 10^3$, $\bar{k} = 2.36 * 10^3$, and $a_2 = 590.1$

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The transfer function of the second PD controller is as follows:

$$G_{C2}(s) = \overline{k}\ \frac{(s+590.1)}{(s+2.35*10^3)} \qquad (8)$$

The transfer function of the second PD controller, denoted as $G_{C2}(s)$, is incorporated into the series connection of G(s) in the following manner:

$$G(s) = \quad 2.3 * 10^3\ \frac{(s+590.1)(s+275.3)}{s^2\ (s+787.9)} \qquad (9)$$

The overall system diagram is shown in the following figure:

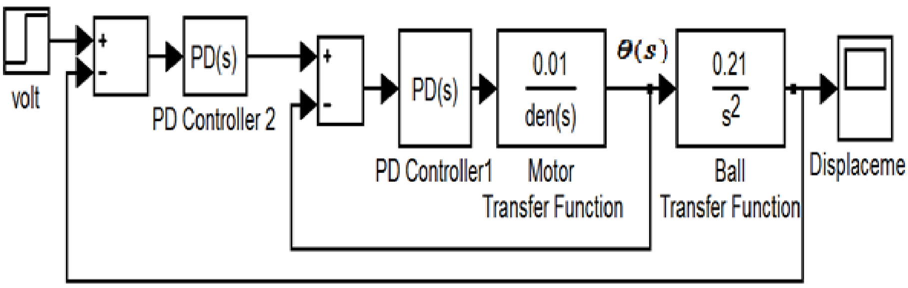


Figure 11: BBS with PD controllers

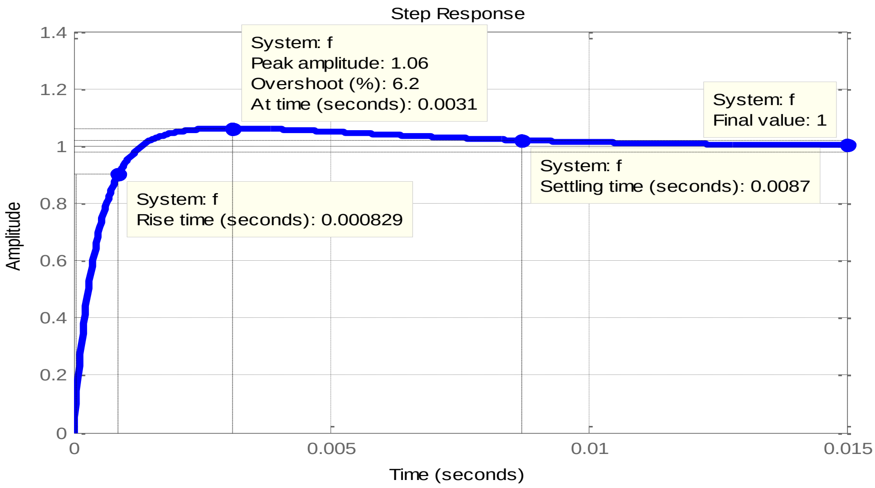


Figure 12: Step Response of the Controlled System

Table 3: Specifications for the time of transient response and steady state response

| Rise time $(T_r)$ | Settling Time( $T_s$ ) | Peak time($T_p$ ) | Overshoot $M_P$ | *steady state errors (ss)* |
|---|---|---|---|---|
| 0.000829 (*second*) | 0.0087( *second*), | 0.0031 | 6.2 % | 0% |

## VI.CONCLUSION

In conclusion, this paper introduces a modelling approach to analyse the dynamics and control of a ball and beam system. The results presented in this paper has successfully demonstrated the application and effectiveness of two PD controller schemes for stabilizing ball position tracking within the ball and beam system. By harnessing the principles of proportional and derivative control, the system's inherent instability and nonlinearity have been effectively managed, leading to accurate and reliable ball positioning along the beam. The implementation of these PD controller schemes not only validates their utility in real-world control scenarios but also contributes to advancing the field of control engineering. As a result, this study provides valuable insights into the practical application of control algorithms in

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addressing complex and dynamic systems like the ball and beam setup. The achieved success underscores the potential for further exploration and refinement of controller designs to enhance the stability and performance of similar systems across various domains.

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