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Control of Quadruple System using Modified Model Reference Adaptive Control

C.Praveen Kumar[1], Dr.K. Ayyar[2]

Assistant Professor (Senior Grade), Department of EIE, SRM Valliammai Engineering College, Chennai, India[1]

Associate Professor, Department of EIE, SRM Valliammai Engineering College, Chennai, India [2]

Abstract: The Quadruple System is a typical control system with nonlinear, coupling and time delay characteristics which can be used to test the applications of different control algorithms on complex systems. The aim of the process is to keep the liquid level in the tanks at the desired values. Here, Modified Model Reference Adaptive Control is proposed and is applied to the four tank system to test its performance. Time domain parameters like Settling time, Rise time and Mean Square Error are used for evaluating the performance of the controllers. The response of the controller is verified and is compared with other control algorithms through simulation. The performance of the closed loop system is simulated using LabVIEW software. The method used here is validated using the simulated results.

Keywords: Quadruple system, Model Reference Adaptive Control, Performance, Proportional - Integral - Derivative Control.

I. INTRODUCTION

Complex control system involves the control action with Multiple Inputs, Multiple Outputs and variables. In general, Mathematical modeling of a complex system is used to represent real world problem and in the analysis of control systems in Laplace domain. To get the desired output of the system, Controllers are used.

These controllers are tuned using specific tuning methods. Many Optimized tuning algorithms and traditional methods are generally preferred to tune the controller. In this paper, a Quadruple System (Four Tank System) is analyzed and its nonlinear model was obtained using Mass Balance Equation and Bernoulli's Principle. The behaviour of the Quadruple System (Four Tank System) varies with the range and specifications of its System components i.e. based on the no of tanks and size of the tank.

A Quadruple Tank System is a Complex System to control as all the Tanks are interconnected to each other. The level of every tank directly depends on one inlet Pump and indirectly depends on another inlet pump. A suitable controller is designed for this Complex Control System to maintain the level of lower two tanks.

II. SYSTEM MODELING

The Quadruple system is a non-linear complex system. The steps for linearizing the non-linear model are as follows. The first step is to construct the transfer function model for the given non-linear model. The next step is to design a decentralized controller for getting the stabilized output for the given input. To reduce the interaction between the four tank process, a decouple is used to get the linearized output. Hence the coupling effect of the tank process is reduced by using decouplers. The final step is to implement the model and selecting the best control input values.

The aim is to control the levels in the lower tanks with two centrifugal pumps. The system has two inputs and two outputs. There is a reservoir under the tanks to accumulate the outgoing water from tank 1 and tank 2.

The process inputs are v_1 and v_2 (input voltages to the pumps) and the outputs are y_1 and y_2 (voltages from the level measurement devices). In Fig. 1, h_i is the level of liquid in tank i where i =1,..., 4. The Quadruple tank is a laboratory process with four interconnected tanks and two pumps and two three port control valves as shown in figure.1.

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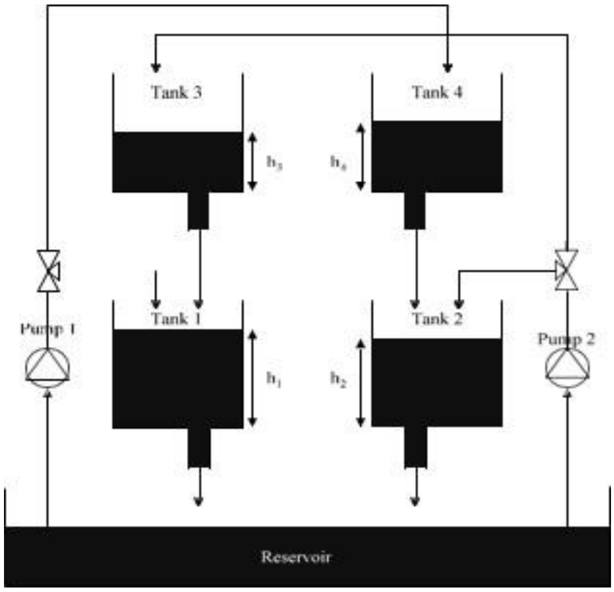

Figure.1. Block diagram of Quadruple System

Bernoulli's law is used for flows out of the tanks. The nonlinear model of the FTS is obtained by using Mass balance equation and Bernoulli's law which are shown in Eq.s (1) - (4) as

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_a k_1}{A_1} v_1 \qquad -(1)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_b k_2}{A_2} v_2 \qquad -(2)$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_b)k_2}{A_3} v_2 \qquad -(3)$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_a)k_1}{A_4} v_1 \qquad -(4)$$

Where A_i = Area of cross-section of Tank i, i =1, ..., 4
a_i = Area of cross-section of outlet hole,
h_i = Level of liquid in tanks

The voltage applied to pump i is v_i and the corresponding flowis $k_i v_i$. The parameters (γ_a, γ_b) are determined from the valve settings of the system. It can be shown that a multivariable right half plane zero will be present when ($\gamma_a + \gamma_b$) < 1 for the nonlinear system. The flow to tank 1 is $\gamma_a k_1 v_1$and the flow to tank 4 is (1- γ_a) $k_1 v_1$. Similarly, the flow to tank 2 is $\gamma_b k_2 v_2$ and the flow to tank 3 is (1- γ_b) $k_2 v_2$. The acceleration of gravity is denoted by g. The parameter values of the process are given inTable I [3].

Table I: Parameter values of the FTS model

Parameter	Value	Parameter	Value
a_1, a_2, a_3, a_4	2.3 cm^2	k_1	5.51 cm^3/s
A_1, A_2, A_3, A_4	730 cm^2	k_2	6.58 cm^3/s
$\bar{v_1}$	60%	γ_a	0.333
$\bar{v_2}$	60%	γ_b	0.307

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The model and control of the Quadruple system are studied at minimum-phase characteristics. The variables $H_i = h_i - h_i'$ and $u_i = v_i - v_i'$ are the deviation variables where h_i' and v_i' are the steady-state values of h_i and v_i respectively. The linearised model equations for the Quadruple system are

$$\frac{dH}{dt} = \begin{bmatrix} -\frac{C_1}{A_1} & 0 & \frac{C_3}{A_1} & 0 \\ 0 & -\frac{C_2}{A_2} & 0 & \frac{C_4}{A_2} \\ 0 & 0 & -\frac{C_3}{A_3} & 0 \\ 0 & 0 & 0 & -\frac{C_4}{A_4} \end{bmatrix} H + \begin{bmatrix} \frac{\gamma_a k_1}{A_1} & 0 \\ 0 & \frac{\gamma_b k_2}{A_2} \\ 0 & \frac{(1-\gamma_b)k_2}{A_3} \\ \frac{(1-\gamma_a)k_1}{A_4} & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} H \qquad - (5)$$

where, $C_i = a_i\sqrt{\frac{g}{2h_i'}}$, i = 1,….,4

the equation (5) can be written as

$$\frac{dH}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} H + \begin{bmatrix} \frac{\gamma_a k_1}{A_1} & 0 \\ 0 & \frac{\gamma_b k_2}{A_2} \\ 0 & \frac{(1-\gamma_b)k_2}{A_3} \\ \frac{(1-\gamma_a)k_1}{A_4} & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} H \qquad - (6)$$

where, $T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i}{g}}$ i = 1,….,4

The inputs are pseudorandom binary sequences (PRBS) with low amplitudes, so that the dynamics are captured by a linear model. The model outputs match with the responses of the real process. The four tanks in the FTS are of Acrylic type. It has also four numbers of smart level transmitters (DPT) to sense the level of each tank. Two numbers of control valves are mounted in the mechanical rigid frame to control the flow rate of the water. The storage tank has the capacity of 75 liters. Centrifugal pumps are provided to circulate the water from the storage tanks. Four numbers of rotameters are connected in the inlet of the process tank to visualize the flow rate which is (10-100) liters per hour (LPH). For simulating the FTS, its mathematical model [1] is necessary and has developed using Mass balance equation and Bernoulli's law which are shownin Eq.s (1) – (4). The system is designed according to the mathematical model. For developing the mathematical model for FTS, the density of liquid in the inlet, in the outlet and in the tank is assumed to be same.

III. CONTROLLER DESIGN

The use of adaptive controllers such as model reference adaptive controller (mrac) and self-tuning regulator (str) are due to the nonlinear and non-stationary nature ofthe system. in this paper, mrac strategy is employed due to the nonlinear nature of the level process. To design an MRACwith equally good transient as well as steady-state performance is a challenging task. The aim is to design an MRAC with very good steady-state and transient performance for a nonlinear process such as the hybrid tank process. In this case, it is assumed that the response of the reference model represents the set point of the process in a standard feedback loop. In MRAC, a reference model is used to adjust the regulator parameters. The reference model is a part of a control system. Adjustment of system parameters in an MRAC can be obtained in two ways:

i. Gradient Method (MIT Rule)
ii. Lyapunov Stability Theory

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A MRAC which employs the famous MIT rule for tuning of the parameters is developed and applied is shown in Figure 2. The controller has two adjustable parameters θ_1 and θ_2. The controller output u is calculated from these parameters, the command signal u_c and the output y of the process as

$$u = \theta_1 u_c - \theta_2 y \qquad - (7)$$

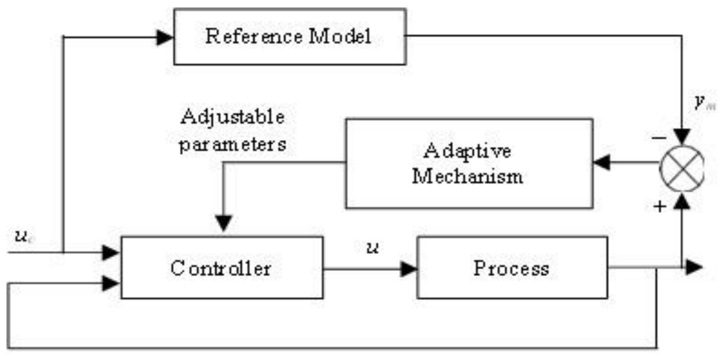

Figure.2. Block diagram of Model Reference Adaptive Control

The change in the value of the adjustable parameters of thecontroller with respect to time as per the MIT rule is given as

$$\frac{d\theta_1}{dt} = -\gamma_1 e \frac{\partial e}{\partial \theta_1} \qquad - (8)$$

And $\frac{d\theta_2}{dt} = -\gamma_2 e \frac{\partial e}{\partial \theta_2}$

Where γ_1 *and* γ_2 are the adaption gains for θ_1 and θ_2 respectively. The speed of convergence relies on the value of the adaptation gain. If the adaptation gain is too small, then it gives a stable response, but it requires a long time for the output to converge with the reference model. When the adaptation gain is too large, the output oscillates. Hence, thereis always a trade-off needed between the stability and the speed of convergence while choosing the value of adaptation gain.For the improvement of transient performance of the system, a modification can be done in the MRAC method. A PID controller is used in the MRAC method and it is called as Modified MRAC which is shown in Fig. 3.

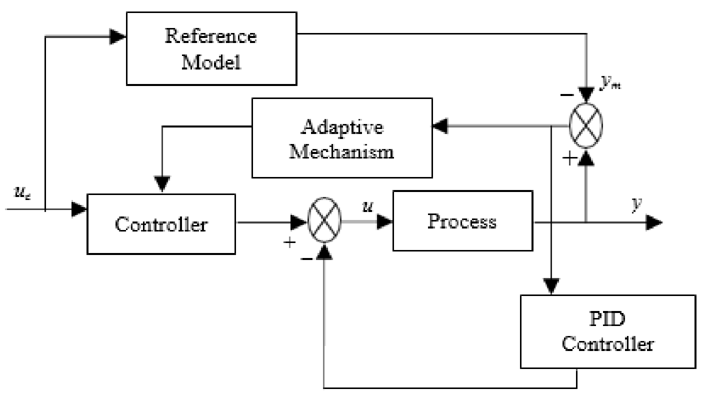

Figure.3. Block diagram of Model Reference Adaptive Control

Thus, the output ofthe controller u in Modified MRAC controller is given as

$$u = \theta_1 u_c - \theta_2 y - (K_p e + K_i \int edt + K_d \frac{de}{dt}) \qquad - (9)$$

The transfer function of the coupled tank system is given by

$$G_p(s) = \frac{0.0001626}{s^2+0.044s+0.0004834} \qquad - (10)$$

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The FTS setup in the coupled tank process is estimated as an over-damped second order system with zero delay. The damping factor is 1.0143 and the time constant of the processis 45.483 sec. The coupled tank process is more sluggish due to the interaction between the two tanks. Initially, the reference model of the system can be selected as the model of the FTS system without interaction. The transfer function of the reference models m_1 and m_2 are taken as

$$G_{m1}(s) = \frac{0.00087}{s^2 + 0.477s + 0.000461} \quad -(11)$$

$$G_{m2}(s) = \frac{0.000153}{s^2 + 1.051s + 0.00051} \quad -(12)$$

The adaptation rules for the MRAC parameters θ_1 and θ_2 are calculated as

$$\frac{d\theta_1}{dt} = -\gamma_1 e \frac{0.477s + 0.000461}{(s^2 + 0.477s + 0.000461)} u_c \quad -(13)$$

$$\frac{d\theta_2}{dt} = -\gamma_2 e \frac{1.051\,s + 0.00051}{(s^2 + 1.051\,s + 0.00051)} y \quad -(14)$$

Where γ_1 *and* γ_2 are the adaption gains for θ_1 and θ_2 respectively. The adaptation gains are either too small or too large. The adaptation gains are chosen in such a way that the system is stable and the output response tracks the desired set point value. The values of adaptation gains used here are 0.00015 and 0.0038, respectively. When the adaptation gains are large, the output responses of the system have oscillations and overshoot. Hence, small values are used for the adaptation gains to get the zero overshoot.

IV. SIMULATION RESULTS AND OBSERVATION

The performance criteria for all the four controllers using reference model m_1 are calculated and shown in table ii. It makes a comparison of the performance indices such as settling time (t_s), rise time (t_r) and mse obtained from step response analysis of all the models. Here, the aim of the process is to track the reference model in an optimal manner. The relative values of t_r and t_s have to be considered ratherthan the absolute values in order to understand the effect of all the four controllers. It is because the output of the process has to be compared with the output of the reference model rather than the input signal, u_c.

In order to obtain the relative values, the values of the reference model are subtracted from the values of the respective controller models. The comparison of performance indices with respect to that of reference model $m_1$is described in table iv. The performance

Table II. Comparison of performance indices of different controllers using reference model m_1

Type	Reference Model	PID Controller	MRAC	Modified MRAC
t_r (sec)	15.9849	22.2688	105.92	21.3996
t_s (sec)	23.7534	31.0929	175.18	27.3326
MSE	0	0.0810	2.2472	0.0032

Table III. Comparison of performance indices of different controllersusing reference model m_2

Type	Reference Model	PID Controller	MRAC	Modified MRAC
t_r (sec)	23.9804	23.9815	110.44	25.8439
t_s (sec)	34.7306	98.5468	165.58	39.1428
MSE	0	0.2759	2.1117	0.0108

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Table IV. Comparison of performance indices with respect to that of reference model m_1

Type	PID Controller	MRAC	Modified MRAC
t_r (sec)	6.2839	9.9339	5.4147
t_s (sec)	7.3395	151.43	3.5792
MSE	0.0810	2.2472	0.0032

The performance criterion for the four controllers using reference model m_2 is shown in Table III. From the Table, it is observed that the proposed FMMRAC method has given the superior performance than the other three controllers in terms of settling time, rise time and MSE. The comparison of performance indices of the four controllers with respect to thatof reference model m_2 is explained in Table V.

Table V. Comparison of performance indices with respect to that of reference model m_2

Type	PID Controller	MRAC	Modified MRAC
t_r (sec)	0.0011	86.4595	1.8635
t_s (sec)	63.8162	130.8398	4.4122
MSE	0.2759	2.1117	0.0108

V. CONCLUSION

The Quadruple System (four tank system) is analyzed and modeled using Mass balance and Bernoulli's law. The linearized model of the FTS is derived. The simulations are carried out using the linearized model of the FTS. Numerical simulation indicates that the Modified MRAC controller has more advantages than the PID controller and MRAC. The Modified Model Reference Adaptive Controller has fast response,good robustness and low settling time. Also, it has a strong ability to adapt to the changes of the system parameters and anti-disturbance performance. The Modified MRAC controller has performed very well even when the reference model order and parameters are different from the process model parameters which indicate the robustness of the design.

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