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A REVIEW PAPER ON RUNGE KUTTA 4TH ORDER METHOD

Karan Mane[1], Vishal V. Mehtre[2]

Student Of B. Tech, Department Of Electrical Engineering, Bharti Vidypeeth (Deemed To Be University) College Of Engineering, Pune, Maharashtra, India[1]

Assistant Professor, Department of Electrical Engineering, Bharati Vidyapeeth (Deemed to Be University) College Of Engineering, Pune, Maharashtra, India[2]

Abstract: This is achieved by using the SIR model to solve the system, two numerical methods are used, namely 4th order Runge-Kutta. In this paper, we study the performance and comparison of both methods in solving the model. The result in this paper that in the running process of solving it turns out that using the euler method is faster than using the 4th order Runge-Kutta method and the differences of solutions between the two methods are large.

Keywords: Fourth order Runge Kutta Method, Derivation, Stability Analysis

INTRODUCTION

In the end of 2019, there is a novel worldwide outbreak of a new type of coronavirus (2019-nCoV)(Paolucci et al., 2020). It was first reported in Wuhan, China for the occurrence of the corona virus caused by Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2) or better known as Coronavirus Disease-19 (COVID-19). The disease has spread to other countries and has reached to 196 countries (Ahmed et al., 2020). In February there was the largest positive COVID-19 case in Italy with 235.270 cases, and on February 21st, 2020 worldwide there had been 7.151.000cases and 407.145 had died from coronavirus (Paolucci et al., 2020). Up to now there are 50 million of people infected with corona virus and 1.4 million of people had died from COVID-19, this data is taken based on per November 21st, 2020 (Ahmed et al., 2020; Grassin-Delyle et al., 2021). To understand the disease, various diciplines are involved, including mathematics. In mathematics, a tool to know how the disease spreads, is called the epidemiological model. The SIR model is a simple dynamic model that describes the spread of disease between populations (Kolokolnikov & Iron, 2021). There are many researches using this approach (Xu et al., 2013; Lede and Mungkasi, 2019; Peng et al., 2020; Salim etal., 2020; Shao et al., 2020; Zhu and Zhu, 2020).

4 TH ORDER RUNGE-KUTTA METHOD

The most popular numerical approach in terms of accuracy, stability, and can be easily programmed is the Runge Kutta method (Salim et al., 2020). The general forth order differential equations (ODEs) of the form y (4) = $f(x, y, y', y'', y''')$, $0 \leq x \leq L$, can be solved by reducing it to its equivalent first order system as mentioned (Ahamad and Charan, 2019). Here is a common form of the4 th order Runge-Kutta method $k1 = hf(ti , ui)$ $k2 = hf\ (ti + h\ 2 , ui + 1\ 2\ k1)$ $k3 = hf\ (ti + h\ 2 , ui + 1\ 2\ k2)$ $k4 = hf(ti{+}1, ui , k3)$ $ui{+}1 = ui + 1\ 6\ (k1 + 2k2 + 2k3 + k4\ )$, for $i = 0,1, \ldots , n - 1$ They can be used to solve the first-order initial-value problem given below: $du\ dt = f(t, u)$, $a \leq t \leq b$, $u(a) = \alpha$. Ashgi et al. / International Journal of Global Operations Research, Vol. 2, No. 1, pp. 37-44, 2021 41 The Runge Kutta method of order 4 turns into Simpson's rule for numerical integration on $[ti ,ti{+}1\ ]$, when there is no dependency f on u. The Runge-Kutta method can be considered to act similarly to the Euler method, where it uses the weighted sum of the slope for each time and value of the function to obtain a more accurate estimate (Molthrop, 2018). Here is the 4 th order Runge-Kutta method written in different notations $k1 = F(t, yn)$ $k2 = F\ (t + h\ 2 , yn + k1\ 2\ )$ $k3 = F\ (t + h\ 2 , yn + k2\ 2\ )$ $k4 = F(t + h, yn)$ $yn{+}1 = yn + $ h 6 $(k1 + 2k2 + 2k3 + k4\ )$, $for\ n = 0,1,2, \ldots$ (6 To solve the SIR model (1), the equation can be written as $dy\ dt = F(t, y)$ (7 with $y = (\ S(t)\ I(t)\ R(t)\ )$, $F(t, y) = (\ f(t, S,I, R)\ g(t, S,I, R)\ j(t, S,I, R)\ )$

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MATLAB PROGRAM

```
%Runga Kutta Method
 function a=runge_kutta(df)
%asking initial conditions
 x0=input('Enter initial value of x:');
 y0=input('Enter initial value of y:');
x1=input('Enyer value of x at which y is to be calculated:');
 tol=input('Enter desired level of accuracy in the final result:');
 n=ceil((x1-x0)/tol);
 h=(x1-x0)/n;
for i=1:n
X(1,1)=x0;Y(1,1)=y0;
k1=h*feval(df,X(1,i),Y(1,i));
k2=h*feval(df,X(1,i)+h/2,Y(1,i)+K1*h/2); k
3=h*feval(df,x(1,i)+h/2,Y(1,i)+k1*h/2);
k4=h*feval(df,X(1,i)+h,Y(1,i)+k3*h);
k=1/6*(k1+2*k2+2*k3+k4);
 X(1,i+1)=x(1,i)+h;
 Y(1,i+1)=Y(1,i)+k;
 end
%displaying results
fprintf('for x=%g\ny=%g\n',x1,Y(1,n+1)) %
displaying graph
 x=1:n+1
; y=Y(1,n+1)*ones(1,n+1)-Y(1,:);
 plot(x,y,'r')
title('XvsY');
```

```
OUTPUT >>df
(x+y)
 Enter initial value of x:
x0
0
Enter initial value of y:
```

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y0

1

Enter desired level of accuracy in the final result :

Tol

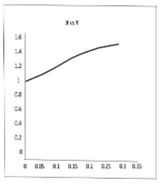
0.01

x=

0.3

y=

1.29405



4 TH ORDER RUNGE-KUTTA METHOD

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RK4 METHOD EXAMPLES

Example 1:

Consider an ordinary differential equation dy/dx = x2 + y2, y(1) = 1.2. Find y(1.05) using the fourth order Runge-Kutta method.

Solution:

Given,

dy/dx = x2 + y2, y(1) = 1.2
So, f(x, y) = x2 + y2
x0 = 1 and y0 = 1.2
Also, h = 0.05

Let us calculate the values of k1, k2, k3 and k4.
k1 = hf(x0, y0)
= (0.05) [x02 + y02]

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= (0.05) [(1)2 + (1.2)2]
= (0.05) (1 + 1.44)
= (0.05)(2.44)
= 0.122
k2 = hf[x0 + (½)h, y0 + (½)k1]
= (0.05) [f(1 + 0.025, 1.2 + 0.061)] {since h/2 = 0.05/2 = 0.025 and k1/2 = 0.122/2 = 0.061}
= (0.05) [f(1.025, 1.261)]
= (0.05) [(1.025)2 + (1.261)2]
= (0.05) (1.051 + 1.590)
= (0.05)(2.641)
= 0.1320
k3 = hf[x0 + (½)h, y0 + (½)k2]
= (0.05) [f(1 + 0.025, 1.2 + 0.066)] {since h/2 = 0.05/2 = 0.025 and k2/2 = 0.132/2 = 0.066}
= (0.05) [f(1.025, 1.266)]
= (0.05) [(1.025)2 + (1.266)2]
= (0.05) (1.051 + 1.602)
= (0.05)(2.653)
= 0.1326
k4 = hf(x0 + h, y0 + k3)
= (0.05) [f(1 + 0.05, 1.2 + 0.1326)]
= (0.05) [f(1.05, 1.3326)]
= (0.05) [(1.05)2 + (1.3326)2]
= (0.05) (1.1025 + 1.7758)
= (0.05)(2.8783)
= 0.1439

By RK4 method, we have;

y1 = y0 + (⅙) (k1 + 2k2 + 2k3 + k4)

y1 = y(1.05) = y0 + (⅙) (k1 + 2k2 + 2k3 + k4)

By substituting the values of y0, k1, k2, k3 and k4, we get;

y(1.05) = 1.2 + (⅙) [0.122 + 2(0.1320) + 2(0.1326) + 0.1439]

= 1.2 + (⅙) (0.122 + 0.264 + 0.2652 + 0.1439)

= 1.2 + (⅙) (0.7951)

= 1.2 + 0.1325

= 1.3325

RESULTS AND DISCUSSION

In this paper, we simulate our model with data case in Wuhan, China from Roda et al. (2020)research Molthrop (2018). It is assumed that the values of the parameters for the SIR model (1) are $\beta = 9.906e - 8$, $\rho = 0.24$ and $= 0.1$, with the starting value $S0 = 6e + 6$, $I0 = 245$, $R0 = 0$. The step size we choose to simulate the model is $h = 0.1$. The solution of the model using the Euler method with a value of $h = 0.1$, is as follow:

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