

Review paper on Newtons Raphson method

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Abstract: This method is used for solving algebraic equation. This method reduce the problem to solving a second degree polynomial equation . This x-intercept will typically be a enhanced approximation to the function's root than the original guess, and the method can be iterated Based on collinear scaling and local quadratic approximation, quasi-Newton methods have improved for function value is not fully used in the Hessian matrix.one of the most important thing is that these method is not applicable for equation which has complex rule . this deficiency,obtaining a third order polynomial equation which has always real root . The Advantages of using Newton's method to approximate a root rest primarily in its rate of convergence. When the method converges, it does so quadratically. Also, the method is very simple to apply and has great local convergence. And the disadvantages of using this method are numerous. First of all, it is not guaranteed that Newton's method will converge if we select an x_0 that is too far from the exact root. Likewise, if our tangent line becomes parallel or almost parallel to the x-axis, we are not guaranteed convergence with the use of this method. Also, because we have two functions to evaluate with each iteration

$f(x_k)$ and $f'(x_k)$, this method is computationally expensive. Another disadvantage is that we must have a functional representation of the derivative of our function, which is not always possible if we working only from given data.

INTRODUCTION

The newtons Raphson method is also known as newtons method this method will help us to find a good approximation for the root of a real value function $f(x)=0$ suppose we need to find the root of continuous ,differential function $f(x)$ and you know the root you are looking for is near the point $x = x_0$ $x_1 = x_0 - f'(x_0)/f(x_0)$.

In general, for any x_n -value x_n , the next value is given by $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

DERIVATION

Let approximate root of $f(x)=0$ is x_0

And let $x_1 = x_0 + h$ be the correct root Then

$$f(x_0+h)=0 \dots\dots\dots(1)$$

by expanding the above equation using Taylors theorem we get

$$f(x_0)+hf'(x_0)+\dots\dots\dots=0 \quad h = -f(x_0) / f'(x_0)\dots\dots\dots$$

Therefore, $x_1 = x_0 - f(x_0) / f'(x_0)$

Now, x_1 is the better approximation than x_0 .

Similarly, the successive approximations x_2, x_3, \dots, x_{n+1} are given by $x_{n+1} = x_n - f(x_n) / f'(x_n)$

$$f(x_n)$$

This is called Newton Raphson formula.

Program for newton raphson method

```
clear all close all clc
```

```
f = @(x) x^3 + 12*x + 7 df = @(x) 3*x^2 + 12 a=0 ; b=1; x=a; for i=1:1:4 x1=x-(f(x)/df(x)); x=x1; end
```

```
sol=x;
```

```
fprintf('Approximate root is %0.4f',sol) a=0; b=1; x=a; er(4)=0; for i=1:1:4 x1=x-(f(x)/df(x)); x=x1; er(i)=x1-sol; end
```

```
plot(er)
```

```
xlabel('Number of iterations') ylabel('Error') title('Error vs. Number of output = function_handle with value:
```

```
@(x)x^3+12*x+7
```

```
df = function_handle with value: @(x)3*x^2+12
```

```
Approximate root is -0.5681 Example-1
```

```
Find a root of an equation  $f(x)=x^3-x-1$  using Newton Raphson method
```

Solution:

Here $x^3-x-1=0$

Let $f(x)=x^3-x-1$

∴ $f'(x)=3x^2-1$

Here

x	0	1	2
f(x)	-1	-1	5

Here $f(1)=-1 < 0$ and $f(2)=5 > 0$

∴ Root lies between 1 and 2

$x_0=1+2=1.5$

1st iteration :

$f(x_0)=f(1.5)=0.875$

$f'(x_0)=f'(1.5)=5.75$

$x_1=x_0-f(x_0)/f'(x_0)$

$x_1=1.5-0.875/5.75$

$x_1=1.34783$

2nd iteration :

$f(x_1)=f(1.34783)=0.10068$

$$f'(x_1) = f'(1.34783) = 4.44991$$

$$x_2 = x_1 - f(x_1) / f'(x_1)$$

$$x_2 = 1.34783 - 0.100684 / 4.44991$$

$$x_2 = 1.3252$$

3rd iteration :

$$f(x_2) = f(1.3252) = 0.00206$$

$$f'(x_2) = f'(1.3252) = 4.26847$$

$$x_3 = x_2 - f(x_2) / f'(x_2)$$

$$x_3 = 1.3252 - 0.00206 / 4.26847$$

$$x_3 = 1.32472$$

4th iteration :

$$f(x_3) = f(1.32472) = 0$$

$$f'(x_3) = f'(1.32472) = 4.26463$$

$$x_4 = x_3 - f(x_3) / f'(x_3)$$

$$x_4 = 1.32472 - 0 / 4.26463$$

$$x_4 = 1.32472$$

Approximate root of the equation $x^3 - x - 1 = 0$ using Newton Raphson method is 1.3247

n	X0	F(x0)	F'(x0)	X1	Update
1	1.5	0.875	5.75	1.34783	X0=x1
2	1.34783	0.10068	4.44991	1.3252	X0=x1
3	1.3252	0.00206	4.26847	1.32472	X0=x1
4	1.32472	0	4.26463	1.32472	X0=x1

APPLICATION OF NEWTON-RAPHSON METHOD

In the univariate cost-head loss ratio criterion method correction to assumed HGL value at each node is obtained independently by considering HGL values at other nodes as constant. This results in simplification of the problem for solution by hand calculation at the cost of increase in number of iterations for final solution. Since high speed computers are now easily available, NewtonRaphson method is proposed for obtaining corrections to assumed HGL values.

NR method is used to solve simultaneous non-linear equations iteratively. It expands the non-linear terms in Taylor's series, neglects the residues after two terms and thereby considers only the linear terms (Bhave 1991). Thus, NR method linearizes the non-linear equations through partial differentiation and solves. Naturally, the solution is approximate, and therefore is successively corrected. The iterative procedure is continued until satisfactory accuracy is reached. Thus, while

applying NR method for obtaining correction in cost-head loss ratio criterion method, all correction equations would be considered simultaneously and solved at a time NR method is used to solve simultaneous non-linear equations iteratively. It expands the non-linear terms in Taylor's series, neglects the residues after two terms and thereby considers only the linear terms (Bhave 1991). Thus, NR method linearizes the non-linear equations through partial differentiation and solves. Naturally, the solution is approximate, and therefore is successively corrected. The iterative procedure is continued until satisfactory accuracy is reached. Thus, while applying NR method for obtaining correction in cost-head loss ratio criterion method, all correction equations would be considered simultaneously and solved at a time

Advantages

1. **Fast convergence:** It converges fast, if it converges. Which means, in most cases we get root (answer) in less number of steps.
2. It requires only one guess.
3. Formulation of this method is simple. So, it is very easy to apply.
4. It has simple formula so it is easy to program.
5. Derivation is more intuitive, which means it is easier to understand its behaviour, when it is likely to converge and when it is likely to diverge.

Disadvantages

1. It's convergence is not guaranteed. So, sometimes, for given equation and for given guess we may not get solution.
2. Division by zero problem can occur.
3. Root jumping might take place thereby not getting intended solution.
4. Inflection point issue might occur.
5. Symbolic derivative is required.
6. In case of multiple roots, this method converges slowly.
7. Near local maxima and local minima, due to oscillation, its convergence is slow.

CONCLUSION

From the referenced research papers we have concluded that the ,The convergence rate of Newton method is fast as compared to other methods .However the current injection method has simple Jacobian matrix and smaller computation in every iteration, which can make the programming easier and reduce the time of the computation Secant method is the most effective method it has a converging rate close to that of the Newton Raphson method but it requires only a single function evaluating per iteration. We have also researched that the convergence rate of bisection method is very slow and it's difficult to extend such kind of systems equations. So in comparison Newton method have a fast converging rate.

The effectiveness of using scientific calculator in solving non-linear equations using Newton Raphson method also reduces the time complexity for solving nonlinear equations. With the help of the built in derivative functions in calculator now we can able to calculate the nonlinear functions faster. This research also shows that the common mistakes made by the participants had been reduced after they were taught the technique to solve the problem using the calculator. In this work a sequence of i t e r a t i v e methods for solving nonlinear equation $f(x) = 0$ with higher-order convergence is developed. The method can be continuously applied to generate an iterative scheme with arbitrarily specified order of convergence. We also concluded that the Newton Raphson method can be used very effectively to determine the intrinsic value based on its measured permittivity.

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