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REVIEW PAPER ON COMPRISION OF RESULT IN TRAPEZIODAL AND SIMPSON'S RULE OF NUMERICAL INTERGATION

Aditya Shivam[1], Ajit Kumar[1], Vishal Mehtre[2]

Student of B. Tech, Department of electrical engineering, Bharti Vidyapeeth (Deemed to Be University), College of Engineering, Pune, Maharashtra, India[1]

Assistant Professor, Department of Electrical Engineering, Bharti Vidyapeeth (Deemed to Be University), College of Engineering, Pune, Maharashtra, India[2]

Abstract: Many different methods are applied and used in an attempt to solve numerical integration. Trapezoidal and Simpson's rule are widely used to solve numerical integration problems. Our paper mainly concentrates on identifying the method which provides more accurate result for numerical integration. with suitable example which solved by trapezoidal method and Simpson's rule and compare its result and error.

Keywords: Trapezoidal rule, Simpson's rule

INTRODUCTION

Numerical integration is the approximate computation of an integral using numerical techniques. In trapezoidal we take every interval as it is . In Simpson's we further divide it into 2 parts and then apply the formula. Hence Simpson's is more précis . The reason behind this is that Simpson's Rule makes use of the quadratic approximation instead of linear approximation. Simpson's Rule as well as Trapezoidal Rule give the approximation value, but the result of Simpson's Rule has an even more accurate approximation value of the integrals. Difference between Trapezoidal and Simpson's Rule. In trapezoidal, the boundary between the ordinates is considered straight. In Simpson's, the boundary between the ordinates is considered parabolic. . In trapezoidal, there is no limitation, it is applicable for any number of ordinates.

TRAPEZOIDAL RULE

In mathematics, the trapezoidal rule, also known as the trapezoid rule or trapezium rule is a technique for approximating the definite integral in numerical analysis. The trapezoidal rule is an integration rule used to calculate the area under a curve by dividing the curve into small trapezoids. The sum of all the areas of the small trapezoids will give the area under the curve.

Let $y = f(x)$ be continuous on $[a, b]$. We divide the interval $[a, b]$ into n equal subintervals, each of width, $h = (b - a)/n$,
such that $a = x_0 < x_1 < x_2 < \cdots < x_n = b$

Area = $(h/2) [y_0 + 2 (y_1 + y_2 + y_3 + \ldots + y_{n-1}) + y_n]$

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Area = $(h/2) [y_0 + 2 (y_1 + y_2 + y_3 + \ldots + y_{n-1}) + y_n]$

where,

- $y_0, y_1, y_2 \ldots$ are the values of function at x = 1, 2, 3….. respectiv

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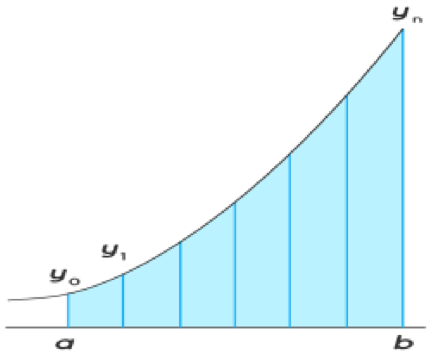

$$\text{Area} = \int_a^b y\,dx \approx \frac{1}{2} h [y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n]$$

$$\text{where } h = \frac{b-a}{n}$$

Fig no:-1 Area under the curve (in part of small rectangle)

Derivation of Trapezoidal Rule Formula

We can calculate the value of a definite integral by using trapezoids to divide the area under the curve for the given function.

Trapezoidal Rule Statement: Let f(x) be a continuous function on the interval (a, b). Now divide the intervals (a, b) into n equal sub-intervals with each of width,

$\Delta x = (b - a)/n$, such that $a = x_0 < x_1 < x_2 < x_3 < \ldots < x_n = b$

Then the Trapezoidal Rule formula for area approximating the definite integral $\int_a^b f(x)dx$ is given by:

${}^b\!\int_a f(x)\,dx \approx T_n = \triangle x/2\ [f(x_0) + 2f(x_1) + 2f(x_2) + \ldots 2f(x_{n-1}) + f(x_n)]$

where, $x_i = a + i\triangle x$

If $n \to \infty$, R.H.S of the expression approaches the definite integral ${}^b\!\int_a f(x)dx$

Proof:

To prove the trapezoidal rule, consider a curve as shown in the figure above and divide the area under that curve into trapezoids. We see that the first trapezoid has a height Δx and parallel bases of length y_0 or $f(x_0)$ and y_1 or f_1. Thus, the area of the first trapezoid in the above figure can be given as,

$(1/2)\ \Delta x\ [f(x_0) + f(x_1)]$

The areas of the remaining trapezoids are $(1/2)\Delta x\ [f(x_1) + f(x_2)]$, $(1/2)\Delta x\ [f(x_2) + f(x_3)]$, and so on.

Consequently,

$\int^b_a f(x)\,dx \approx (1/2)\Delta x\ (f(x_0)+f(x_1)) + (1/2)\Delta x\ (f(x_1)+f(x_2)) + (1/2)\Delta x\ (f(x_2)+f(x_3)) + \ldots + (1/2)\Delta x\ (f(_{n-1}) + f(x_n))$

After taking out a common factor of (1/2)Δx and combining like terms, we have,

$\int^b_a f(x)\,dx \approx (\Delta x/2)\ (f(x_0)+2\,f(x_1)+2\,f(x_2)+2\,f(x_3)+ \ldots +2f(_{n-1}) + f(x_n))$

Simpson's rule

Simpson's rule is one of the formulas used to find the approximate value of a definite integral.[1] A definite integral is an integral with lower and upper limits. Usually, to evaluate a definite integral, we first integrate (using the integration techniques) and then we use the fundamental theorem of calculus to apply the limits. But sometimes, we cannot apply any integration technique to solve an integral, and sometimes, we do not have a specific function to integrate, instead, we have some observed values (in case of experiments) of the function. In such cases, Simpson's rule helps in approximating the value of the definite integral.Simpson's rule is used to find the value of a definite integral (that is of the form ${}^b\!\int_a f(x)\,dx$) by approximating the area under the graph of the function f(x). While using the Riemann sum, we calculate the area under a curve (a definite integral) by dividing the area under the curve into rectangles whereas while using Simpson's rule, we evaluate the area under a curve is by dividing the total area into parabola Simpson's rule is also known as Simpson's 1/3 rule (which is pronounced as Simpson's one-third rule. If we have f(x) = y, which is equally spaced between [a,b], the Simpson's rule formula is:

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Simpson's 1/3 Rule Derivation

Let us derive Simpson's 1/3 rule where we are going to approximate the value of the definite integral $^b\int_a f(x)\,dx$ by dividing the area under the curve f(x) into parabolas. For this let us divide the interval [a, b] into n subintervals $[x_0, x_1]$, $[x_1, x_2]$, $[x_2, x_3]$, ..., $[x_{n-2}, x_{n-1}]$, $[x_{n-1}, x_n]$ each of width 'h', where $x_0 = a$ and $x_n = b$

The interval [a,b] is divided into subintervals of width h

Now let us approximate the area under the curve by considering every 3 successive points to lie on a parabola. Let us approximate the area under the curve lying between x_0 and x_2 by drawing a parabola through the points x_0, x_1 and x_2. Of course, all three may not come on a single parabola. But let us try to draw an approximate parabola through these three points.

Let us make this parabola symmetric about the y-axis. Then it becomes something like this:

Let us assume that the equation of the parabola be $y = ax^2 + bx + c$. Then the area between x_0 and x_2 is approximated by the definite integral:

Area between x_0 and $x_2 \approx {}_{-h}\int^h (ax^2 + bx + c)\,dx$
$= (ax^3/3 + bx^2/2 + cx)\ {}_{-h}|^h$
$= (2ah^3/3 + 0 + 2ch)$
$= h/3\ (2ah^2 + 6c)$ ... (1)

Let us have another observation from the above figure.

- $f(x_0) = a(-h)^2 + b(-h) + c = ah^2 - bh + c$
- $f(x_1) = a(0)^2 + b(0) + c = c$
- $f(x_2) = a(h)^2 + b(h) + c = ah^2 + bh + c$

Now, $f(x_0) + 4f(x_1) + f(x_2) = (ah^2 - bh + c) + 4c + (ah^2 + bh + c) = 2ah^2 + 6c$.

Substitute this in (1):

Area between x_0 and $x_2 \approx h/3\ (f(x_0) + 4f(x_1) + f(x_2))$

Similarly, we can see that:

Area between x_2 and $x_4 \approx h/3\ (f(x_2) + 4f(x_3) + f(x_4))$

Calculating the other areas in a similar way, we get

$^b\int_a f(x)\,dx$
$= h/3\ (f(x_0) + 4f(x_1) + f(x_2))$
$+ h/3\ (f(x_2) + 4f(x_3) + f(x_4))$
$+ \ldots$
$+ h/3\ (f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$
$\approx (h/3)\ [f(x_0)+4\ f(x_1)+2\ f(x_2)+ \ldots +2\ f(x_{n-2})+4\ f(x_{n-1})+f(x_n)]$

The like terms are combined here.

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left\{f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right\}$$

Hence we have derived Simpson's rule formula.

Now we take an example of numerical integration and solve each by SIMPSON rule and TRAPEZIODAL and compare its answer which give more accuracy

EXAMPLE: evaluate $\int_0^6 (2x^2 + 3)\,dx$ by taking 10 step

(1)By trapezoidal rule

Solution :

Here number of step given (n) =10

sub-interval width, h

$h = \frac{upear\ limit - lower\ limit}{number\ of\ step} = \frac{6-o}{10} = 0.6$

now .

x1 = x0 +h = 0+0.6 = 0.6 Xn = Xn-1 + h

X2 = X1 +h = 0.6+ 0.6 = 1.2

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X	0	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4	6
y	3	3.72	5.88	9.48	14.52	21	28.92	38.28	49.08	61.32	75

$\int_{xo}^{xn} ydx = \frac{h}{2}$ [(y0+ yn) + 2(y1 + y2 +y3 +y4+ y3 + y4 ….+yn-1)

Now putting the value in formula

$=\frac{0.6}{2}$[(3+75)+2(3.72+5.88+9.48+14.52+21+28.92+38.28+49.08+61.32)]

=0.3[78+464.4]

=162.72

Now by simpson 1/3 rule

$\int_{x0}^{xn} ydx = \frac{h}{3}$[(y0 + yn) + 4(y1 + y3 + y5 + y7 + y9)+2(y2 + y4 + y6 +y8)]

$=\frac{0.6}{3}$[(3+75) + 4(3.72 + 9.48 + 21 + 38.28 + 61.32) + 2(5.88 + 14.52 + 28.92 + 49.08)]

=0.2[78 + 535.2 + 196.8]

= 162

In this way we solve number of integral and find thers solution and compare its result with trapezoidal and simpson's rule which rules give more accurate solution .comapression with its each result by trapezoidal and simpson's with each exact result. In trapezoidal and simpson rule if the number of sub-interval which denoted by (h)increase then tere is possibility its result get near about to exact value of solution which shown in table no 1

NOW SEE THE RESULT OF DIFFERENT INTERGATION EXAMPALE WITH ITS COMPRESSION.

Integral	Exact Value	Trapezoidal	Simpson's	Error	
				Trapezoidal	Simpson's
$\int_0^6 (2x^2 + 3)$	161.847	162.72	162	0.873	0.153
$\int_0^6 \frac{1dx}{1+x}$	1.9459	2	1.999	0.0541	0.0004
$\int_0^6 dx/1+x^\wedge 2$	1.4056	1.083778	1.083759	0.321822	1.083759
$\int_{-3}^{3} x^4 dx$	97.2	98.0000	97.99999	0.8	0.799
$\int_0^1 \sqrt{1-x^2}\, dx$	0.78539816	0.77834373	0.78262639	0.00705443	0.00277178
$\int_0^2 \left(e^{x^2} - 1\right)dx$	14.45262777	14.9331133	14.47143621	0.48048553	0.01880844
$\int_{0.1}^{2.5} (3logx + 2x^2)dx$	10.77895602	10.72289932	10.76522243	0.05605670	0.01373358
$\int_0^2 \sqrt{1+3sin^2x}\, dx$	3.26107456	3.35966472	3.26108019	0.00140984	0.00000563

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$\int_0^1 (1+e^{-x}\cos(4x))dx$	1.00745963	1.00882686	1.00749796	0.00136722	0.00003833

Table-1: A comparison among Trapezoidal rule, Simpson's rule and for n = 12

CONCLUSION

In this article, to find the a numerical approximate value of a definite integral, $\int_{x0}^{xn} f(X)dx$Trapezoidal rule, Simpson's rule are used and it is seen that the simpson rule gives more accuracy .Here we see that the result of by simpson rule get more accurate result The main objective of our work is to determine better numerical integration result. Therefore, we apply Trapezoidal rule, Simpson's rule to solve various numerical problems and compare the result with their exact solution. We have found that Simpson's rule gives better result than any other numerical method for numerical intergration.

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