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A REVIEW OF NEWTON'S METHOD FOR NON-LINEAR TWO VARIABLE EQUATIONS

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Abstract: In this paper solution of non-linear system of equations is examined by Newton's method in one variable and in two variables. Rate of convergence is examined and conclude that Newton's method converges rapidly When a good approximation is available. At last, we found that a method with a higher rate of convergence may Reach the solution of a system in less iteration in comparison to another method with a slower convergence.

Keywords: Rate of convergence

INTRODUCTION

Numerical methods are the study of methods in which we compute the numerical data. In this we find a general Sequence of approximations with repeating the process again and again.

When we use the numerical methods for solving any problem .We want

- The rate of convergence.
- Accuracy of the answer of the question.
- The completeness of the response.

In mathematics we study two types of equations-

- Linear system of equations
- Non- linear system of equations.

Linear System of Equations:

Linear System of equation is represented by $\sum_{i=1} a_i x_i + b$ having n variables with degree 1.

Non-Linear System of Equations : Non-linear equation is also an algebraic equation which is not linear.

These are of two types:

- Polynomial Equations
- Transcendental Equations

Solutions of Non-Linear System of Equations:

We obtain the solution of non-linear system of equations by the following methods:

- Direct method
- Iterative method

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Direct Method : When we find the roots of equation in a finite number of steps then it is called direct method Like factorization, discriminate etc. Direct Method gives us an exact root of the equations.

Iterative Method : Iterative Methods based on successive approximations. In mathematics, an iterative Method is a mathematical procedure which generates a sequence of improving approximate solutions. An Iterative method is called convergent if any sequence converges for given initial approximations. An iterative Method uses an initial guess to generate successive approximations to a solution.

In Numerical Methods, we can solve the equation with different methods:

• Bisection method

•Secant method & False position method

• Newton Raphson method

In this paper we will discuss Newton Raphson method in one variable as well as in two variables.

II. HISTORICAL BACKGROUND OF NEWTON RAPHSON METHOD

According to the Articles, N.Kollerstorm [11] or T.J.Ypma [12]. The following facts seem to be agreed upon

Literature Review:-

Newton explained his method of approximation to the basic causes of numerical equations in a tract, In1600, Francois Vieta (1540–1603) had designed a perturbation technique for thesolution of the scalar polynomial Equations, which supplied one decimal place of the unknown solution per step via the explicit calculation of Successive polynomials of the successive perturbations. In modern terms, the method converged linearly. It Seems that this method had also been published in 1427 by the Persian astronomer and mathematician Al-Kashi (1380–1429). The Key to Arithmetic based on much earlier work by al-Biruni (973–1048); it is not clear to Which extent this work was known in Europe. Around 1647, Vieta‟s method was simplified by the English.Mathematician Oughtred (1574–1660).

In 1664, Isaac Newton (1643–1727) got to know Vieta‟s method. Up to 1669 he had improved it by linearizing The successively arising polynomials.Newton explained his method of approximation to the basic causes of numerical equations in a tract, De analysi Per aequations numeroterminaraun infinitas. This is known as the first announcement of the principle of Fluxions and binomial theorem.

In 1669, Newton placed it in the hands of his teacher, Isaac Barrow, then Barrow sent it to john Collins, he had a Great desire for collecting and diffusing scientific information. John Collin was a member of Royal Society.The earliest attempt of Newton‟s method of approximation became noticeable in Wallis Algebra, London, 1685 Chapter 94.Wallis discusses Newton‟s method of solving the equation.Some correspondents of Collins and friends of Newton knew about the tract but it was not printed until 1704 And 1711. Essentially, Newton gave same explanation of his method of approximation in his second tract, The Methodus fluxionum et serierum infinitarum. This planed for publication in 1671, but it was not printed until 1736.

The extension of Newton-Raphson method to irrational and transcendental equations appear to have been made For the first time by Thomas Simpson with his Essays on Several Curious and Useful Subjects in Speculates and Mixed Mathematics‟, in London in 1740. In this he does not mention Newton and Raphson, he calls his Procedure a „New Method‟. Thomas Simpson introduced derivatives in his book „Essays on Several Curious and Useful Subjects in Speculates and Mixed Mathematics‟. He described Newton‟s method as an Iterative method for solving general non-linear equations for one equation using fluxional calculus. Simpson Also gives the generalization to system of two equations in two unknowns and shows that Newton method can Also be used for solving optimization problems by setting the gradient to zero.All 18th and 19th century writers discriminate between the methods of Newton and that of Raphson. Then the Writers like Euler, Laplace, Lacroix, and Legendre who explains the Newton-Raphson process. Finally, in Different publications of writers distributed to Newton. Then popularity of Fourier‟s writings led to universal Adoption of "Newton‟s method" for the Newton-Raphson process.

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III. NEWTON METHOD

Newton's method is also called the Newton Raphson method. It is a root finding algorithm that uses the first few

Terms of the Taylor series of a function f (x) in the vicinity of a suspected root. Newton's method is sometimes

Also known as Newtons iteration.

Newton Raphson Method In One Variable

Newton Raphson method is one of the fast iterative methods in Numerical Analysis. Newton Raphson method Converges faster than false position method and secant method.

Let $f(x) = 0$ be the given equation.

Let x_k be an initial approximation to the root of the equation $f(x) = 0$.

Let Δx be an increment in x such that $x_k+\Delta x$ is an exact root where Δx is small.

therfore $f(x+\Delta x)= 0$

Expanding $f(x+\Delta x)$ by Taylor series about the point x_k ,

$f(x_k + \Delta x) = f(x_k) + \Delta x f^{|}(x_k)+ 1/2!(\Delta x)^2 f^{|}(x_k)+\ldots=0$

Now Δx is small so that neglecting square and higher powers of Δx, where $f^{|}(x_k)\neq 0$

$f(x_k)+ \Delta x f^{|}(x_k) = 0$

$\Delta x=-\{f(x_k)/f^{|}(x_k)$

Then the next approximation to the root is

$X_{k+1}=x_k+\Delta x=x_k+\{f(x_k)/f^{|}(x_k)\}$

$=x_k- \{f(x_k)/f^{|}(x_k)\},k=0,1,2,3\ldots\ldots$

This formula is known as Newton Raphson Formula, its Rate of Convergence is 2.

Newton Raphson Method in Two Variables

Newton's method is one of the most popular numerical methods, and is even referred by Burden and Faires [13]As the most powerful method which is used for solving the equation f (x) = 0. Newton"s method is that methodWhich is related to a quadratic function. This approximate function is minimized exactly.We can approximate function f at a given point xk by a Taylor series.

$f(x)= f(x_k)+ \nabla f(x_k)(x-x_k)+ 1/2(x-x_k)^{|} H^*(x_k)(x-x_k)$.

RESULT AND DISCUSSION

From the result a newton raphson method is fast as compared to other methods we can make the program easier and The effectiveness of using scientific calculator in solving non-linear equations using Newton-Raphson method also reduces the time complexity for solving nonlinear equations. With the help of the built-in derivative functions in calculator now we can able to calculate the nonlinear functions faster. The method can be continuously applied to generate an iterative scheme with arbitrarily specified order of convergence. We also concluded that the Newton Raphson method can be used very effectively to determine the intrinsic value based on its measured permittivity.

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