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Trapezoidal Method and Gauss Seidel Method in Numerical Computation

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Abstract: Gauss Seidel and Trapezoidal Methods: Derivation, Mathematical Solution, Differentiation between Trapezoidal and Gauss Seidel Methods, Application Used in Real Life, Advantages & Disadvantages are all covered in detail in this article. When using the trapezoidal approach, more iterations result from increasing the number of integration. If we use the Gauss-Seidel technique with more iterations, the resultant value increases after the decimal. Digital computers employ the Gauss-Seidel technique for computation, whereas geological formations use the trapezoidal approach.

Keywords: Trapezoidal Method, Gauss Seidel Method, Application.

INTRODUCTION

An equations are integrated, and the liner system equation is solved using the trapezoidal and gauss-Seidel methods. Systems of linear equations can be found in many different contexts, either directly (for example, in the modelling of real-world events) or indirectly (for example, in the numerical solutions of other mathematical models). Engineering, biological, physical, social science, and other domains are just a few that these applications may be found in[1]. Even the sign for integration, a stylised capital "S," indicates that summation and mathematical integration are closely related to one another [2].

Trapezoidal Rule:- One of the key integration principles is the "Trapezoidal Rule". Because little trapezoids rather than rectangles are used to divide the overall area when the area under the curve is calculated, this shape is known as a trapezoid. a technique for raising the contrast of low-contrast photographs. The procedure specifies a unique discrete integration approach using a trapezoidal rule-based contrast enhancement algorithm. [3] The procedure specifies an unique discrete integration approach using a trapezoidal rule-based contrast enhancement algorithm. [4] The suggested method's efficiency is demonstrated by the findings, which include linear phase response throughout almost 0% - 80% of the whole Nyquist frequency range and a significantly reduced percentage absolute magnitude relative error (PARE). For more reading on applications of numerical integration[5–6],

Gauss seidel Method:- Both Philipp L. Seidel and Carl Friedrich Gauss (1777–1855) created the Gauss-Seidel (GS) method (1821– 1896) As soon as a new value is determined, the variables' values are changed.

The Gauss-Seidel method is a point-wise iteration technique that, with one notable exception, is very similar to the Jacobi technique. [7] Each iteration of the Gauss-Seidel method, in essence, yields a fresh approximation of the solution[8]. A linear system with the variables x1, x2,..., and xn can be solved using the Gauss-Seidel method, which involves making an initial guess at the solution and repeatedly substituting new values for x1, x2,..., and xn to arrive at the original values. The approaches are effective if the values for x1, x2,..., xn eventually stabilize, leading to the actual solution. [9] New component values are used by Gauss-Seidel as soon as they are computed. Consequently, it is typically more accurate. SOR (successive overrelaxation) determines the weighted average of the method[10]

DERIVATIONS

Trapezoidal Rule: The Newton-Cotes formula is the foundation of the trapezoid rule, which states that if the integrand is approached by an nth order polynomial, the integral of the function will be approximated by the integral of that nth order polynomial. It is easy to integrate polynomials and is based on the calculus formula.

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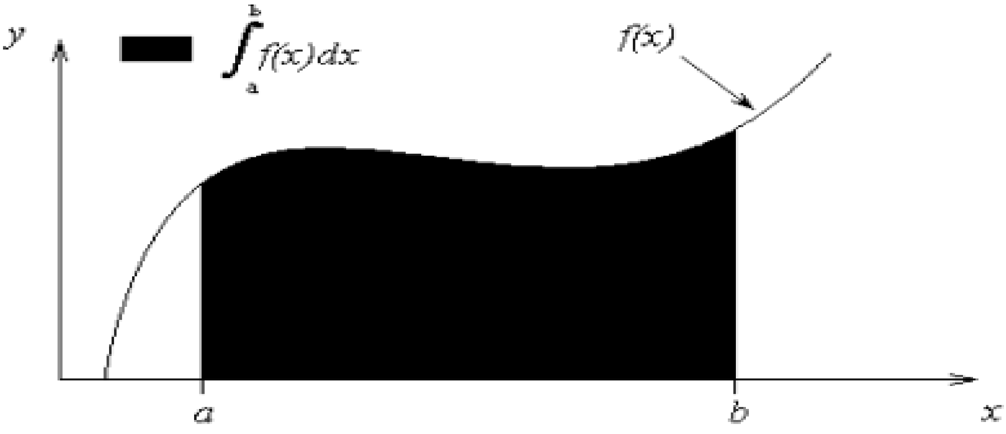

Figure 1 Integration of a function y = f(x)

$_a\int^b f(x)dx \approx {}_a\int^b f_1(x)dx$

$= {}_a\int^b (a_0+a_1x)dx$

$=a_0(b-a)+a_1[(b^2-a^{2)}/2]$ ....(1)

But what is a_0 and a_1 ?

Now if one chooses, (a, f (a)) and (b, f (b)) as the two points to approximate f(x) by a straight line from a to b

$f(a)=f_1(a)=a_0+a_1a$ ....(2)

$f(b)=f_1(b)=a_0+a_1b$ ....(3)

Solving the above two equations for a_1 and a_0

$$a_1 = \frac{f(b)-f(a)}{b-a}$$

$$a_0 = \frac{f(a)b-f(b)a}{b-a}$$

Hence from Equation (1)

$_a\int^b f(x)dx \approx$ {[f(a)b-f(b)a]/b-a}*(b-a)+{[f(b)-f(a)](b-a)}*$[(b^2-a^a)/2]$

=(b-a)[(f(b)+f(a))/2]

Formula

$_{x0}\int^{xn} y.dx = h/2\ [(y_0+y_n)+2(y_1+y_2+y_3+\ldots.y_{(n-1)}]$

Here h=Height Between x_0 and x_1

No. of steps (n)=($[x_n-x_{0)}/h]$

N= (0,1,2,3,4,5….)

Gauss Seidel Method :-

The linear equations of the above system are solved mathematically using the Gauss-Seidel technique. Its name is a tribute to two renowned German mathematicians, Carl Friedrich Gauss and Philipp Ludwig von Seidel. The iterative procedure known as the Gauss-Seidel method is what allows us to solve several linear equations[11]. The Jacobi technique is

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comparable to the Gauss-Seidel method. The sequential displacement technique and the Liebmann method are other names for the Gauss-Seidel[12] approach. The practise of solving equations by applying successive linear values is known as an iterative approach (like 1, 2, 3, 4. . .).

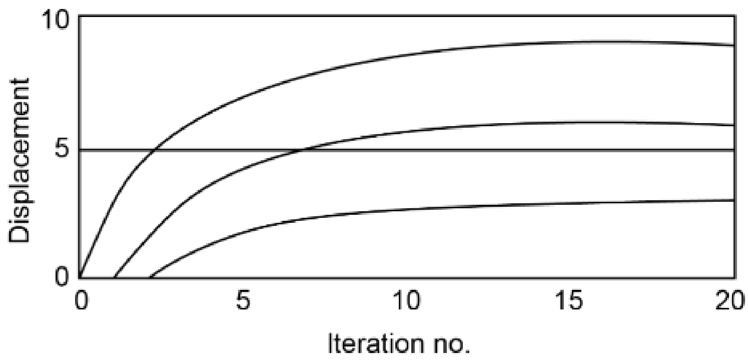

Figure: 2 Displacement Vs. Iteration number

Gauss-Seidel method is the iteration-based method that solves a number of linear equations having unknown variables.

Basic form- Ax = bx=b

Here, *x* represents the unknown variable of the equation.

This general equation is defined by the general iteration form

$Lx^{(k+1)}=b-Ux^k$ ………..(1)

Here, *k* represents the approximation (i.e. 1,2,3,4,5,...) and *k*+1 shows the proceeding value.

It shows the iteration of the unknown variable *x*.

If equation (1) is rearranged in the form of *x*, i.e., unknown variable, then it will be

$X^{(k+1)} = L^{-1}b-Ux^k$

In general form, without using the iteration, it will be

$x=L^{-1}b-Ux$

Here, all the proceeding iterations are removed (i.e., *k* or *k*+1).

The important part is that *A* must be diagonally dominant because the Gauss-Seidel method is dependent on the matrix *A* or the values of matrix *A*.

Their findings shown that GS surpasses Jacobi approach in terms of accuracy and the amount of iterations required to reach convergence; GS is also quicker. The GS and Jacobi techniques' convergence requirements [13–14].

Formula :-

- $A_{11}X_1 + B_{12}X_2 + C_{13}X_3 + D_{14}X_4 = B_1$
- $A_{21}X_1 + B_{22}X_2 + C_{23}X_3 + D_{24}X_4 = B_2$
- $A_{31}X_1 + B_{32}X_2 + C_{33}X_3 + D_{34}X_4 = B_3$
- $A_{41}X_1 + B_{42}X_2 + C_{43}X_3 + D_{44}X_4 = B_4$

$X_1^{(K+1)} = 1/A_{11}[B_1 - A_{12}*X_2^{(k)} – A_{13}*X_3^{(k)} – A_{14}*X_4^{(k)}]$

$X_2^{(K+1)} = 1/A_{22}[B_2 – A_{21}*X_1^{(k)} – A_{23}*X_3^{(k)} – A_{24}*X_4^{(k)}]$

$X_3^{(K+1)} = 1/A_{33}[B_3 – A_{31}*X_1^{(k)} – A_{32}*X_2^{(k)} – A_{34}*X_4^{(k)}]$

$X_4^{(K+1)} = 1/A_{44}[B_4 – A_{41}*X_1^{(k)} – A_{42}*X_3^{(k)} – A_{43}*X_3^{(k)}]$

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MATLAB Program :-

```
 %Trapezoidal rule
function I = trapezoidal_f1(f)
f = @(x) (x/sqrt(4+x^2))
% for calculating integrals using trapezoidal rule when function is known
%asking for the range and desired accuracy
R= input('enter the limits of integrations [ x_min,x_max]:/n')
tol = input(' error allowed in the final answer should be of an order: /n');
a=R(1,1);b=R(1,2);
%initial h and n
n=100;
h=(b-a)/100;
%for calculating maximum of h^2 *f''(x) in the given region
for k = 0:100
x(1,k+1)=a+k*h;
   y2(1,k+1)=feval(f,x(1,k+1)+2*h)-2*feval(f,x(1,k+1)+h)+feval(f,x(k+1));
end
[y i]=max(y2);
 x_opt=x(1,i);
%for calculating the desired value of h
m=0;
while abs((feval(f, x_opt+2*h)-2*feval(f, x_opt+h)+feval(f,x_opt))*(b-a)/12)>tol%global error for trapezoidal rule
   m=m+1;
   h=h*10^-m;
end

%calculating n
n=ceil((b-a)/h);
h=(b-a)/n;
%forming matrix X
for k=1:(n+1)
   X(k,1)=a+(k-1)*h;
   X(k,2)=feval(f,X(k,1));
end
%trapezoidal formula
I=h/2*(2*sum(X(:,2))-X(1,2)-X(n,2));
%displaying final result
disp(sprintf(' the area under this curve in the interval(10f,10fis 10,6f.',a,b,I))
```

Output :

```
>> trapezoidal
enter the limits of integrations [ x_min,x_max]:/
```


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```
[0,3]
R=
0    2
Error allowed in final should be of an order: /n
0.001
the area under this curve in the interval (10f, 10fis 10,6f.
area
 1.0234155
```

Gauss Seidel Method :-

```
%Gauss-Seidel Method in MATLAB
function x = gauss_seidel (A,B)
disp('Enter the system of linear equations in the form of AX=B')
%Inputting matrix A
A=input ('Enter the matrix A: \n')
%check if the entered matrix is a square matrix
[na,ma]=size (A);
if na~=ma
   disp('Error: matrix B must be a column matrix')
   return
end
%Inputting matrix B
B=input ('Enter the matrix B:')
% check if B is a column matrix
[nb,mb]=size (B);
if nb ~=na || mb ~=1
   disp('ERROR:Matrix B must be a column matrix')
   return
end
%separation matrix A into lower triangular and upper triangular matrices
%A=D+L+U
D=diag(diag(A));
L=tril(A)-D;
U=triu(A)-D
%check for convergence condition for gauss-seidel method
e=max(eig(-inv(D+L)*(U)));
if abs (e) >=1
   disp('since the modulus of largest eigen value of iterative matrix is not than assumed number')
   disp('this process is not convergent.')
   return
end
% initial guess for X..?
```

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```
% default guess is [1 1 .... 1]
r=input ('any initial guess for X? (y/n):','s');
switch r
   case 'y'
      %asking for initial guess
      X0=input('enter initial guess for X :\n')
      %check for initial guess
      [nx,mx]=size(X0);
      if nx~=na||mx~=1
         disp('ERROR:Check input')
         return
      end
   otherwise
      X0=ones(na,1);
end
%allowable error in final answer
t=input('enter the error allowedin final answer:');
tol=t*ones(na,1);
k=1;
X(:,1) = X0;
err=1000000000*rand(na,1);%initial error assumption for looping while sum (abs(err)>=tol)~=zeros(na,1)
X(:,k+ 1 )=-inv (D+L)*X(:,k)+ inv(D+L)*B; %gauss -seidel formula
err=X( :,k+1)-X(:,k); % finding error
k=k+1;
fprintf('the final answer obtained after %g iterations is\n',k)
X( :,k)
```

Output :

```
>> Enter the system of linear equation in the form of AX=B
Enter the matrix A:
A  =
      10   -2    -1
       2    10    -5
      3    -4     10
Enter the matrix B:
[23;33;41]
B  =
      23
      33
      41
U  =
0     -2     -1
```

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2 0 -5

3 -4 0

Any initial guess for x? (y/n);

1

Enter the error allowedin finial answer

0.5

The finial answer obtained after 2 iteration is

Ans =

1.331

-2.337

3.293

0.8447

-1.9197

2.7659

Mathematical solution of Trapezoidal rule and Gauss Seidel Method

Examples :

Trapezoidal Method -

Q. Evaluate using trapezoidal rule with four steps to estimate the value of integeral

$_0\int^2$[$x/\sqrt{(2+x^2)}$].dx

Solution. Here n is given i.e n=4

So we have to find h we know that h= [$(x_n$-$x_0)$/n]

Here x_0= 0 , x_1 = 2 [(2-0)/4] =0.5

X	0	0.5	1.0	1.5	2.0
Y=f(x)	0	0.3333	0.57735	0.727606	0.81649
	Y0	Y1	Y2	Y3	Y4

According to formula

$_{x0}\int^{xn}$y.dx =h/2 [(y_0+y_4)+2(y_1+y_2+y_3]

= (0.5/2)*[(0+0.81649) +2(0.3333+0.57735+0.727606)]

=0.5*2.046831

=1.0234155

Gauss Seidel Method –

Q. Solve the following set of linear equation using Gauss Seidel Method. Upto 4 iteration.

$10x_1 - 2x_2 – x_3 = 23$

$2x_1 + 10x_2 – 5x_3 = 33$

$3x_1 - 4x_2 + 10x_3 = 41$

According to formula.

$X_1^{(K+1)} = 1/A_{11}[B_1 - A_{12}*X_2^{(k)} – A_{13}*X_3^{(k)} – A_{14}*X_4^{(k)}]$

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$X_2^{(K+1)} = 1/A_{22}[B_2 – A_{21}*X_1^{(k)} – A_{23}*X_3^{(k)} – A_{24}*X_4^{(k)}]$

$X_3^{(K+1)} = 1/A_{33}[B_3 – A_{31}*X_1^{(k)} – A_{32}*X_2^{(k)} – A_{34}*X_4^{(k)}]$

$X_4^{(K+1)} = 1/A_{44}[B_4 – A_{41}*X_1^{(k)} – A_{42}*X_3^{(k)} – A_{43}*X_3^{(k)}]$

1st iteration

Assume k = 0 $X_1^0 = X_2^0 = X_3^0 = 0$

X_1^1 =[1/10]*[23 +0+0]

=2.3

X_2^1 = 1/10[-33+0+0]

=-3.3

X_3^1 = 1/10[41+0+0]

= 4.1

2nd iteration k=1

X_1^2 = 1/10[23+2(-3.3) -3(4.1)]

=1/10[4.1]

=0.41

X_2^2 = [-33-2(2.3)+5(4.1)]

= 1/10[-17.1]

=-1.71

X_3^2 = 1/10[41-3(2.3)+4(3.3)]

= 1/10[20.9]

=2.09

3rd iteration k=2

X_1^3 = 1/10[23+2(-1.71)-3(2.09)]

= 1/10[13.31]

= 1.331

X_2^2 = [-33-2(0.41)+5(2.09)]

= 1/10[-23.37]

= - 2.337

X_3^3 = 1/10[41-3(0.41)+4(-1.71)]

= 1/10[32.93]

= 3.293

4th iteration k=3

X_1^4 = 1/10[23+2(-2.337)-3(3.293)]

= 1/10[08.447]

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= 0.8447

X_2^2 = [-33-2(1.331)+5(3.293)]

=1/10[-19.197]

=-1.9197

X_3^4 = 1/10[41-3(1.331)+4(-2.337)]

= 1/10[27.659]

= 2.7659

Differentiate Between Trapezoidal rule and Gauss Seidel Method

Trapezoidal rule	Gauss Seidel Method
• The definite integral is approximated using the trapezoidal rule utilising trapezoidal approximations. • The region beneath the graph of the function f(x) is approximated as a trapezoid in the trapezoidal rule, and its area is calculated. . • The difference between the integral's value and the numerical result is the composite trapezoidal rule error. :	• Gauss-Seidel technique As soon as a new value is determined, the variables' values are changed. • The Gauss-Seidel method converges to the actual solution with a certain level of accuracy with fewer iterations. • An iterative method known as the Gauss-Seidel method can be used to solve a square system of n linear equations with an unknown x.

Applications –

Trapezoidal rule:

This approach is used in numerical to determine an approximate value for a defined integral.

It is employed to calculate the size of various geological structures..

Gauss seidel method

It is applied to the solution of unknown variable linear equations.

It is utilised for computing in digital computers.

Advantages -

Trapezoidal rule:

More precisely, use the trapezoidal rule.

It is easy to understand and works well for many integration tasks.

Gauss seidel method :

This approach's computation is straightforward.

It's simple to programme this method.

Less memory space is required.

For the small system of linear equations, this technique is helpful.

Disadvantages

It is less accurate than Simpson's 1/3rd approach since it employs linear approximations rather than quadratic estimates.

The integral is either overestimated or underestimated depending on whether the function is concave up or down.

Gauss seidel method:

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To obtain convergence, several iterations are necessary.

It is ineffective for a big system.

The system's size has an impact on how quickly it converges.

RESULT & DISCUSSION

Trapezoidal rule is simpler and more accurate where as Gauss seidel method in also simple and memory requirement is less but iteration for gauss seidel is longer as compare to trapezoidal. Trapezoidal rule is easier and can solved easily and there is not long calculation, but gauss seidel method is longer method it takes time to solving the problem and in this method is easy but time taken long calculation. If we increase the steps size then number of iterations increases and resulted value also increases after decimal. if we decrease step size then number of iterations decreases and resulted value get decreased after decimal. If we increase integration value then number of iteration increases. If we increase (k) factor then also number of iteration increases.

CONCLUSION

From the above explanation and derivation, we came to know that trapezoidal and Gauss Seidel Method are used to solve linear system equation. According to me both methods are used to solve there questions and both the method are good. Hence trapezoidal used to estimate the volume of different geological structures and gauss seidel method used in digital computers for computing.

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