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Stability Robustness Bounds for State-Space Models with Dependent Uncertainty: Application to a Microgrid System

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Abstract: In this paper stability robustness bounds for linear state space models with dependent uncertainty based on the Lyapunov stability method and time-invariant perturbation method are applied to a microgrid system. In contrast to previous observations, it is shown that the Lyapunov stability bound gives better result than the time-invariant perturbation bound for a dependent uncertainty for the open-loop microgrid system. It is also shown that the Lyapunov stability bound gives larger value for the closed-loop microgrid system with the linear quadratic regulator control.

Keywords: Robust Control, Microgrid, Lyapunov Method, Stability, State-Space Model, Dependent Uncertainty.

I. INTRODUCTION

Modeling, analysis and design of controllers for a microgrid system have been active areas of research [1, 2]. A microgrid is a low voltage electric distribution grid consisting of renewable energy sources (RES), and energy storage system, which has challenging control requirements [3, 4]. Existing robust control methods have been applied to the microgrid system in recent years to address some of these challenges [5]. In this paper, stability robustness bounds on parameter variations in state-space models [6] are applied for analysis and design of controllers for a microgrid system model. Two different types of robustness bounds are considered: the Lyapunov stability method [7-9] and the stability radius-based time-invariant perturbation method [10-14].

The bounds based on the time-invariant perturbation method were shown to give larger values than the Lyapunov stability method with examples in the past [6]. While these are valid for unstructured and structured perturbations [6], it is shown in this paper that this is not the case for some dependent perturbations for the microgrid system model [1]. In particular, the Lyapunov stability method-based dependent perturbation bound [9] gives better result than the time-invariant perturbation method-based bound [14] for a dependent perturbation for the open-loop microgrid system. It is also shown that the Lyapunov stability bound gives larger bound value for the closed-loop microgrid system with the Linear Quadratic Regulator (LQR) control [4, 6, 15]. The Lyapunov stability bound also provides an insight that a parameter of this system can vary very large value and still maintain the stability of the system.

II. STABILITY ROBUSTNESS BOUNDS FOR DEPENDENT UNCERTAINITY

Consider the following linear continuous-time system with linear perturbation [6]:

$$\dot{x} = (A + E)x \qquad (1)$$

where A is an nxn asymptotically stable matrix, x is the state vector, and E is a perturbation matrix. The matrix E is in the following dependent perturbation form:

$$E = \sum_{i=1}^{m} k_i E_i \qquad (2)$$

where E_i are constant matrices and k_i are m uncertain parameters.

A. The Lyapunov Stability Bound

It is shown in [9] that the system (1) is stable if

$$|k_l| < \frac{1}{\sigma_{max}(\sum_{i=1}^{m} |P_i|)}, \quad l = 1,2,\ldots,m \qquad (3)$$

where

$$P_i = \frac{(E_i^T P + P E_i)}{2}, \qquad (4)$$

P is the solution of the Lyapunov equation

$$PA + A^T P + 2I = 0, \qquad (5)$$

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$\sigma_{max}(.)$ is the largest singular value of (.) which is the spectral norm ‖(.)‖, the notation |(.)| represents the matrix with absolute value elements of (.), $(.)^T$ is the transpose of the matrix, and I is the identity matrix. This is an improved bound derived based on the unstructured perturbation bound (6) [7]

$$\sigma_{max}(E) < \frac{1}{\sigma_{max}(P)}, \quad (6)$$

and the structured perturbation bound [8] by considering the dependent perturbation model (2).

B. Time-Invariant Perturbation Bound

An improved bound is obtained by Yedavalli [14] for time-invariant perturbations E. It is shown that (1) is stable if

$$|k_l| < \frac{1}{\sup\limits_{\omega\geq 0}\rho(\sum_{i=1}^{m}|(j\omega I-A)^{-1}E_i|)}, \quad l = 1,2,\dots,m, \quad \text{for } m > 1 \quad (7)$$

$$|k_1| < \frac{1}{\sup\limits_{\omega\geq 0}\rho[(j\omega I-A)^{-1}E_1]} \quad \text{for } m = 1 \quad (8)$$

where $\rho(.)$ denotes the spectral radius of (.). This improved bound is derived based on the unstructured perturbation bound (9)

$$\|E\| < \frac{1}{\sup\limits_{\omega\geq 0}\|(j\omega I-A)^{-1}\|}, \quad (9)$$

and the structured perturbation bound [10-13] from the stability radius concepts by considering the dependent perturbation model (2). Through examples, it was shown in the past that the bounds (7), (8), gives better result than (3) [6]. However, for the microgrid system model considered, it is shown in the next sections that (3) gives better result than (8) for a dependent parameter variation.

III. MICOGRID SYSTEM MODEL

In a microgrid, RES are integrated by using voltage source converters (VSC) [3]. Fig.1 shows VSC connected to grid [4], where U_{dc} represents a RES.

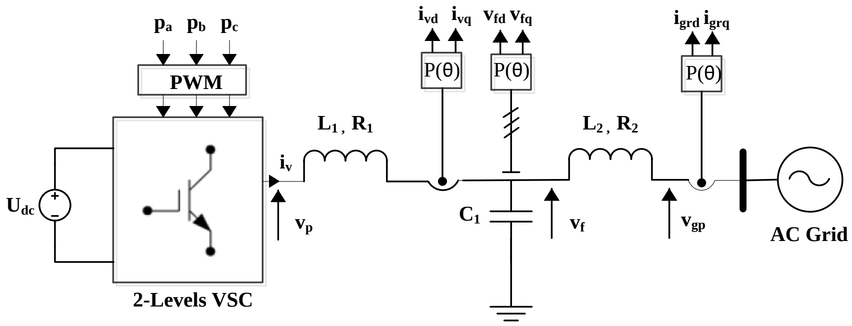

Fig. 1 VSC connected to grid [4]

For a three-phase VSC system in microgrid, the state-space model in the d-q frame is represented in the form of (10) [1, 3, 4]

$$\dot{x} = Ax + Bu + Fw \quad (10)$$

where

$$A = \begin{bmatrix} -\frac{R_1}{L_1} & \bar{\omega} & -\frac{1}{L_1} & 0 & 0 & 0 \\ -\bar{\omega} & -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} & 0 & 0 \\ \frac{1}{C_1} & 0 & 0 & \bar{\omega} & -\frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_1} & -\bar{\omega} & 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} & \bar{\omega} \\ 0 & 0 & 0 & \frac{1}{L_2} & -\bar{\omega} & -\frac{R_2}{L_2} \end{bmatrix}, \quad (11)$$

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$$B^T = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{L_1} & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (12)$$

A is the state matrix, B is the input matrix, F is the disturbance input matrix, u is the input vector, and w is the disturbance vector. L_1 and L_2 are filter inductances, R_1 and R_2 are filter resistances, and C_1 is filter capacitance. ϖ is the angular frequency. Table 1 gives various parameters for a sample system from [1].

Table I Parameters of microgrid system model

Parameter	Value
R_1	0.1 Ω
L_1	1.35 mH
C_1	50 μF
R_2	0.03 Ω
L_2	0.35 mH
ϖ	314 rad/s

For this VSC model in a microgrid, stability robustness bounds given in Section II are applied and important observations are made in the next section.

IV. STABILITY ROBUSTNESS BOUNDS FOR MICROGRID SYSTEM

The eigenvalues of the open-loop system A matrix in (11), with the parameter values in Table 1, are found to be

$$-76.5 \pm j314, \qquad -41.7 \pm j8168.7, \qquad -41.7 \pm j8796.7$$

using MATLAB, which have negative real parts, and it is asymptotically stable.
It can be noted that the A matrix in (11) has a unique dependent perturbation model when the parameter ϖ changes resulting in the E matrix in (1) with an uncertain parameter k_1 and E_1 in (2) given by (13)

$$E_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}. \qquad (13)$$

For this E_1 matrix, the Lyapunov stability bound (3) on k_1 is found to be 8.1180×10^{14} using MATLAB which is a very large number. To assure the validity of this large bound, the eigenvalues of the A+E for several k_1 values are checked. For example, the eigenvalues of A+E with $k_1 = 8.1180 \times 10^{14}$ are found to be

$-76.391 \pm j8.118 \times 10^{14}$, $-41.688 \pm j8.118 \times 10^{14}$, $-41.875 \pm j8.118 \times 10^{14}$

using MATLAB, which show that A+E is stable.

To calculate the time-invariant perturbation bound (8), Fig. 2 is obtained. The peak values of $\rho[(j\omega I - A)^{-1}E_1]$ occur at ω = 314, 8168.7, and 8796.7, which correspond to the imaginary parts of the eigenvalues of the open-loop system A matrix in (11) [13]. The bound on k_1 is found to be 41.6589 which is much smaller than the Lyapunov stability bound (3). To assure the validity of this bound, the eigenvalues of the A+E for several k_1 values are checked.

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For example, the eigenvalues of A+E with $k_1 = 41.6589$ are found to be

$$-76.5 \pm j355.7, \quad -41.7 \pm j8127.0, \quad -41.7 \pm j8838.3$$

using MATLAB, which shows that A+E is stable.

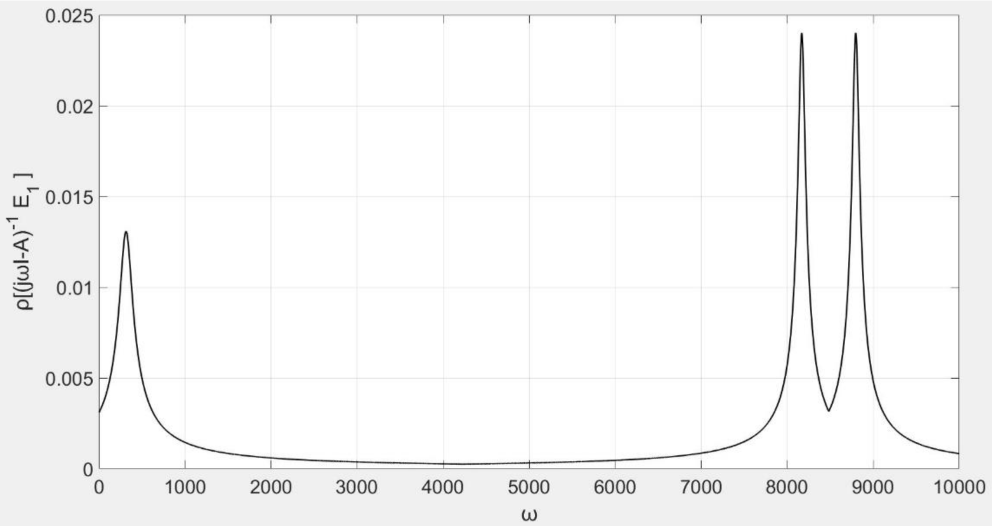

Fig. 2 ω vs $\rho[(j\omega I - A)^{-1}E_1]$ for time-invariant perturbation bound

Remarks: While the Lyapunov stability bound for dependent perturbations for the microgrid system is larger than the time-invariant perturbation bound, for the unstructured perturbations [6] time-invariant perturbation bound (9) is larger than the Lyapunov stability bound (6). For unstructured perturbations, the Lyapunov stability bound (6) is found to be 6.4709 for the microgrid system, and the time-invariant perturbation bound (9) is found to be 22.6050. This fact of time-invariant perturbation bound being larger than the Lyapunov stability bound for unstructured perturbations case was proved in Theorem 2 of [10]. For the microgrid system, the parameter ϖ can vary very large values (8.1180×10^{14}) and still keep the system stable, which is identified by the Lyapunov stability-based dependent perturbation bound (3).

C. Closed-loop System with the LQR Control

For the microgrid system model in (10), a controller is designed using the LQR method that minimizes the performance index J in (14) [4, 6]

$$J = \int_0^{\infty} (x^T Q x + u^T R u)\, dt . \qquad (14)$$

With unit state weighting matrix Q=I and control weighting matrix R=I of appropriate dimensions, for the parameter values given in Table 1, the control gain matrix G in (15) for the state feedback controller u = - G x,

$$G = R^{-1} B^T Z , \qquad (15)$$

after solving the algebraic Riccati equation (16) for Z

$$A^T Z + ZA + Q - ZBR^{-1}B^T Z = 0 , \qquad (16)$$

is found to be [4, 6]

$$G = \begin{bmatrix} 3.358 & -3.93\text{x}10^{-14} & 0.203 & 1.47\text{x}10^{-15} & -2.074 & 4.48\text{x}10^{-14} \\ -4.99\text{x}10^{-14} & 3.358 & -4.89\text{x}10^{-15} & 0.203 & 2.93\text{x}10^{-14} & -2.074 \end{bmatrix}.$$

The eigenvalues of the closed-loop system matrix $A_c = A - BG$ are

$$-815.98 \pm j314, \quad -915.66 \pm j8221.1, \quad -915.66 \pm j8849.1$$

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and A_c is asymptotically stable. For the dependent perturbation with E_1 matrix in (13), the Lyapunov stability bound (3) on k_1 is found to be 3.4324 $x10^{15}$ for the closed-loop matrix A_c using MATLAB which is a very large value. To calculate the time-invariant perturbation bound (8) for the closed-loop system matrix A_c, a figure similar to Fig. 2 is obtained. The bound (8) is found to be 815.98, which is much smaller than the Lyapunov stability bound (3). Similar Remarks as the open-loop system are found for the closed-loop system in case of unstructured perturbations [6] for the Lyapunov stability and time-invariant perturbation bounds.

V. CONCLUSION

Stability robustness bounds for dependent uncertainty based on the Lyapunov stability method and time-invariant perturbation method are applied to a VSC-based microgrid system model. For this system, the Lyapunov stability method gives larger bound than the time-invariant perturbation method for the considered dependent uncertainty. The parameter $\overline{\omega}$ for this system can vary large values and still keep the system stable, which is identified by the Lyapunov stability-based dependent perturbation bound (3). The P_i matrix in (4) for the E_1 matrix in (13) provides this unique feature. Further research may lead to other important observations for this microgrid system model.

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