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An Analysis of Numerical Solutions and Errors with Euler's Method

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Abstract—The Euler technique for solving initial value problems (IVP) for ordinary differential equations is the focus of this work (ODE). The proposed strategy is both efficient and practical for resolving these issues. We compare numerical results to precise solutions in order to ensure correctness. The precise answers and the numerical solutions are in good agreement. The step size must be very tiny in order to attain more precision in the solution. Finally, we look into and calculate the errors of the system. For varied step sizes, use Euler's technique.

Keywords— ODE, IVP, Euler method

I. INTRODUCTION

An equation comprising independent variables, dependent variables, and the derivatives of dependent variables with regard to independent variables is known as a differential equation.

A differential equation's order is the order of the highest derivative in the equation, whereas its degree is the degree of the highest order derivative in the equation after fractional and negative exponents of the derivatives are removed. If no product of the dependent variable with itself or any of its derivatives appears in the equation, the ODE is linear; otherwise, it is non linear. The nth order equation's solutions are determined by n parameters. There are n requirements that must be satisfied in order to find these parameters. If these n criteria are only specified at one location, the differential equation and the conditions are referred to as IVP of nth order. The problem is known as a boundary value problem when the n conditions are specified at two or more sites (BVP).

Numerical techniques are commonly employed to solve mathematical issues posed in science and engineering where exact answers are difficult or impossible to find. Analytical solutions are only possible for a small number of differential equations. The solution of ordinary differential equations can be found using a variety of analytical approaches. Even yet, there are many ordinary differential equations whose solutions cannot be derived in closed form using well-known analytical methods, and we must rely on numerical methods to find an approximate solution of a differential equation under the specified initial condition or conditions. For solving initial value issues for ordinary differential equations, there are a variety of viable numerical approaches. We offer the Euler Method for solving IVP of ODEs in this study.

Several studies on numerical solutions of initial value problems utilising the Euler method have been carried out, according to the literature survey. Many writers have attempted to solve IVP quickly and accurately using a variety of approaches, including the Euler method. [1]-[13] also looked at utilising the Euler technique and other numerical approaches to solve initial value issues for ordinary differential equations.

II. EULER'S METHOD

The slope at the start of the period is used to approximate the average slope for the whole interval. This method is known as Euler's Method.

Though it is theoretically possible to use Taylor's method of any order for any IVP to obtain good approximations, there are a few drawbacks, such as the scheme's assumption that all higher order derivatives for the given function f(x,y) exist, which is not a requirement for the existence of the solution for any first order initial value problem.

Even if the existence of these higher derivatives is assured, computing them for any given f may be difficult (x,y). Because the formula uses higher order derivatives, it is difficult to construct computer programmes, hence the approach is better suited to manual computations. Euler devised a strategy for overcoming these challenges by approximating y' in the specified IVP.

The first-order derivative term IVP $y' = f(x, y)$, $y(x0) = y0$

To numerically integrate Eq., we now employ Euler's technique.

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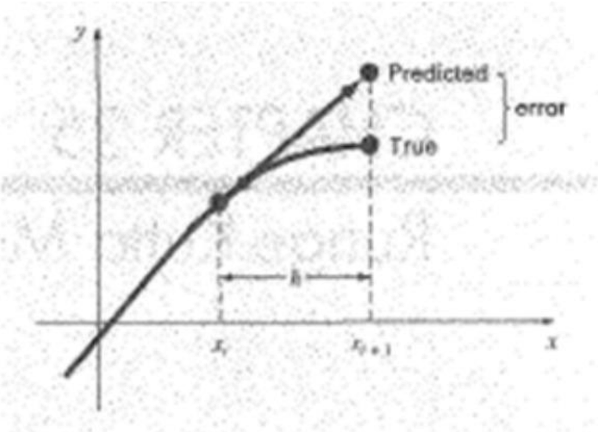
III. ERROR ANALYSIS FOR EULER'S METHOD

There are two forms of errors in numerical solutions of ODEs: Truncation and Round-off errors. The inaccuracy caused by truncating an innite sum and approximating it with a nite sum is known as truncation error in numerical analysis and scientific computing. Round-off mistakes are caused by a computer's limited ability to preserve significant digits.

There are two types of truncation mistakes. The first is a local truncation mistake that occurs when the procedure in question is applied in a single step. The second type of mistake is a propagated truncation error, which is caused by the approximations made in the preceding phases. The total, or global truncation, error is the sum of the two.



The Taylor series up to and including the phrase f(x,y)*h corresponds to Euler's technique. Because we approximate the correct solution with a finite number of terms from the Taylor series, the comparison shows that a truncation mistake occurs. As a result, we omit or truncate a portion of the genuine answer. The remaining terms in the Taylor series expansion that were not included, for example, are to blame for the truncation mistake in Euler's technique. Only the local truncation error—that is, the mistake caused during a single step of the method—is estimated using the Taylor series. It does not offer a measure of the propagated truncation error, and hence the global truncation error.
We frequently deal with functions that are more intricate than basic polynomials in real-world applications. As a result, obtaining the derivatives required to assess the Taylor series expansion would not always be simple. Despite the fact that these constraints prevent accurate error analysis for most practical applications, the Taylor series nevertheless gives useful insight into Euler's method's behaviour. The local inaccuracy is proportional to the square of the step size and the differential equation's first derivative.

IV. CONCLUSION

The more you travel away from the beginning value, the less accurate the approximation becomes. When the points in the approximation are closer together, the accuracy improves. If the function is concave down, your approximation will be above it, and if the function is concave up, it will be below it. By reducing the step size, the error can be decreased. Because the second derivative of a straight line is zero, the approach will produce error-free predictions if the differential equation solution is linear. Because Euler's approach approximates the solution using straight-line segments, the latter result makes intuitive sense. As a result, Euler's approach is known as a first-order method.

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