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RUNGE-KUTTA METHODS TO STUDY NUMERICAL SOLUTIONS OF INITIAL VALUE PROBLEMS IN ORDINARY DIFFERENTIAL EQUATIONS

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Abstract—The major goal of this paper is to look back on the work done on Runge-Kutta Methods to analyse Numerical Solutions of Initial Value Problems in Ordinary Differential Equations from 1983 to 2020. With thanks and heartfelt acknowledgment, the needed material from 1983 to 1996 for this study work was acquired freely from Hull et al. The current writers have gathered the state of this subject from 1983 to 2020 for the benefit of the new authors. newcomers to this field of study

Keywords—Runge-Kutta Methods, Initial Value Problem, Numerical Solutions, Ordinary Differential Equations

I. INTRODUCTION

A differential equation is a mathematical formula that connects one or more functions and their derivatives. Differential equations aid in the comprehension of rate-of-change events. The most significant mathematical tools in the world are these equations. Creating models in the fields of engineering, biology, and physics. The differential equations are used in the following way: Many real-world problems are represented mathematically. Differential equations, for example, assist us in comprehending Disease transmission, weather and climate forecasting, traffic flow, financial markets, population expansion, and water pollution are all issues that need to be addressed. chemical processes, suspension bridges, brain function, tumour growth, radioactive decay, electrical circuits, and planetary systems are some of the topics covered in this course. Guitar string motion and vibrations When studying ordinary problems, this is an essential form of problem that we must solve. An initial value issue is a differential equation. An "Initial Value Problem" is a single number that contains an ordinary differential equation, the constraints placed on the unknown function, and the values of its derivatives. When the starting value problem becomes too difficult to answer precisely, one of two options becomes available. To approximate the answer, many methodologies are used. The first step is to reduce the differential equation to one that can be solved. be answered accurately, and then the simplified equation's solution be used to estimate the original equation's solution. In research and engineering, numerical methods are most commonly utilized to solve mathematical issues. Only a few differential equations can be solved analytically in cases when exact solutions are difficult to get. Ordinary differential equations can be solved using a variety of analytical approaches. Many ordinary differential equations, however, cannot be solved analytically. To solve these types of ordinary differential equations, we must utilise numerical techniques at that point. Initial value issues can be solved using a variety of numerical approaches. During our research, we came across various publications that used the Runge-Kutta first, second, third, and fourth-order techniques to solve Initial Value Problems numerically. Several writers have attempted to address initial value issues utilising the approaches indicated above in order to get good accuracy. Hull et al. [1996] finished the literature on Runge-Kutta techniques from 1957 to 1996, and the current writers have attempted to update the literature.

II. NUMERICAL METHODS

The numerical technique is crucial for addressing initial value issues in ordinary differential equations, especially when no closed form solution exists. Numerical techniques are essential for solving differential equations for which analytic methods fail to provide accurate solutions. Numerical techniques can be explicit or implicit, and they can be computed in one or several phases. The numerical solution at the next time point is computed utilizing the numerical solution at the previous time point in an explicit fashion. An implicit approach, on the other hand, evaluates a function by employing the numerical answer at the next time point. Runge-Kutta procedures [2015] were created by C. Runge and M.W. Kutta. C. Runge published the first and second order Runge-Kutta techniques in 1895, while K. Heun published the third order Runge-Kutta method in 1996. The fourth and fifth order Runge-Kutta procedures were first introduced in 1901 by W. Kutta. Butcher pioneered the sixth order Runge-Kutta technique in 1964. Curits [1996] introduced the seventh and eighth

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order approaches. For numerous years, Runge-Kutta procedures have been used in research. In a number of studies, researchers examined and contrasted lower and higher order Runge-Kutta techniques in terms of performance.

III. REVIEW OF LITERATURE FROM 1983 TO 2020

Enrightet al. [1994], Owen and Zennaro [1991,1992], Muir and Owren [1993], and Verner [1993] have done the most work on Runge-Kutta techniques during this time period. Interpolants for Runge- Kutta formulae were studied by Enright et al. (1986). Enright [1993] investigated the relative efficacy of several defect control strategies for high Runge-Kutta systems.Enright and Suhartanto [1992] also used formulae to solve single initial-value issues.[1986] Peterson and For explicit Runge-Kutta procedures, Higham [1991] investigated global error utilising defect correction approaches. Detecting rigidity Robertson [1986] created explicit Runge-Kutta procedures, which resulted in the development of certain unique Runge-Kutta formulations. Muir (1984) and Enright and Muir (1986) implemented it for boundary-value issues. Iterative linear equation solvers were employed by Chan and Jackson [1986] in algorithms for large systems of stiff initial value problems. Sharp [1987, 1989] investigated the Runge-Kutta-Nystrom integrator for second-order initial value problems and novel low-order explicit Runge-Kutta pairs. Sharp [1989] designed and implemented others for first-order issues, and Sharp and Fine [1987,1992] for second-order problems. Enright [1989, 1991] looked at error control techniques for continuous Runge-Kutta methods and a new error control strategy for initial value issues. Sharp [1991] has created new formats for displaying and reporting comparative findings. Jackson and Norsett [1995] investigated the possibility for parallelism in typical Runge-Kutta Methods and gave both negative and positive results. A theorem that confines the order of a Runge-Kutta formula in terms of the smallest polynomial associated with its coefficient matrix underpins many of the negative results. The favourable results are mostly prototype formulas that show the possibility of effective "coarse-grain" parallelism on computers with a few processors. Enenkel [1988] described and Jackson, Kvarno, and Norsett [1994] analysed Runge-Kutta predictor-corrector approaches. Broderick et al. (1994) used free carrier effects in their coupled mode calculations.

Suhartan proposed a new method for detecting singular points in the numerical solution of initial value problems in [1990]. Nguyen [1995] used continuous implicit Runge-Kutta techniques to develop interpolation and error control strategies for algebraic differential equations. Jackson [1991] reviewed parallel numerical approaches for ODE initial value issues. Jackson and Pancer [1992] investigated the parallel solution of the ABD system that arises in numerical techniques for BVPs for ODEs. The practical consequences of order reduction for implicit Runge-Kutta algorithms were studied by Enright and Macdonald [1992]. Hayashi [1996] used continuous Runge-Kutta techniques to solve retarded and neutral delay differential equations numerically.

Enright and Muir [1996] also worked on a defect-control Runge-Kuttatype boundary value ODE solver. Enright et al. [1988] investigated the use of Runge-Kutta formulae and interpolants to solve discontinuous IVPs. Enright [1986,1996] looked at parallel defect control and convergence analysis for continuous numerical algorithms to solve retarded and neutral delay differential equations. The findings in the past two studies are useful not only for developing parallel ODE programmes, but also for implementing implicit Runge-Kutta techniques on ordinary sequential computers in an efficient, reliable, and resilient manner. Continuous finite difference approximations for solving differential equations were researched by Onumanyiet.al. in 1999. Gander [1999] explored the development of numerical algorithms using computer algebra in depth. "The computation of global error for initial value problems of ordinary differential equations," Hong [2000] worked on. Linget al. [2000] investigated the Runge-Kutta–Merson Algorithm's applicability to creep damage assessments. Boyce [2000] also worked on boundary value issues and elementary differential equations. Goekenand Johnson [2000] published a work on Runge–Kutta with higher order derivative approximations. For convection-dominated situations, Cockburn [2001] investigated the RungeKutta discontinuous Galerkin Method. Pimenov [2001] presents generic linear techniques for solving functional-Differential Equations numerically. For convection-dominated issues, Bernardo and Wang [2001] discussed Runge-Kutta discontinuous Galerkin Methods.

For generic second order Ordinary Differential Equations, Awoyemi [2001] devised a novel sixth-order method. Fredebeul et al. [2002] studied Runge-KuttaFehlberg formulas with multiple orders and double outputs, as well as some techniques for their efficient implementation. Murugesan [2002] investigated a fourth-order embedded Runge-Kutta method with error control based on arithmetic and centroidal means. Gerald [2002] worked on numerical analysis that was applied. Berdan (2002, 2003, 2008) and Butcher (2003, 2008) investigated numerical methods for ordinary differential equations. P-stable linear multistep approach for solving generic third order ordinary differential equations was investigated by Awoyemi [2003]. Butcher [2003] also goes into great length about numerical approaches for ordinary differential equations. Biazaret al. [2004] used the Adomian decomposition approach to solve a system of ordinary differential equations. In his book Numerical Methods for Mathematics, Mathews [2005] provided several findings. Majid [2006] worked on the block backward differentiation formula for solving first-order ordinary differential equations in higher order systems. Chapra [2006] used MATLAB to examine numerical methods.

"Variable step block backward differentiation formula for solving first-order stiff ODEs" was developed by Ibrahim [2007]. The propagation of mistakes in the Euler technique was studied by Akanvi [2010]. Numerical solutions to ordinary differential equations were provided by Atkinson [2009]. Hossain [2013, 2019] looked at how the modified Euler approach and the Runge-Kutta method compare in solving initial value issues. In their well-known study "on certain numerical

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approaches for solving initial value problems in ordinary differential equations," Bosedeet al. [2012] presented various numerical methods. In addition, Ogunrinde [2012] investigated numerical approaches for addressing initial value issues in ordinary differential equations. For solving ordinary differential equations, Rabiei [2012] enhanced the fifth order Runge-Kutta technique. The accuracy of numerical solutions to initial value problems (IVP) for ordinary differential equations was investigated by Islam [2015]. (ODE). He also used the fourth order Runge-Kutta technique to provide correct starting value solutions for ordinary differential equations. Islam [2015] investigated numerical solutions to initial value problems in depth. In addition, Islam [2015] conducted a comparison study of Euler and Runge-Kutta techniques for numerical solutions of initial value problems (IVP) for ordinary differential equations (ODE).

The Runge-Kutta fourth order approach with differential equations and its application were explored by Gowri et al. [2017]. The Runge-Kutta fourth order approach was used to solve differential equation issues, and the application problem was explored using Runge-Kutta fourth order. He came to the conclusion that the Runge-Kutta fourth order method produced more precise findings. "Research on the numerical solutions of second order initial value problems (IVP) for ordinary differential equations with fourth order and Butcher's fifth order Runge-Kutta techniques," according to Hossain [2017]. For first order ordinary differential equations with beginning conditions, Hamed [2017] presented the accuracy of Euler and modified Euler techniques. Samsudin et al. [2018] worked on cube arithmetic: enhancing the Euler technique for ordinary differential equation analysis and comparing the Euler and Runge-Kutta methods for numerical solutions of initial value problems (IVP) in ordinary differential equations. The numerical solutions of initial value issues were analysed and compared by Anthony [2019] et al. They examined the performance and computing effort of the two approaches and concluded that the step size needed to be very short to attain more precision in the results. Jamali [2019] conducted a comparison and analysis of numerical approaches for solving ordinary differential equations with initial value issues. Murad Hossen et al. (2019) investigated the numerical solution of the initial value issue using modified Euler's technique and Runge-Kutta method in a comparative study. The numerical solutions of ordinary differential equations with initial value issues were found using Modified Euler's technique and RungeKutta methods in this article. MATLAB is used. Mohammad et al. [2020] investigated several numerical approaches for solving ordinary differential equations in a comparative study.

IV. CONCLUSIONS

We have attempted to read every study publication on this topic that has been published too yet. The majority of the work on this issue comes from workers in other countries, and they have mostly worked with accuracy and comparative analysis using the Runge-Kutta first, second, third, and fourth order methods. In the Indian context, this review reveals that there are little reports of work accessible. As a result, there is a great need to do extensive research on these issues in order to solve the initial value concerns.

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