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Newton Backward Interpolation Method

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Abstract: This paper offers us the primary statistics of "Newton backward interpolation method" In order to lessen the numerical computations associated to the repeated utility of the prevailing interpolation components in computing a massive quantity of interpolated values, a formula has been derived from Newton's backward interpolation formula for representing the numerical facts on a pair of variables by means of a polynomial curve. application of the components to numerical records has been proven inside the case of representing the records on the total populace of India corresponding as a feature of time. The system is appropriate in the scenario wherein the values of the argument (i.e. unbiased variable) are at equal c language.

Keywords: Interpolation, newton's backward interpolation formula, polynomial curve, representation of numerical data

INTRODUCTION

Interpolation, that is the system of computing intermediate values of a function from the set of given values of the function {Hummel (1947), Erdos & Turan (1938) et al}, plays tremendous function in numerical research almost in all branches of technological know-how, humanities, commerce and in technical branches. some of interpolation formulas specifically Newton's forward Interpolation method, Newton's Backward Interpolation system, Lagrange's Interpolation formula, Newton's Divided difference Interpolation components, Newton's vital distinction Interpolation method, Stirlings components, Bessel's components and some others are available within the literature of numerical analysis {bathe & Wilson (1976) [1], Jan (1930), Hummel (1947) et al}.

In case of the interpolation by means of the prevailing formulae, the value of the dependent variable similar to each cost of the unbiased variable is to be computed afresh from the used formulation putting the fee of the independent variable in it. this is if it is wanted to interpolate the values of the established variable similar to some of values of the independent variable by means of a suitable current interpolation formula, it is required to use the formulation for every fee separately and accordingly the numerical computation of the fee of the established variable based totally at the given information are to be achieved in each of the cases. so as to get rid of those repeated numerical computations from the given records, you can actually think of an approach which consists of the representation of the given numerical records by way of a appropriate polynomial after which to compute the price of the based variable from the polynomial similar to any given cost of the impartial variable. however, a method/components is vital for representing a given set of numerical data on a pair of variables by using a appropriate polynomial. One such system has been evolved on this observe. The method has been derived from Newton's backward interpolation components. The method obtained has been carried out to symbolize the numerical statistics, on the entire population of India considering the fact that 1971, by a polynomial curve.

Newton's Backward Interpolation Formula

Then Newton's backward interpolation formula is

Then Newton's backward interpolation formula is -:

$$f\ (x) = f(x_n) + v\nabla f(x_{n-1}) + \frac{v\,(v+1)}{2!}\nabla^2 f(x_{n-2}) + \frac{v\,(v+1)(v+2)}{3!}\nabla^3 f(x_{n-3}) + \frac{v\,(v+1)(v+2))(v+3)}{4!}\nabla^4 f(x_{n-4}) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{v\,(v+1)(v+2)\ldots\ldots\ldots\ldots(v+n-1)}{n!}\nabla^n f(x_0)$$

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where $v = \frac{x-x_n}{h}$

$$= f(x_n) + (x-x_n)\frac{\nabla f(x_{n-1})}{h} + (x-x_n)(x-x_{n-1})\frac{\nabla^2 f(x_{n-2})}{2!h^2} + (x-x_n)(x-x_{n-1})(x-x_{n-2})\frac{\nabla^3 f(x_{n-3})}{3!h^3} + (x-x_n)(x-x_{n-1})(x-x_{n-2})(x-x_{n-3})\frac{\nabla^4 f(x_{n-4})}{4!h^4} + \ldots\ldots + (x-x_n)(x-x_{n-1})(x-x_{n-2})(x-x_{n-3})\ldots\ldots(x-x_1)\frac{\nabla^n f(x_0)}{n!h^n}$$

This formula can be expressed as

$$f(x) = C_n + C_{n-1}(x-x_n) + C_{n-2}(x-x_n)(x-x_{n-1}) + C_{n-3}(x-x_n)(x-x_{n-1})(x-x_{n-2}) + C_{n-4}(x-x_n)(x-x_{n-1})(x-x_{n-2})(x-x_{n-3}) + \ldots\ldots + C_0(x-x_n)(x-x_{n-1})(x-x_{n-2})(x-x_{n-3})\ldots\ldots(x-x_1)$$

$\rightarrow$ (2.2)

where $C_n = f(x_n)$

$C_{n-1} = \frac{\nabla f(x_{n-1})}{h}$

$C_{n-2} = \frac{\nabla^2 f(x_{n-2})}{2!h^2}$

$C_{n-3} = \frac{\nabla^3 f(x_{n-3})}{3!h^3}$

$C_{n-4} = \frac{\nabla^4 f(x_{n-4})}{4!h^4}$

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$C_0 = \frac{\nabla^n f(x_0)}{n!h^n}$

Representation of Numerical Data by Polynomial Curve:

By algebraic expansion, one can obtain that

$(x-x_n) = (x-x_n)$

Also,

$(x-x_n)(x-x_{n-1}) = x^2 - x.x_{n-1} - x.x_n + x_n x_{n-1}$

$= x^2 - (x_{n-1} + x_n)x + x_n x_{n-1}$

$= x^2 - (\sum_{i=n-1}^{n} x_i)\,x + x_n x_{n-1}$

Again,

$(x-x_n)(x-x_{n-1})(x-x_{n-2})$

$= (x^2 - x.x_{n-1} - x.x_n + x_n x_{n-1})(x-x_{n-2})$

$= x^3 - x^2.x_{n-1} - x^2.x_n + x\,x_n x_{n-1} - x^2.x_{n-2} + x.x_{n-2}x_{n-1} + x.x_n x_{n-2} - x_{n-2}x_{n-1}x_n$

$= x^3 - (x_{n-2} + x_{n-1} + x_n)x^2 + (x_{n-2}x_{n-1} + x_{n-2}x_n + x_{n-1}x_n)x - x_{n-2}x_{n-1}x_n$

$= x^3 - (\sum_{i=n-2}^{n} x_i)\,x^2 + (\sum_{i=n-2}^{n-1}\sum_{j=n-1}^{n} x_i x_j)\,x - x_{n-2}x_{n-1}x_n$

Similarly,

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$(x-x_n)(x-x_{n-1})(x-x_{n-2})(x-x_{n-3})$

$= \{x^3-(x_{n-2}+x_{n-1}+x_n)x^2+(x_{n-2}x_{n-1}+x_{n-2}x_n+x_{n-1}x_n)x-x_{n-2}x_{n-1}x_n\}(x-x_{n-3})$

$= x^4-(x_{n-2}+x_{n-1}+x_n)x^3+(x_{n-2}x_{n-1}+x_{n-2}x_n+x_{n-1}x_n)x^2-x_{n-2}x_{n-1}x_n x-x_{n-3}x^3$

$+(x_{n-2}+x_{n-1}+x_n)x^2x_{n-3}-(x_{n-2}x_{n-1}+x_{n-2}x_n+x_{n-1}x_n)x_{n-3}x+x_{n-3}x_{n-2}x_{n-1}x_n$

$= x^4-(x_{n-3}+x_{n-2}+x_{n-1}+x_n)x^3+(x_{n-3}x_{n-2}+x_{n-3}x_{n-1}+x_{n-2}x_{n-1}+x_{n-2}x_n+x_{n-3}x_n+x_{n-1}x_n)x^2-(x_{n-3}x_{n-2}x_{n-1}+x_{n-3}x_{n-2}x_n+x_{n-3}x_{n-1}x_n+x_{n-2}x_{n-1}x_n)x+x_{n-3}x_{n-2}x_{n-1}x_n$

$= x^4-(\sum_{i=n-3}^{n}x_i)x^3+(\sum_{i=n-3}^{n-1}\sum_{j=n-2}^{n}x_ix_j)x^2-(\sum_{i=n-3}^{n-2}\sum_{j=n-2}^{n-1}\sum_{k=n-1}^{n}x_ix_jx_k)x+$

$x_{n-3}x_{n-2}x_{n-1}x_n$

In general,

$(x-x_n)(x-x_{n-1})(x-x_{n-2})(x-x_{n-3})\ldots\ldots\ldots(x-x_1)$

$= x^n-(\sum_{i=n-(n-1)}^{n}x_i)x^{n-1}+(\sum_{i=n-(n-1)}^{n-1}\sum_{j=n-(n-2)}^{n}x_ix_j)x^{n-2}-$

$(\sum_{i=n-(n-1)}^{n-(n-2)}\sum_{j=n-(n-2)}^{n-(n-1)}\sum_{k=n-(n-1)}^{n}x_ix_jx_k)x+\ldots\ldots\ldots+(-1)^n(x_nx_{n-1}x_{n-2}\ldots\ldots\ldots x_1)$

$= x^n-(\sum_{i=1}^{n}x_i)x^{n-1}+(\sum_{i=1}^{n-1}\sum_{j=2}^{n}x_ix_j)x^{n-2}-(\sum_{i=1}^{2}\sum_{j=2}^{1}\sum_{k=1}^{n}x_ix_jx_k)x+\ldots\ldots\ldots\ldots$

$+(-1)^n(x_nx_{n-1}x_{n-2}\ldots\ldots\ldots x_1)$

Now, equation (2.2), can be expressed as

$f(x)=C_n+C_{n-1}(x-x_n)+C_{n-2}[x^2-(\sum_{i=n-1}^{n}x_i)x+x_nx_{n-1}]+C_{n-3}[x^3-(\sum_{i=n-2}^{n}x_i)x^2+(\sum_{i=n-2}^{n-1}\sum_{j=n-1}^{n}x_ix_j)x-x_{n-2}x_{n-1}x_n]+C_{n-4}[x^4-(\sum_{i=n-3}^{n}x_i)x^3+(\sum_{i=n-3}^{n-1}\sum_{j=n-2}^{n}x_ix_j)x^2-(\sum_{i=n-3}^{n-2}\sum_{j=n-2}^{n-1}\sum_{k=n-1}^{n}x_ix_jx_k)x+x_{n-3}x_{n-2}x_{n-1}x_n]$

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$C_0[x^n-(\sum_{i=1}^{n}x_i)x^{n-1}+(\sum_{i=1}^{n-1}\sum_{j=2}^{n}x_ix_j)x^{n-2}-(\sum_{i=1}^{2}\sum_{j=2}^{1}\sum_{k=1}^{n}x_ix_jx_k)x+$
$+\ldots\ldots\ldots+(-1)^n(x_nx_{n-1}x_{n-2}x_{n-3}\ldots\ldots\ldots x_1)] \rightarrow$ **(3.1)**

This is of the form

$f(x)=A_0+A_1x+A_2x^2+A_3x^3+\ldots\ldots\ldots+A_nx^n \rightarrow$ **(3.2)**

Where

$A_0=C_n-C_{n-1}x_n+C_{n-2}x_nx_{n-1}-C_{n-3}x_{n-2}x_{n-1}x_n+C_{n-4}x_{n-3}x_{n-2}x_{n-1}x_n-$
$\ldots\ldots\ldots+(-1)^nC_0(x_nx_{n-1}x_{n-2}x_{n-3}\ldots\ldots\ldots x_1)$

$A_1=C_{n-1}-C_{n-2}(\sum_{i=n-1}^{n}x_i)+C_{n-3}(\sum_{i=n-2}^{n-1}\sum_{j=n-1}^{n}x_ix_j)-$
$C_{n-4}(\sum_{i=n-3}^{n-2}\sum_{j=n-2}^{n-1}\sum_{k=n-1}^{n}x_ix_jx_k)+\ldots\ldots\ldots-C_0(\sum_{i=1}^{2}\sum_{j=2}^{1}\sum_{k=1}^{n}x_ix_jx_k)$

$A_2=C_{n-2}-C_{n-3}(\sum_{i=n-2}^{n}x_i)+C_{n-4}(\sum_{i=n-3}^{n-1}\sum_{j=n-2}^{n}x_ix_j)-\ldots\ldots$
$+C_0(\sum_{i=2}^{3}\sum_{j=3}^{2}\sum_{k=2}^{1}\sum_{l=1}^{n}x_ix_jx_kx_l)$

$A_3=C_{n-3}-C_{n-4}(\sum_{i=n-3}^{n}x_i)+\ldots\ldots\ldots\ldots\ldots+$
$C_0(\sum_{i=3}^{4}\sum_{j=4}^{3}\sum_{k=3}^{2}\sum_{l=2}^{1}\sum_{m=1}^{n}x_ix_jx_kx_lx_m)$

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$A_n=C_n$

Equation (3.2), with the coefficients

$A_0, A_1, A_2, A_3, \ldots\ldots\ldots, A_n,$

,as defined above, is the required formula for representing a given set of numerical data on a pair of variables by a suitable polynomial we have aimed at.

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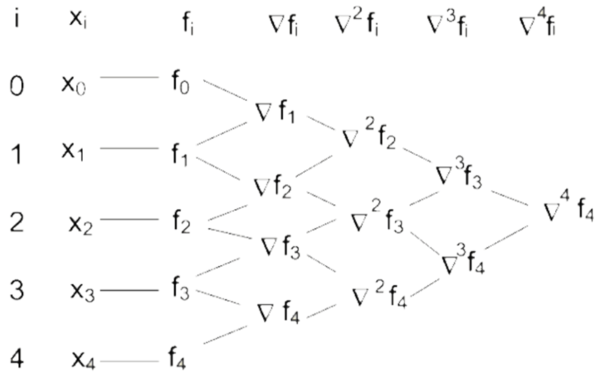

Note: The components is legitimate for representing a given set of numerical facts on a couple of variables by a appropriate polynomial underneath the following two conditions: (i) values of the argument are at identical interval (ii) the price of x corresponding to which the cost of y is to be interpolated is in the closing half of the series

EXAMPLE OF NEWTON BACKWARD INTERPOLATION METHOD

Estimate f(42) from the following data using newton backward interpolation.

x:	20	25	30	35	40	45
f(x):	354	332	291	260	231	204

Solution

The difference table is:

x	f	∇f	$\nabla^2 f$	$\nabla^3 f$	$\nabla^4 f$	$\nabla^5 f$
20	354					
		- 22				
25	332		- 19			
		- 41		29		
30	291		10		-37	
		- 31		- 8		45
35	260		2		**8**	
		- 29		**0**		
40	231		**2**			
		- 27				
45	**204**					

Here $x_n = 45$, $h = 5$, $x = 42$ and $p = -0.6$

Newton backward formula is:

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$$P(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n$$

$$P(42)=204+(-0.6)(-27)+\frac{(-0.6)(0.4)}{2}\times 2+\frac{(-0.6)(0.40(1.4)}{6}\times 0+\frac{(-0.6)(0.4)(1.4)(2.4)}{24}\times 8+\frac{(-0.6)(0.4)(1.4)(2.4)(3.4)}{120}\times 45 = 219.1430$$

Thus, f(42) = 219.143

2. Find Solution using Newton's Backward Difference formula(x = 1925)

x	1891	1901	1911	1921	1931
y	46	66	81	93	101

Solution:
Newton's backward difference table is

X	Y	ΔY	Δ^2Y	Δ^3Y	Δ^4Y
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

$$\Delta x = x_1 - x_0 = 1901 - 1891 = 10$$

$$S = \frac{x - x_n}{\Delta x}$$

$$S = \frac{1925 - 1931}{10}$$

$$S = -0.6$$

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Newton’s backward difference interpolation formula is

$$P_n(x) = f_n + s\Delta f_n + \frac{s(s+1)}{2!}\Delta^2 f_n + \frac{s(s+1)(s+2)}{3!}\Delta^3 f_n + \cdots + \frac{s(s+1)(s+2)\cdots(s+(n-1))}{n!}\Delta^n f_n$$

$$= 101 + (-0.6)\times 8 + \frac{-0.6(-0.6+1)}{2}\times(-4) + \frac{-0.6(-0.6+1)(-0.6+2)}{6}\times(-1)\frac{-0.6(-0.6+1)(-0.6+2)(-0.6+3)}{24}\times(-3)$$

$$= 101 - 4.8 + 0.48 - 0.056 + 0.1008$$

$$= 96.8368$$

USE OF NEWTON FORWARD AND BACKWARD INTERPOLATION METHOD

with other difference formulation, the degree of a Newton interpolating polynomial may be elevated by using including extra phrases and factors without discarding current ones. Newton's form has the simplicity that the brand new points are usually added at one quit: Newton's backward formula can add new points to the left. The accuracy of polynomial interpolation depends on how close the interpolated factor is to the middle of the x values of the set of points used. glaringly, as new points are delivered at one give up, that middle turns into farther and farther from the primary facts point. therefore, if it isn't recognized how many points can be wished for the preferred accuracy, the center of the x-values is probably far from wherein the interpolation is achieved.

CONCLUSION

The method defined by equation (3.2) can be used to symbolize a given set of numerical statistics on a pair of variables, via a polynomial. The degree of the polynomial is one much less than the quantity of pairs of observations. The polynomial that represents the given set of numerical facts may be used for interpolation at any function of the unbiased variable mendacity within its two severe values. The method of interpolation, defined right here, can be suitably carried out in inverse interpolation also. Newton’s backward interpolation formula is legitimate for estimating the fee of the structured variable below the following situations: (i) Values of the argument are at equal c program languageperiod (ii) The value of x similar to which the cost of y is to be interpolated is inside the closing 1/2 of the collection therefore, the components derived here is legitimate for representing a set of numerical records on a pair of variables via a polynomial beneath those two conditions most effective. consequently, there is necessity of looking for a few components for representing a set of numerical information on a couple of variables with the aid of a polynomial if the cost of the independent variable corresponding to which the value of the dependent variable is to be expected lies inside the final 1/2 of the series of the given values, which are at same c language, of the impartial variable. moreover, there is additionally necessity of attempting to find some formulation for representing a hard and fast of numerical facts on a pair of variables with the aid of a polynomial if the given values of the unbiased variable aren't at identical c language

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