

Gauss-seidel Method

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Abstract : We shall provide a Gauss-Seidel method for solving linear equations and explore its convergence in this work. The purpose of this work is to describe and report on an experiment that was conducted to assess students' understanding of the Gauss-Seidel method. To demonstrate its efficacy, some samples are taken. I've included tasks to investigate the learner's ability to state the theorem and apply it to reasoning tasks, the impact of several concept images on their reasoning about the theorem, and the learner's ability to examine the relationship between the Gauss-Seidel method and other related mathematical concepts.

INTRODUCTION :

The user can regulate round off error with this approach. The majority of these approaches have a rounding error. This is a modified version of Jacob's iteration method. The Gauss Seidel technique, also known as the Liebmann method or the method of successive displacement in numerical linear algebra, is an iterative method for solving a system of linear equations. The German mathematicians Carl Friedrich Gauss and Phillip Ludwig Von Seidel are credited with inventing this method. It can be used on any matrix with non-zero diagonal components, but convergence is only guaranteed if the matrix is either strictly diagonally dominating or symmetric and positive definite.

Calculus is a topic that is rich in abstraction and requires a high level of intellectual comprehension, which many students struggle with. According to Ferrini-Mundy and Graham (1991), students' knowledge of basic calculus concepts is extremely primitive. When teaching calculus to a diverse group of students, it's appropriate to emphasise students' intuitive comprehension of mathematical concepts and ideas, building on what they need to know without placing undue demands on their meticulous mathematical understandings. Some researchers argue that an introductory calculus course should be informal, intuitive [1], and abstract, primarily based on graphs and functions (Koirala, 1997); formulas and rules should be intentionally and intuitively taught to the student's previous tasks in addition to arithmetic, calculation, and different sciences (Held, 1988; Orton 1983).

NEED OF GAUSS-SEIDEL METHOD :

Gauss-Seidel Method is used to solve the linear system Equations. This method is named after the German Scientist **Carl Friedrich Gauss** and **Philipp Ludwig Siedel**. It is a method of iteration for solving an linear equation with the unknown variables. This method is very simple and uses in digital computers for computing.

The Gauss-Seidel method is the modification of the gauss-iteration method. This modification reduces the number of iteration. In this methods the value of unknown instantly reduces the number of iterations, the calculated value restore the earlier value only at the end of the iteration. Because of it, the gauss-seidel methods intersect much faster than the Gauss methods. In gauss seidel methods the number of iteration method requires acquiring the solution is much less as compared to Gauss method. We see the solution of the linear system .

Why we used this method?

We used this method it allows the user the control round off error.

Elimination methods such as Gauss'sian Elimination and LU Decomposition are prone to round off error.

ALSO: If the physics of the problem are understood a close initial guess can be made decreasing the number of iterations needed.

Basic Procedure :

At first we have to algebraically solve each linear equation for X^i .

Then assume an initial guess solution array.

Solve for each X^i and repeat.

And Lastly use absolute relative approximation error after each iteration to check if error is within a pre-specified tolerance.

Algorithm : Suppose, A set of **n equations** with an **n no. of unknowns** :

$$A_{11}X_1 + A_{12}X_2 + A_{13}X_3 + \dots + A_{1N}X_N = B_1$$

$$A_{21}X_1 + A_{22}X_2 + A_{23}X_3 + \dots + A_{2N}X_N = B_2$$

⋮
⋮
⋮

$$A_{N1}X_1 + A_{N2}X_2 + A_{N3}X_3 + \dots + A_{NN}X_N = B_N$$

If there are non-zero diagonal elements. Each equation should be rewritten to solve for the relevant unknown.

Solve the first equation for X1.

Solve the second equation for X2.

Calculate the Relative Absolute Approximate Error...

$$|\epsilon_a|_i = \left| \frac{X_i^{new} - X_i^{old}}{X_i^{new}} \right| \times 100$$

So, when is the answer going to be found?

When the absolute relative approximate error for all unknowns is less than a predetermined tolerance, the iterations are ended.

Application of gauss seidel method for power flow studies :

Assume that all buses other than the swing or slack bus are P-Q or load buses for the purpose of illustrating how to apply the Gauss-Seidel method for power flow analyses. Both V and are specified at slack bus, and they remain fixed throughout. There are (n-1) buses that provide P and Q. We assume the magnitudes and angles at these (n-1) buses at first, and update these voltages at each iteration step.

$$\text{From Eq. (6.57a) } I_i = \frac{P_i - jQ_i}{V_i^*} \quad \dots(6.67)$$

$$\text{From Eq. (6.8) } V_i = \frac{1}{Y_{ii}} \left[I_i - \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik} V_k \right] \quad \dots(6.68)$$

Substituting $I_i = \frac{P_i - jQ_i}{V_i^*}$ from Eq. (6.67) in Eq. (6.68), we have

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik} V_k \right]; i = 2, 3, \dots n \quad \dots(6.69)$$

Bus 1 is a slack bus with a specified V, hence V1 is used in the preceding calculation. Eq. (6.69) denotes a set of (n – 1) equations for I = 2, 3,..., n that must be solved simultaneously for V2, V3, V4,..., Vn.

Except for the slack bus, where the voltage magnitude and phase angle are defined and stay fixed, the Gauss technique assumes the voltage for all buses. Normally, we set these buses' voltage magnitude and phase angle to the same as the slack bus and work in a per unit system.

To obtain a new set of bus voltages, the assumed bus voltage and the slack bus voltage, as well as P and Q, are substituted in RHS of Eq. (6.69). The new set of bus voltages is substituted along with the new set of bus voltages when the full iteration is completed.

THE PROCESS IS CONTINUE TILL :

where r is the number of iterations, and is a very small value that relies on the precision of the system and is usually equal to 0.0001 or less.

As a result, the process is repeated until the mod of the bus voltage achieved in the current iteration less the value of the bus voltage obtained in the previous iteration is less than a selected extremely small number, and we obtain the solution, i.e., magnitude and phase angle of voltage.

The above-mentioned Gauss iterative approach is more slower to converge and may occasionally fail.

In the Gauss-Seidel technique, the calculated bus voltages for any bus immediately replace the previous values in the next step, but in the Gauss method, the calculated bus voltages only replace the prior value at the end of the iteration, as previously stated. As a result, the Gauss-Seidel approach converges much faster than other methods. One of the most common approaches for solving power flow equations was the Gauss-Seidel method.

It has the following benefits and drawbacks :

Advantages :

1. Technique simplicity.
2. Small computer memory requirement.
3. Less computational time per iteration.

Disadvantages :

1. Slow rate of convergence resulting in large number of iterations.
2. Increase in number of iterations directly with the increase in the number of buses.
3. Effect on convergence due to choice of slack bus.

CONCLUSION :

We intended to use specific challenges to review students' understanding of the Gauss-Seidel method. In most situations, the instantiations in the context of the Gauss-Seidel technique appear to involve well-known functions or graphs, such as the conic. For a few students, this results in a misinterpretation or misunderstanding of the hypothesis or idea conclusion. Their instantiations are overly specific, with a lack of richness and precision.

To summarise, our experiment highlights possible issues students have in learning Gauss seidel method, which includes making sense of it, as well as the capacity to employ it in situations where the function isn't explicitly presented. tasks. To put it another way, their capacity to translate ordinary thinking into abstract mathematical concepts allows them to have a wide range of concept representations. They are guided to have diverse concept images by abstract mathematical notions.

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