

# NEWTON RAPHSON METHOD

**Prof .Vishal V. Mehtre<sup>1</sup>, Satyam Triyar<sup>2</sup>**

Assistant Professor, Department of Electrical Engineering, Bharati Vidyapeeth( Deemed to be University),

College of Engineering, Pune, India<sup>1</sup>

Department of Electrical Engineering, Bharati Vidyapeeth (Deemed to be University),

College of Engineering, Pune, India<sup>2</sup>

**Abstract:** The paper is prepared Newton Raphson approach this is all-inclusive to resolve the non-square and non-linear problems. The look at additionally objectives to comparing the fee of overall performance, charge of convergence of Bisection technique, root findings of the Newton method and Secant method. It additionally represents a new method of calculation the usage of nonlinear equation and this could be similar to Newton Raphson easy technique and inverse Jacobian matrix can be used for the iteration system and this could be similarly used for allotted power load drift calculation and will also be beneficial in some of the packages. The paper also discusses the distinction between the usage of built in spinoff feature and self-derivative function in fixing non-linear equation in scientific calculator. The derivation Newton Raphson method, set of rules, use and disadvantages of Newton Raphson technique have additionally been discussed.

## INTRODUCTION:

Finding the answer to the set of nonlinear equations  $f(x) = (f_1, \dots, f_n)' = 0$  is been a trouble for the beyond years. Here we take into account this nonlinear equation and attempt locates the solution to it and this could be determined out with the aid of the Newton Raphson method. This approach could be very acquainted for its speedy charge convergence and for improving the convergence property, the Homotopy technique is adopted out of numerous methods. Homotopy works by using reworking an authentic trouble into a clean hassle so one can be afterwards turn out to be clean to be solved[1]. A Homotopy map can also be required in in addition fixing the hassle. Root finding is likewise one of the problems in realistic applications. Newton technique could be very rapid and green compared to the others strategies. In order to examine the overall performance, it's miles therefore very important to take a look at the cost and speed of the convergence. Newton technique requires best one new release and the by-product assessment in step with iteration. The result of evaluating the price of convergence of Bisection, Newton and Secant techniques got here as Bisection technique < Newton approach < Secant technique which in phrases of range is that the Newton approach is 7.678622465 instances better than the Bisection approach while Secant technique is 1.389482397 instances higher than the newton approach. complex systems with better pace processing manipulate are in call for now a days and the answer to that is to divide them into subsystems and in this manner each subsystem might be handled personally and the control and operation can be carried out to each of that subsystem[2]. a brand new technique of dispensed load flow calculation the use of nonlinear is additionally offered in the paper. Finding roots of the nonlinear equation with the help of Newton Raphson technique provides desirable result with speedy convergence velocity and Mat lab also adopted this method for locating the roots and device used for such calculations is medical calculator. Bracketing method is which calls for bracketing of the root via guesses are always convergent as they may be based totally on decreasing the c programming language between two guesses[9]. Bisection method and the false position approach use the bracketing method.

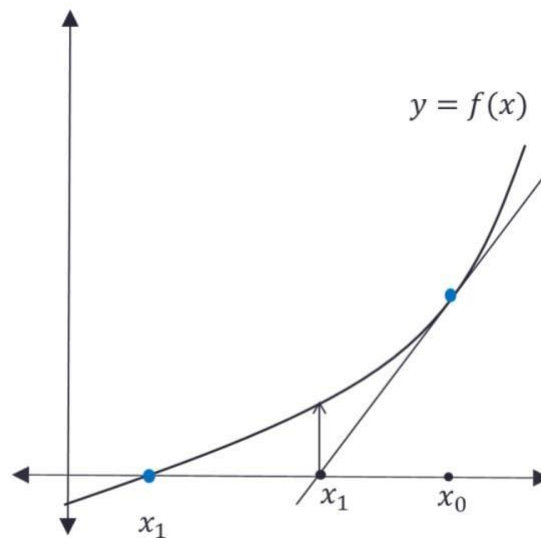
## 2 NEWTON RAPHSON METHOD

### 2.1 DEFINATION

Newton's technique (also recounted as the Newton–Raphson method), named after Isaac Newton and Joseph Raphson, is a method for judgment sequentially superior approximations to the extraction of an actual-valued function

$$x : f(x) = 0.$$

Any 0-finding approach (Bisection method, fake position method, Newton-Raphson, and so on.) also can be used to find a minimum or maximum of the sort of feature, via finding a 0 within the function's first by-product, see Newton's technique as an optimization set of rules.



Newton Raphson approach uses to the slope of the characteristic in some unspecified time in the future to get closer to the foundation. using equation of line  $y = mx_0 + c$  we can calculate the factor wherein it meets x axis, in a wish that the authentic feature will meet xaxis somewhere close to. we can reach the original root if we repeat the equal step for the brand new cost of  $x[3]$ .

**2.2 EXPLANATION**

The concept of the Newton-Raphson method is as follows: one starts off evolved with an initial conjecture that's logically at ease to the actual root, then the reason is approximated by using its digression line and one computes the x-intercept of this digression line. This x-intercept will typically be a better approximation to the characteristic's root than the original wager, and the approach can be iterated primarily based on collinear scaling and nearby quadratic approximation, quasi-Newton techniques have stepped forward for feature value isn't always absolutely used in the Hessian matrix[4]. As collinear scaling issue in paper may seem singular, this paper, a state-of-the-art collinear scaling factor is studied. using local quadratic approximation, a complicated collinear scaling set of rules to enhance the steadiness is offered, and the global convergence of the algorithm is proved. in addition, numerical effects of education neural community with the progressed collinear scaling set of guidelines proven the efficiency of this algorithm is an entire lot advanced to traditional one[8].

**2. DERIVATION**

In numerical analysis, Newton's method, named after Isaac Newton and Joseph Raphson, is an approach for locating successively better approximations to the roots of a real valued feature.

$$x : f(x) = 0 .$$

The Newton–Raphson technique in one variable is implemented as follows:

Given a function  $f$  defined over the reals  $x$ , and its byproduct  $f'$ , we start with a primary bet  $x_0$  for a root of the characteristic  $f$ . supplied the feature satisfies all of the assumptions made within the derivation of the formula, a higher approximation  $x_1$  is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} .$$

Geometrically,  $(x_1, 0)$  is the intersection with the xaxis of the tangent to the graph of  $f$  at  $(x_0, f(x_0))$ .

The process is repeated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

till a sufficiently accurate price is reached.

**Steps**

1. Initial guess
2. Using the formula mentioned above calculate the next value of x
3. Check if x is the root of the function or is in the range of affordable error. In other words check if  $f(x)=0$  or  $|f(x)| < \text{affordable error}$ . Repeat step 2 if not. 3 (option b) If the formula mentioned above gives the same result, x is the root of the polynomial.

**EXAMPLES**

Cube root of a number

Consider the problem of finding the square root of a number. Newton's method is one of many methods of computing square roots.

**For example**

Let's say we're trying to find the cube root of 3. And let's say that x is the cube root of 3. Therefore,

$$x^3=3$$

For the Newton-Raphson method to be able to work its magic, we need to set this equation to zero.

$$x^3-3=0$$

Now we will recall the iterative equation for NewtonRaphson.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Substituting for  $f(x)=x^3-3$  gives us:

$$x_{n+1} = x_n - \frac{(x_n)^3 - 3}{3 \cdot (x_n)^2}$$

Now, we pick an arbitrary number, (the closer it actually is to  $3\sqrt[3]{3}$  the better) for  $x_0$ . Let's use  $x_0=0.5$ . Then we substitute each previous number for  $x_n$  back into the equation to get a closer and closer approximation to a solution of  $x^3-3=0$ .

$$\begin{aligned}x_1 &= 0.5 - \frac{(0.5)^3 - 3}{3 \cdot (0.5)^2} = 4.33333\bar{3} \\x_2 &= x_1 - \frac{(x_1)^3 - 3}{3 \cdot (x_1)^2} \approx 2.94214333 \\x_3 &= x_2 - \frac{(x_2)^3 - 3}{3 \cdot (x_2)^2} \approx 2.07695292 \\x_4 &= x_3 - \frac{(x_3)^3 - 3}{3 \cdot (x_3)^2} \approx 1.61645303 \\x_5 &= x_4 - \frac{(x_4)^3 - 3}{3 \cdot (x_4)^2} \approx 1.46034889 \\x_6 &= x_5 - \frac{(x_5)^3 - 3}{3 \cdot (x_5)^2} \approx 1.44247296 \\x_7 &= x_6 - \frac{(x_6)^3 - 3}{3 \cdot (x_6)^2} \approx 1.4422496 \\x_8 &= x_7 - \frac{(x_7)^3 - 3}{3 \cdot (x_7)^2} \approx 1.44224957\end{aligned}$$

You can see that with only 8 iterations, we've obtained an approximation of  $\sqrt[3]{3}$  which is correct to 8 decimal places!

You can apply this same logic to whatever cube root you'd like to find, just use  $x^3-a=0$  as your equation instead, where a is the number whose cube root you're looking for.

**Example**

Let's approximate the root of the following function with Newton Raphson Method

$$f(x) = e^{-x} - x$$

Solution-

$$f(x) = e^{-x} - x$$

$$\frac{df}{dx} = -e^{-x} - 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n + \frac{e^{-x_n} - x_n}{e^{-x_n} + 1}$$

error < 0.05%

$$x_{n+1} = x_n + \frac{e^{-x_n} - x_n}{e^{-x_n} + 1}$$

	$x_n$	$x_{n+1}$	error
1	0	0.5	
2	0.5	0.5663	
3	0.5663	0.5671	0.0014
4	0.5671	0.5671	0.0000

The relative error is 0 because we have found the exact root and a function.

**3. CONVERGENCE OF NEWTON RAPHSON METHOD**

Suppose  $m_r$  is a root of  $g(m) = 0$  and  $m_k$  is an sortilege of  $m_r$  such that  $|m_r - m_k| = \delta < 1$  and there the Taylor's series expansion:

$$0 = g(m_r) = g(m_k + \delta) = g(m_k) + g'(m_k)(m_r - m_k) + \frac{g''(\xi)}{2}(m_r - m_k)^2 + \dots$$

For others, the spectrum  $\xi\xi$  is between  $m_k$  and  $m_r$ , The Newton Raphson Method system say that that We have,

$$m_{k+1} = m_k - \frac{g(m_k)}{g'(m_k)}$$

*i.e.*  $g(m_k) = g'(m_k) (m_k - m_{k+1}) \dots\dots\dots$

Inside the univariate price-head loss ratio criterion method correction to assumed

Since index of  $E_K$  is 2

Hence, the Newton Raphson methods of convergence is 2

The given equation suggests that if the following conditions are fulfilled, the rate of convergence is at least quadratic.

- i.  $g'(m) \neq 0$ ; for all m belongs to I, the place I is the interval  $[\alpha - r, \alpha + r]$  for some r,  $r \geq |\alpha - m_0|$
- ii. For any m belonging to I,  $G''(x)$  is continuous.
- iii.  $m_0$  Is close enough to the root  $\alpha$ .

Quadratic convergence is a property of the Newton-Raphson method

**NOTE:** Inside the one of a kind hand, the constant – factor precept can be used to prove the quadratic convergence of the Newton-Raphson procedure.

#### APPLICATIONS:

Newton-Raphson technique is extensively used for analysis of glide in water distribution networks. Numerous efficient computer packages, the use of Newton-Raphson approach, also are available for analysis of flow in big length networks. but, green pc software for most reliable layout of multi-source looped water distribution networks are now not comfortably to be had. Bhawe (1978) advised a univariate technique, referred to as value-head loss ratio criterion method, for no computer optimization of water distribution networks. cost-head loss ratio criterion technique is comparable to Hardy go Head Correction approach with the difference that assumed heads are successively corrected to satisfy value-head loss ratio criterion as opposed to pleasing means of thinking about HGL values at different nodes as consistent. These consequences in simplification of the problem for solution through hand calculation at the cost of increase in wide variety of iterations for final solution. Considering the fact that excessive velocity computers are now without problems available, Newton-Raphson technique is proposed for acquiring corrections to assumed HGL values [11]. NR technique is used to remedy simultaneous non-linear equations iteratively. It expands the non-linear phrases in Taylor's series, neglects the residues after terms and thereby considers best the linear terms (Bhawe 1991) [6]. thus, NR approach linearizes the non-linear equations through partial differentiation and solves. Naturally, the solution is approximate, and consequently is successively corrected. The iterative process is sustained until best accuracy is reached [7]. Thus, even as applying NR approach for acquiring correction in fee-head loss ratio criterion approach, all correction equations would be taken into consideration simultaneously and solved at a time.

$$0 = g'(m_k) ((m_r - m_{k+1}) + \frac{g''(\xi)}{2} (m_r - m_k)^2$$

Say,

$$E_k = (m_r - m_k), E_{k+1} = (m_r - m_{k+1});$$

where  $E_k, E_{k+1}$  denote the deviations in the equation at it

$$\text{Therefore, } E_{k+1} = -\frac{g''(\xi)}{2g'(m_k)} E_k^2$$

$$\rightarrow E_{k+1} \propto E_k^2$$

Node-flow continuity equation. The univariate price-head loss ratio criterion technique is changed here in for rapid convergence through the application of Newton-Raphson method. Any to be had software for evaluation of water distribution community using Newton-Raphson method may be without difficulty upgraded for best design of water distribution networks.

HGL Fee at every node is acquired independently by

From the referenced studies papers we've concluded that the ,The convergence charge of Newton technique is fast in comparison to different strategies .but the current injection method has easy Jacobian matrix and smaller computation in each iteration, that can make the programming simpler and decrease the time of the computation Secant approach is the most powerful method it has a converging rate near that of the Newton Raphson approach however it calls for handiest a single characteristic evaluating in step with iteration.

We've got additionally researched that the convergence price of bisection method may be very gradual and it's difficult to extend such sort of structures equations. So in assessment Newton approach have a quick converging charge [10]. The effectiveness of the use of clinical calculator in fixing non-linear equations using Newton Raphson approach additionally reduces the time complexity for fixing nonlinear equations. With the help of the built in spinoff features in calculator now we will capable of calculate the nonlinear capabilities quicker.

This research additionally suggests that the common mistakes made by means of the contributors had been reduced after they were taught the technique to resolve the trouble the use of the calculator. On this work a series of i t e r a t i v e techniques for solving nonlinear equation  $f(x) = 0$  with higher-order convergence is advanced. The method can be constantly carried out to generate an iterative scheme with arbitrarily special order of convergence.

We additionally concluded that the Newton Raphson method can be used very successfully to decide the intrinsic value based totally on its measured permittivity.

**REFERENCES:**

- [1] “Ehiwario J.C &Aghamie S.O”, Comparative Study of Bisection, Newton Raphson and Secant Methods of Root Finding Problems, Volume no 04, Issue no 04, April 2014.
- [2] “Tan Tingting”, “ Li Ying” & “ Jiang Tong”, The Analysis of the Convergence of Newton-Raphson Method Based on the Current Injection in Distribution Network Case”, Volume no 5, Issue no 03, June 2013.
- [3] “Changbum Chun”, Iterative method improving Newton’s method by the decomposition method, March 2005.
- [4] “Cheong Tau Han”, “Lim Kquatiaan Boon” & “ Tay Kim Gaik”, Solving Non Linear Equation by Newton-Raphson Method using Built-in Derivative Function in Casio fx-570ES Calculator, Publication: UniversitiTeknologi Mara, 2 UniversitiPendidikan Sultan Idris, UniversitiTun Hussein Onn Malaysia.
- [5] “MinetadaOsano” & “Miriam A. M Capretz”, A Distributed Method for Solving Nonlinear Equations Applying the Power Load Flow Calculation, Publication: Fukushima, 965-80 Japan.
- [6] “NICHOLAS J. HIGHAM” & “HYUNMIN KIM”, Numerical Analysis for a quadratic matrix Equations, Publication: 5 August 1999 from 13 December 1999.
- [7] “S.W.Ng” & “Y.S.Lee”, Variable Dimension Newton–Raphson Method, volume no 47, Issue no 6, June 2000.
- [8] “R.S.Maciel”, “A.Padilha-Feltrin” & “E.Righeto”, Substitution-Newton Raphson Method forthe Solution of Electric Network Equations, 1-4244-0288- 3/06/\$20.00(©2006IEEE.
- [9] “J.J Yang” & “D.M Zhang”, Newton Raphson method to determine the intrinsic permittivity of XLPE Cable.
- [10] Sawarkar, V. (2002). “Optimal design of water distribution networks”, Thesis submitted to Nagpur University for the award of Master’s Degree in Civil Engineering
- [11] Bhave, P. R. (1985). "Optimal expansion of water distribution systems." J. Environ. Engrg., ASCE, 111(2), 177195
- [12] Higher Engineering Mathematics 44<sup>th</sup> Edition by B.S. Grewal.