IJIREEICE ISSN (O) 2321-2004, ISSN (P) 2321-5526

International Journal of Innovative Research in Electrical, Electronics, Instrumentation and Control Engineering

Impact Factor 7.047 ⁘ Vol. 10, Issue 2, February 2022

DOI: 10.17148/IJIREEICE.2022.10204

APPLICATION OF NUMERICALS INTEGRATION IN SOLVING A REVERSE OSMOSIS MODEL

Prof .Vishal V. Mehtre[1], Abhishek Ranjan[2]

Assistant Professor, Department of Electrical Engineering, Bharati Vidyapeeth(Deemed to be University),

College of Engineering, Pune, India[1]

Department of Electrical Engineering, Bharati Vidyapeeth (Deemed to be University),

College of Engineering, Pune, India[2]

Abstract: Clean water is continually wanted for human residing. due to pollutants, we frequently want to purify water. One manner to do so is the use of the opposite osmosis machine. A mathematical version for the opposite osmosis system has been acquired. in this paper, we display the significance of numerical strategies in fixing the opposite osmosis version. in particular, we cognizance on the application of numerical integration techniques inside the process of solving the model. We recall three kinds of rules in numerical integration, specifically, the Riemann sums, the trapezoidal rule and the Simpson's rule. We present our studies results of those 3 rules referring to the fixing procedure of our reverse osmosis version. The Simpson's rule is the maximum correct, as it has the best order of accuracy in contrast to the Riemann sums and the trapezoidal rule. Our major point in this studies is that the numerical integration has an essential role in fixing the opposite osmosis version.

1. INTRODUCTION:

Water is one supply of lifestyles for residing matters in the world. however, some to be had water has now been heavily polluted through numerous styles of waste and rubbish from the outcomes of human sports
[1]. consequently, a technique is needed to filter out present water. one of the water purification strategies is the reverse osmosis gadget. The opposite osmosis system is the process of separating and eliminating dissolved, organic, pyrogenic, colloidal submicrons, colorations, nitrates, and bacteria from water the usage of a semipermeable membrane
[2]. In reverse osmosis the high strain is given to the focused side of the membrane. when stress is applied to this side, natural water will waft through the semipermeable membrane closer to the opposite aspect of the lower concentration
[3].The technique of distribution of attention with space and time is defined by way of parabolic type partial differential equations called advection-diffusion equations. The advection-diffusion equation is a model that may be used to simulate the spread of pollutants
[4]. This mathematical version for the reverse osmosis gadget has been acquired
[5].In this paper, we display the importance of numerical integration in solving the opposite osmosis mathematical version. Numerical integration has many applications in the discipline of carried out mathematics, specifically in mathematical physics and computational chemistry
[6]. Numerical integration is a way for calculating integrals that are hard to solve analytically. We evaluate 3 styles of guidelines in numerical integration, the Riemann sums, the trapezoidal rule, and Simpson's rules. three styles of Riemann sums are the left Riemann sum, the right Riemann sum, and the middle Riemann sum.
some authors (see [7, 8, 9]) have applied some of the three numerical integration policies to find solutions to a mathematical model. popular formula for the Riemann sum rule is given in [10]:

$$\int_a^b f(x)dx = \sum_{i=1}^{n} f(x_i^*)\,\Delta x \qquad i$$

with $x_i^* = x_i$ for the right Riemann sum, $x* = xi–1$ for the left Riemann sum, and $x* = (xi + xi–1)/2$ for the center Riemann sum. the overall components for the trapezoidal rule is given by way of [10]:

$$\int_a^b f(x)dx = \frac{h}{2}\left(f_o + 2\sum_{i=1}^{n-1} f_i + f_n\right)$$

Where $f_i = f(x_i)$, $x_o = a$, and$x_n = b$. the general formulation for the Simpson's rule is given by way of [10]:

International Journal of Innovative Research in Electrical, Electronics, Instrumentation and Control Engineering

Impact Factor 7.047 ⁙ Vol. 10, Issue 2, February 2022

DOI: 10.17148/IJIREEICE.2022.10204

$$\int_a^b f(x)dx = \frac{h}{3}(f_o + 4\sum_{i=1}^{M} f_{2i-1} + 2\sum_{i=1}^{M-1} f_{2i} + f_{2M}$$

In which n = 2M and M is a high-quality integer. in this paper, we additionally calculate errors from the five rules of numerical integration.
The relaxation of this paper consists of the mathematical version, application of numerical integration, discussion of consequences, and conclusions.

II. MATHEMATICAL VERSIONS

The mathematical version for predicting the attention of salt solutions in semipermeable membranes inside the opposite osmosis gadget is as follows [5]:

$$Y\frac{\partial C}{\partial x} = \alpha\frac{\partial^2 C}{\partial y^{2\prime}}$$

with $\alpha = \frac{Dh}{v_o}$ and the boundary situations are
$C(0,y) = c_o, C(x,\propto) = c_0$
and

$$-D\frac{\&C}{\&y}(x, 0) = qC(x, 0) \quad \text{----- (6)}$$

in which x and y are area variables. C = C(x, y) represents the awareness of salt solution in a semipermeable membrane at factor (x, y). Notations q, D, h, c_0, and v_o are, respectively, water go with the flow prices in semipermeable distribution, salt diffusivity in water, distance from the semipermeable boundary to the center of the channel, attention faraway from semipermeable membranes, and horizontal pace measured at a distance h from the semipermeable boundary. here q, D, h, and v_o are constants.

III. APLLICATION OF OF NUMERICAL INTEGRATION

The answers of equations (4), (5), and (6) are as follows [5]:

$$C(x,0) = \frac{3^{-\frac{1}{3}}qc_0}{DI}\left(\frac{Dh}{v_0}\right)^{1/3} x^{\frac{1}{3}} + c_0,$$

with $= \int_0^\infty e^{-v^3} dv$. Equation (7) is an equation this is used to are expecting the attention of salt answers in semipermeable membranes within the opposite osmosis device.

In equation (7) there's a specific critical I which could be very hard to solve analytically. therefore, we resolve the specific crucial I the usage of the left Riemann sum rule, proper Riemann sum rule, and middle Riemann sum rule, trapezoidal rule, and Simpson's rule.

In all calculations the same c program language period is used, specifically [0, 10]. This c program language period [0, 10] is partitioned into n = 200 subintervals namely $[v_o, v_1]$, $[v_1, v_2]$, $[v_2, v_3]$, … , $[v_{199}, v_{200}]$ with v_o=0 and v_n = 10 where $\Delta v = \frac{10-0}{200} = 0.05$
is uniform, as shown in Fig. 1.

International Journal of Innovative Research in Electrical, Electronics, Instrumentation and Control Engineering

Impact Factor 7.047 ⪤ Vol. 10, Issue 2, February 2022

DOI: 10.17148/IJIREEICE.2022.10204

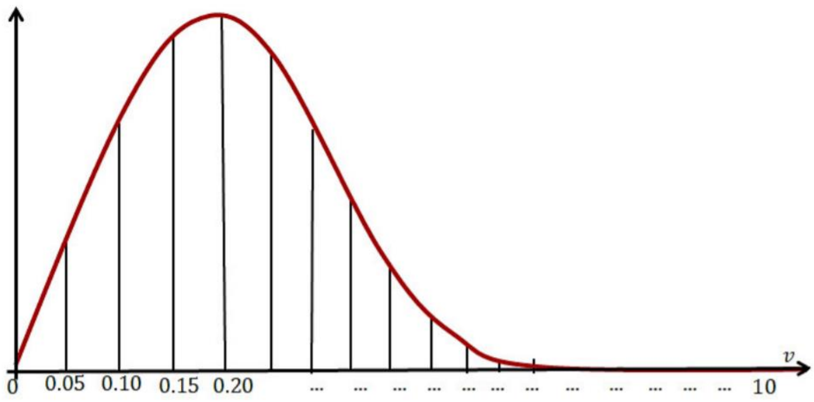

FIGURE 1. Graph of function $f(v)$

Right Riemann Sum

Based totally on the proper Riemann sum formulation, vi = 0.05 is chosen as the right give up factor of the c program language period [0,10] with i = 0.05, 0.10, 0.15, … , 200, in order that we gain

$$\int_0^{10} e^{-v^3} v\,dv \approx \sum_{i=0.05}^{2000} (e^{-v_i{}^3} v_i)(0.05)$$
$$\approx f(x_0^*)\Delta v + f(x_1^*)\Delta v + \cdots + f(x_n^*)\Delta v$$
$$\approx f(0.05)(0.05) + f(0.10)(0.05) + \cdots + f(10)(0.05)$$
$$\approx (e^{-0.05^3}0.05)(0.05) + (e^{-0.10^3}0.10)(0.05) + \cdots + (e^{-100^3}10)(0.05)$$
$$\approx (0.04999375)(0.05) + (0.09990005)(0.05) + \cdots + (0)(0.05)$$
$$\approx (0.002499688) + (0.004995002) + \cdots + (0)$$
$$\int_0^{10} e^{-v^3} v\,dv \approx 0.4511643131.$$

For that reason, the approximation of the I cost is received the use of the proper Riemann sum rule, that is 0.4511643131.

Left Riemann Sum

Primarily based at the right Riemann sum formula, vi = 0.05 is selected as the right end point of the c program language period from each subinterval [0,10] with i = zero.05, 0.10, 0.15, … , 200, in order that we reap

$$\int_0^{10} e^{-v^3} v\,dv \approx \sum_{i=0}^{199} (e^{-v_i{}^3} v_i)(0.05)$$
$$\approx f(v_0^*)\Delta v + f(v_1^*)\Delta v + \cdots + f(v_{n-1}^*)\Delta v$$
$$\approx f(0)(0) + f(0.05)(0.05) + \cdots + f(9.95)(0.05)$$
$$\approx (e^{-0^3}.0)(0.05) + (e^{-0.05^3}0.05)(0.05) + \cdots + (e^{-9.95^3}9.95)(0.05)$$
$$\approx (0)(0.05) + (0.04999375)(0.05) + \cdots + (0)(0.05)$$
$$\approx (0) + (0.002499688) + (0.004995002) + \cdots + (0)$$
$$\int_0^{10} e^{-v^3} v\,dv \approx 0.4511643131.$$

as a result, the approximation of the I price is received the use of the left Riemann sum rule, that's 0.4511643131.

Middle Riemann Sum

Impact Factor 7.047 ⋇ Vol. 10, Issue 2, February 2022
DOI: 10.17148/IJIREEICE.2022.10204

Within the middle Riemann sum rule, $\bar{v}_i$ is chosen because the midpoint of every subinterval at [0,10] with i = 0, 1, 2, … , 200,in order that primarily based at the middle Riemann sum we acquire

$$\int_0^{10} e^{-v^3} v\,dv \approx \sum_{i=0}^{200} f(\bar{v}_i)\Delta v$$

with $\Delta v = 0.05$ and $\bar{v}_i = \frac{1}{2}(v_{i-1} + v_i)$, as listed in Table 1.

TABLE 1. Value of $\bar{v}_i$ for summation of middle Riemann

v_{i-1}	v_i	$\bar{v}_i$
0	0.05	0.025
0.05	0.10	0.075
0.10	0.15	0.125
…	…	…
9.95	10	9.975

furthermore, the price of $\bar{v}_i$ in desk 1 is substituted to the middle Riemann sum formula as follows

$$\begin{aligned}\int_0^{10} e^{-v^3} v\,dv &\approx \sum_{i=0}^{200} (e^{-v_i^3} v_i)(0.05)\\ &\approx f(v_0^*)\Delta v + f(v_1^*)\Delta v + f(v_2^*)\Delta v + \cdots + f(v_{n-1}^*)\Delta v\\ &\approx f(0.025)(0) + f(0.075)(0.05) + \cdots + f(9.975)(0.05)\\ &\approx (e^{-0.025^3}0.025)(0.05) + (e^{-0.075^3}0.075)(0.05) + \cdots + (e^{-9.975^3}9.975)(0.05)\\ &\approx (0.024999609)(0.05) + (0.074968366)(0.05) + \cdots + (0)(0.05)\\ &\approx (0.00124998) + (0.003748418) + \cdots + (0)\\ &\approx 0.451476813.\end{aligned}$$

for this reason, the approximation of the I fee is obtained the usage of the center Riemann sum rule, which is 0.451476813.

Trapezoidal Rule

Primarily based on the trapezoidal rule system, at c program languageperiod [0,10] that's divided into two hundred durations the section [vO, v2OO] as huge as Δv = zero.05 through the use of a partition point this is equidistant, that is, $v_i = 0 + i\Delta v$, i = 0, 1, 2, … , 200, we attain

$$\begin{aligned}\int_0^{10} e^{-v^3} v\,dv &\approx \frac{\Delta v}{2}\left(f(v_0) + 2\sum_{i=1}^{n-1} f(v_i) + f(v_{200})\right)\\ &\approx \frac{0.05}{2}(f(0) + 2f(0.05) + 2f(0.10) + \cdots + f(10))\\ &\approx \frac{0.05}{2}\left((e^{-0^3}0) + 2e^{-0.05^3}0.05 + \cdots + (e^{-10^3}10)\right)\\ &\approx \frac{0.05}{2}((0) + 2(0.04999375) + \cdots + (0))\\ &\approx 0.4511643131.\end{aligned}$$

Consequently, the approximation of the I price is received by using the trapezoidal rule of zero.4511643131.

Simpson's Rule

Primarily based at the Simpson's rule formula 1⁄3, for the c language [0,10] is divided into 2M = 200 intervals, the section $[v_{o,}v_{200}]$ is same to that of Δv = zero.05 and uses partition factors which can be equidistant, that is, $v_i = 0 + ih$, i = zero, 1, 2, … , 2M, then

DOI: 10.17148/IJIREEICE.2022.10204

$$\begin{aligned}\int_0^{10} e^{-v^3} v\,dv &\approx \frac{\Delta v}{3}\left(f_0 + 4\sum_{i=1}^{M} f_{2i-1} + 2\sum_{i=1}^{M-1} f_{2i} + f_{200}\right)\\ &\approx \frac{0.05}{3}\left(f_0 + 4\sum_{i=1}^{100} f_{2i-1} + 2\sum_{i=1}^{99} f_{2i} + f_{200}\right)\\ &\approx \frac{0.05}{3}(f_0 + 4f_1 + 2f_2 + \cdots + 4f_{199} + f_{200})\\ &\approx \frac{0.05}{3}(e^{-0^3}0 + 4e^{-0.05^3}0.05 + \cdots + 4e^{-9.95^3}9.95 + e^{-10^3}10)\\ &\approx (0.016666667)(27.08235879)\\ \int_0^{10} e^{-v^3} v\,dv &\approx 0.4513726464.\end{aligned}$$

As a result, the approximation of the I value is acquired the use of the Simpson 1/3 rule, which is zero.4513726464.

IV. DISCUSSION

On this section, we talk the accuracy of calculating the 5 rules of numerical integration above. the precise fee of I is received the use of the Maple application, as follows:

$I = \int_0^{\infty} e{-}v^3 v\,dv = 0.45137264563$

based totally on the outcomes of the calculation of the definite imperative I in phase three, the mistake of the left Riemann sum rule, the right Riemann sum rule, the middle Riemann sum rule, the trapezoidal rule, and the Simpson's rule are given in Table 2.

TABLE 2. Results of each numerical integration rule

Integration Rule	Exact of Maple	Numerical results	Error
Left Riemann sum rule	0.4513726463	0.4511643131	0.0002083332
Right Riemann sum rule	0.4513726463	0.4511643131	0.0002083332
Middle Riemann sum rule	0.4513726463	0.4514768131	0.0001041668
Trapezoidal rule	0.4513726463	0.4511643131	0.0002083332
Simpson's rule	0.4513726463	0.4513726464	0.0000000001

Based totally on the mistake calculation in desk 2, the mistake of the Simpson's rule is smaller than those of the alternative four numerical integration rules. that is because the Simpson's rule has a better order of accuracy than the opposite four numerical integration guidelines. as a result, we use the value I of the Simpson's rule because the most correct I fee. This price of I is substituted to equation (7), so that we obtain

$$\frac{C(x,0)}{c_0} = 1.5361171751\left(\frac{q}{D}\right)\left(\frac{Dh}{v_0}\right)^{1/3} x^{\frac{1}{3}} + 1\,.$$

Equation (8) is a mathematical equation to are expecting the attention of salt solutions in semipermeable membranes within the opposite osmosis gadget.

V. CONCLUSION

Numerical integration has an crucial role in solving the reverse osmosis model. The more correct the the critical value that we reap, the more correct effects of the mathematical model we are able to get. based totally at the calculation of the numerical integration, the Simpson's rule is the maximum accurate rule compared to the left Riemann sum rule, proper Riemann sum rule, center Riemann sum rule, and trapezoidal rule.

VI. ACKNOWLEDGMENTS

A number of this work become a part of the first writer's thesis [11] written under the supervision of the second author. fees of the studies of the primary creator have been included by Sanata Dharma college. costs of the work of the second one creator had been blanketed by using Direktorat Riset dan Pengabdian Masyarakat, Direktorat Jenderal Penguatan Riset dan Pengembangan, Kementerian Riset, Teknologi, dan Pendidikan Tinggi of The Republic of Indonesia with contract wide variety DIPA-042.06.1.401516/2019.

Impact Factor 7.047 ⋡ Vol. 10, Issue 2, February 2022

DOI: 10.17148/IJIREEICE.2022.10204

VII. REFERENCES

[1] D. Sulaeman, S. Arif and Sudarmadji, "Trash-polluted irrigation: characteristics and impact on agriculture," IOP Conference Series: Earth and Environmental Science, **148**, 012028 (2018).

[2] R. M. Garud, S. V. Kore, V. S. Kore and G. S. Kulkarni, "A short review on process and applications of reverse osmosis," Universal Journal of Environmental Research and Technology, **1**, 233–238 (2011).

[3] S. J. Wimalawansa, "Purification of contaminated water with reverse osmosis: effective solution of providing clean water for human needs in developing countries," International Journal of Emerging Technology and Advanced Engineering, **3**, 75–89 (2013).

[4] F. Sanjaya and S. Mungkasi, "A simple but accurate explicit finite difference method for the advection-diffusion equation," Journal of Physics: Conference Series, **909**, 012038 (2017).

[5] G. R. Fulford and P. Broadbridge, Industrial Mathematics: Case Studies in the Diffusion of Heat and Matter (Cambridge University Press, Cambridge, 2002).

[6] D. H. Baileyaa and J. M. Borwein, "High-precision numerical integration: progress and challenges," Journal of Symbolic Computation, **46**, 741–754 (2011).

[7] M. Aigo, "On the numerical approximation of Volterra integral equations of second kind using quadrature rules," International Journal of Advanced Scientific and Technical Research, **1**, 558–564 (2013).

[8] J. Li and D. H. Yu, "Error expansion of classical trapezoidal rule for computing Cauchy principal value integral," Computer Modeling in Engineering & Sciences, **93**, 47–67 (2013).

[9] B. Grier, E. Alyanak, M. White, J. Camberos and R. Figliola, "Numerical integration techniques for discontinuous manufactured solutions," Journal of Computational Physics, **278**, 193–203 (2014).

[10] J. Stewart, Calculus, Eight Edition (Cengage Learning, Boston, 2016).

[11] O. P. Maure, Aspek Matematis dan Aspek Pendidikan pada Suatu Model Pemurnian Air dalam Sistem Osmosis Terbalik (Unpublished master's thesis in Indonesian language, Sanata Dharma University, Yogyakarta, 2019).