



Direction of Arrival Estimation for 1D Antenna Array Using MUSIC Algorithm

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Abstract: Promoting wireless communication by estimating the direction of arrival of multiple waves incident on the sensor array is a well-known problem in array signal processing. DOA estimation is a widely studied issue in many fields, including wireless communications, astronomical observations, radar, and sonar. One of the main research trends of DOA estimation is to improve the accuracy and super-resolution, and improve the adaptability to harsh scenes such as limited snapshots and low signal-to-noise ratio. Various methods have been proposed to meet these requirements, such as beamformers, subspace-based methods, scarcity induction methods, and maximum likelihood methods. The performance estimated by the DOA has undergone lasting development. In this work, the method based on the MUSIC subspace is used for the estimation of DOA. The subspace-based technique is based on the use of the characteristic structure of the data covariance matrix. The purpose is to analyze the DOA estimation algorithm under low signal-to-noise ratio, array element spacing, number of array elements, and changes in the number of snapshots, signal incidence angle differences, and coherent signals. The performance of the subspace-based DOA estimation algorithm is performed on a 1D linear array. The simulation results show that the influence of different parameters will affect the DOA estimate. The simulation results show that the MUSIC algorithm is accurate in recognizing signals.

Keywords: DoA, MUSIC, ULA, Antenna

INTRODUCTION

Array signal processing involves collecting, processing, enhancing useful signals, preventing or avoiding noise signals, and estimating certain signal parameters to perform different tasks, such as source location, estimation, and separation. Sensor array signal processing has always been an important part of analyzing the data received by the sensor [1][2]. Direction of Arrival (DOA) estimation is an important application of matrix signal processing, and this field has proven to be one of the most important research fields. In radar applications, they can be used for air traffic control and target acquisition. Intelligence agencies use them for secret transmission positioning and signal interception [3]. An example of a business application is to identify the direction of an emergency mobile phone call so that the rescue team must be dispatched to the right place at the right time. In signal processing, the direction of arrival (DOA) refers to the process of retrieving the direction information of various electromagnetic waves from the output of the various receiving antennas that constitute the sensor array. Basically, it involves estimating the direction/angle of an electromagnetic source relative to a set of receivers [4].

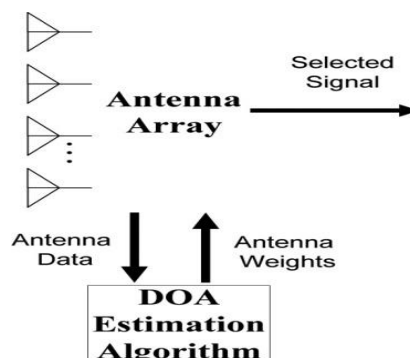


Fig..1. Direction of Arrival (DoA) Estimation



The operating principle of the MUSIC algorithm is to perform its own decomposition on the covariance matrix of any output data matrix to generate a signal subspace orthogonal to the noise subspace corresponding to the signal component [6]. The noise subspace and the signal subspace form a spectrum function, and the DOA of the signal is detected by looking for the peak of the spectrum. The MUSIC algorithm has attracted many scholars for its high resolution, precision, and stability. It also has the advantages of measuring multiple signals with high precision and accuracy, and can perform real-time processing after using high-speed processing technology [6].

The multiple signal classification algorithm (MUSIC) was proposed by Schmidt et al. In 1979, ushering in a new era of spatial spectrum estimation algorithms. Promotion and development characterized by structural algorithms has become the key algorithm of the spatial spectrum theory system. Before this algorithm was proposed, some related algorithms directly processed the data received from the covariance matrix of the matrix.

I. DATA MODEL

AN Consider K complex-valued narrow-band signals striking an antenna array composed of N isotropic elements. It is assumed that the source emitting these signals is in the far field of the array and that the antenna elements of the transmitter and receiver are co-polarized. $x_n(t)$ is the sample received by the nth element, that is, $n = 1, \dots, N$, in the nth time instance, the data vector $X(T) = [X_1(T), \dots, X_N(T)]^T$ IS MODELEDD AS

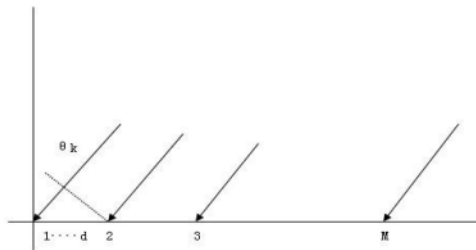


Fig.2. Uniform Linear array

$$X(t) = As(t) + n(t) \quad (1)$$

Where $x(t) \in \mathbb{C}^N$, $A \in \mathbb{C}^{N \times K}$ is the array manifold, $s(t) \in \mathbb{C}^K$ is a vector containing complex amplitudes of the transmitted signal and $n(t) \in \mathbb{C}^N$ is a vector containing the additive noise per antenna element.

The array manifold is given by

$$A = [a_1, a_2, \dots, a_K] \quad (2)$$

$a_k \in \mathbb{C}^N$ is the steering vector associated with the kth source, i.e. $k = 1, \dots, K$. The kth steering vector is given by

$$a_k = [a_{1,k}, a_{2,k}, a_{3,k}, \dots, a_{N,k}]^T \quad (3)$$

and depends on the positions of the array elements relative to a reference point, the direction information of the signals, and the wavelength λ . The nth element of the kth steering vector is given by,

$$a_{n,k} = \frac{e^{-j2\pi r_n w_k}}{\lambda} \quad (4)$$

The vector $r_n \in \mathbb{R}^3$ contains the Cartesian coordinates of the nth array element R_{xn} relative to the reference point

$$r_n = [x_n, y_n, z_n]^T \quad (5)$$

and $w_k \in \mathbb{R}^3$ is composed of the Cartesian coordinates of the unit-vector pointing from the reference point towards the kth source T_{xk} . These Cartesian coordinates are computed from the azimuth angle ϕ_k and the elevation angle θ_k as follows

$$w_k = \begin{bmatrix} \cos \theta_k \cos \phi_k \\ \cos \theta_k \sin \phi_k \\ \sin \theta_k \end{bmatrix}$$

When T snapshots are available, i.e. $t = 1, \dots, T$, equation (1) can be written as the matrix equation

$$X = AS + N \quad (6)$$

with, $X = [x(1), \dots, x(T)]$, $S = [s(1), \dots, s(T)]$ and

$$N = [n(1), \dots, n(T)]$$



eigen decomposition of array covariance for array output x , corresponding calculations can get its covariance matrix R_x :

$$R_x = E[XX^H] \tag{7}$$

If we assume that noise at each sensor is uncorrelated and independent from both incident signals and noise at other sensors, then the array covariance matrix is written as:

$$\begin{aligned} R_{xx} &= E[x(t) \cdot x^H(t)] \\ &= A E[s(t) \cdot s^H(t)] A^H + E[n(t) \cdot n^H(t)] \\ &= ASA^H + \sigma^2 I \end{aligned} \tag{8}$$

To define spatial spectrum $P_{\text{mu}}(\theta)$ can be defined

$$P_{\text{mu}}(\theta) = \frac{1}{a^H(\theta)E_n E_n^H a(\theta)} = \frac{1}{\|E_n^H a(\theta)\|^2} \tag{9}$$

Thus, to define spatial spectrum, where the denominator of the formula is an inner product of the signal vector and the noise matrix. When $P_{\text{mu}}(\theta)$ is orthogonal with each column of E_n , the value of this denominator is zero, but because of the existence of the noise, it is actually a minimum. $P_{\text{mu}}(\theta)$ has a peak. By this formula, make θ change and estimate the arrival angle by finding the peak [3].

II. RESULTS

1. Basic simulation of the MUSIC algorithm for DOA estimation.

As shown in the figure, these two signals are generated by musical algorithms. There are two independent narrowband signals, the angles of incidence are 30° and 80° respectively, these two signals are uncorrelated

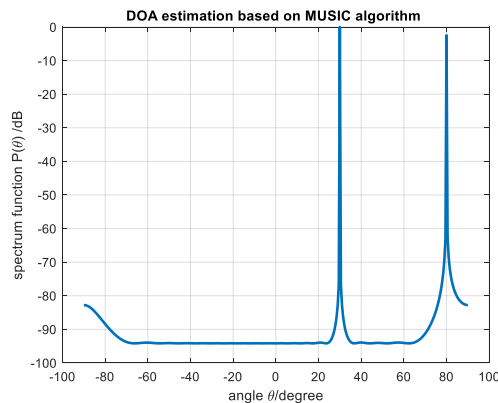


Fig.2 Basic simulation for MUSIC algorithm

Ideal white Gaussian noise has a signal-to-noise ratio of 50 dB, the cell spacing is half the wavelength of the input signal, the number of elements in the array is 20, and the number of snapshots is 1000.

As shown in Fig.2, under the assumption of two independent signals, the needle spectrum peak algorithm can be constructed using MUSIC. Using the MUSIC algorithm, the number and direction of the incident signal can be estimated, and this DOA (direction of arrival) can be effectively estimated. As a MUSIC algorithm, it overcomes all shortcomings because it provides high resolution and high precision for multiple signals, with better performance and higher efficiency.

2. The relationship between DOA estimation and the number of array elements.

Fig.3 shows that the two signals are generated using musical algorithms. There are two independent narrowband signals, the angles of incidence are 30° and 80° respectively. The two signals are uncorrelated and have ideal Gaussian white noise. The signal-to-noise ratio is 50 dB. Cell spacing is half the wavelength of the input. Different array elements Numbers 15,80,150, the number of snapshots (input) is 1000.

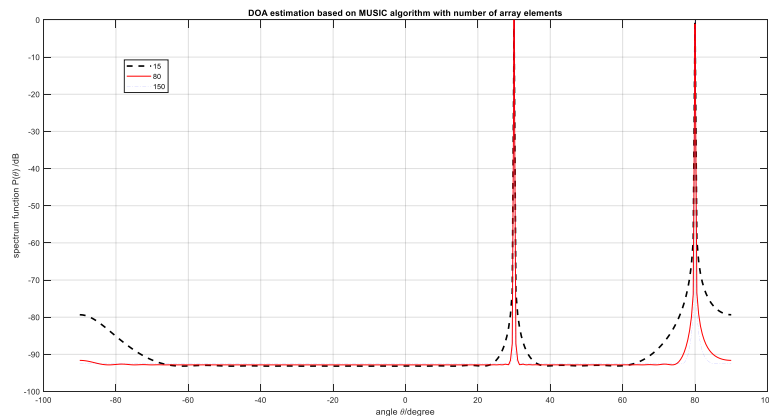


Fig.3 simulation for the relationship between MUSIC algorithm and the number of array elements

As shown in Fig. 3, the dotted line indicates that the number of elements is 15, the solid line indicates that the number of elements is 80, and the dotted line indicates that the number of elements is 150. With increasing number of elements array elements, the beamwidth of the DOA estimation spectrum becomes narrower, and the directivity of the array elements is accurate. A more accurate DOA estimate can be obtained by increasing the number of elements in the array.

3. The relationship between DOA estimation and the array element spacing.

The fig.4 shows that these two signals are generated using music algorithms. There are two independent narrowband signals, the incident angles are 30° and 80° respectively. These two signals are not correlated and have ideal Gaussian white noise. The signal-to-noise ratio is 50dB. The distance between different matrix elements is $\lambda/6$, $\lambda/2$.The number of elements in the array is 20, and the number of snapshots (input) is 1000.

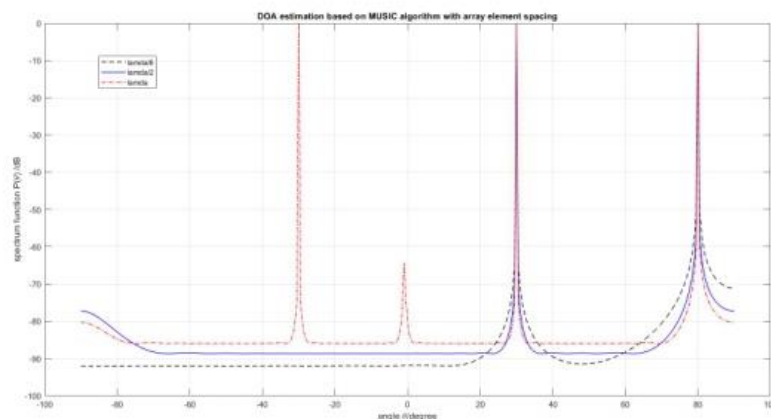


Fig.4. Simulation for the relationship between MUSIC algorithm and array element spacing

As shown in Fig.4, the dotted line indicates that the spacing of matrix elements is $\lambda/6$, the solid line indicates that the spacing of matrix elements is $\lambda/2$, and the dotted line indicates that the spacing of matrix elements is $\lambda/2$ and the matrix is λ . As the distance between the array elements increases, the beam width of the DOA estimation spectrum becomes narrower, and the directivity of the array elements becomes better. The resolution of the MUSIC algorithm increases with the increase of the element spacing, but is limited to half of the wavelength.

4. The relationship between DOA estimation and the number of snapshots.

As shown in the fig. 5, these two signals are generated using music algorithms. There are two independent narrowband signals with incident angles of 30° and 80° . These two signals are uncorrelated and have ideal Gaussian white noise.

The signal-to-noise ratio is 50dB. The cell spacing is half the length of the input signal waveform, The number of



elements in the array is 20 and the number of snapshots (input) is 10,80,300.

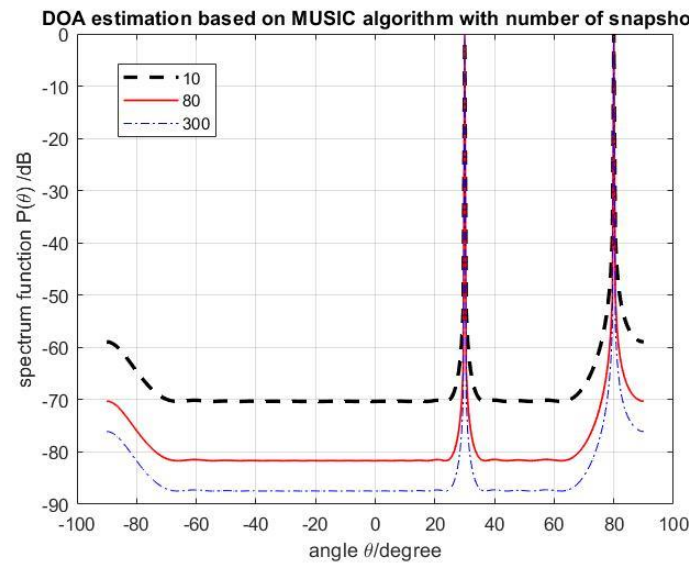


Fig.5 Simulation for relationship between MUSIC algorithm and the number of snapshots

As shown in Fig.5, the dotted line indicates that the number of snapshots is 10, the solid line indicates that the number of snapshots is 80, and the dotted line indicates that the number of snapshots is 300. The estimated spectrum becomes narrower, the directivity of the array element becomes better, and the accuracy of the MUSIC algorithm is also improved. Choose reasonable sample snapshots to ensure the accuracy of DOA estimates.

5. The relationship between DOA estimation and SNR.

As shown in the fig.6, these two signals are generated using music algorithms. There are two independent narrowband signals, the incident angles are 30° and 80° respectively. The two signals are not correlated, have ideal Gaussian white noise, and have different SNR values of 10dB, 0dB, and 10dB. The interval between components is 1/2 input signal.

Wavelength, the number of array elements is 20, and the number of snapshots (input) is 1000.

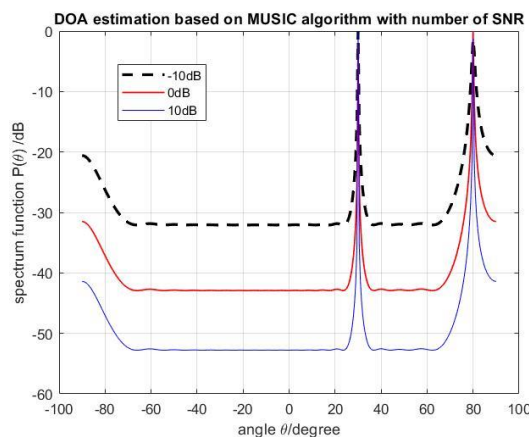


Fig.6 Simulation for the relationship between MUSIC algorithm and SNR

As shown in Fig.6, the dashed line indicates that the SNR is 10dB, the solid line indicates that the SNR is 0dB, and the dashed line indicates that the SNR is 10dB. As the number of signal-to-noise ratios increases, the beam width of the DOA estimation spectrum becomes narrower, the directivity of the array element becomes clearer, and the accuracy of the MUSIC algorithm also improves. SNR directly affects the performance of the high-resolution DOA estimation algorithm, reduces the SNR value performance of the MUSIC algorithm, and reduces the Sharpley.

6. The relationship between DOA estimation and the signal incident angle difference.



As shown in the fig.7, these two signals are generated by musical algorithms. There are two independent narrowband signals at different angles of incidence of 5100° and 200° . The two signals are uncorrelated and have ideal Gaussian white noise. The

SNR is 50dB. The space between elements is half the wavelength of the input signal. The number of array elements is 20 and the number of snapshots(inputs) is 1000

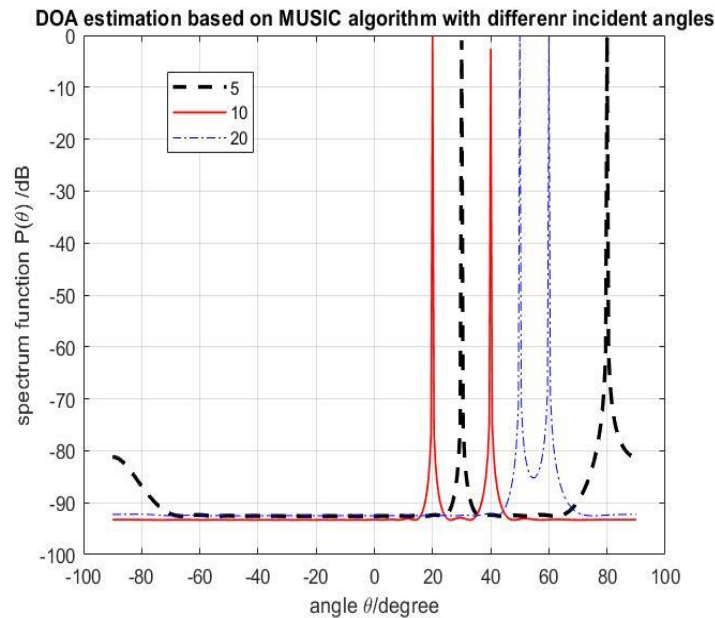


Fig.7 Simulation for the relationship between MUSIC algorithm and incident angle difference

As shown in Fig.7, the dotted line indicates that the incident angle is 5° , the solid line indicates that the incident angle is 10° , and the dotted line indicates that the incident angle is 20° . As the number of incident angles increases, the beam width of the DOA estimation spectrum becomes narrower, the directivity of the array element becomes clearer, and the resolution of the MUSIC algorithm increases. When the signal angle space is very small, the algorithm cannot estimate the number of signal sources.

7. The MUSIC algorithm and improved MUSIC algorithm for coherent signals.

As shown in the figure, these two signals are generated using music algorithms. There are two independent narrowband signals at different incident angles of 30° and 80° . The two signals are uncorrelated and have ideal Gaussian white noise. The SNR is 5dB, and the element spacing is half the wavelength of the input signal. The number of element is 150 and the number of snapshots is 1000.

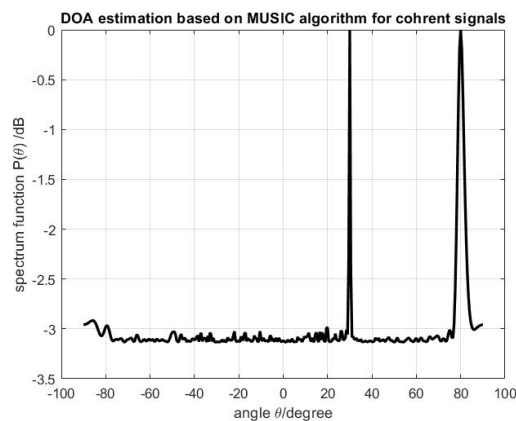


Fig.8 Simulation for MUSIC algorithm when the signals are coherent



8. Improved MUSIC algorithm

As shown in the fig.9 , these two signals are generated by musical algorithms. There are two independent narrowband signals at different angles of incidence of 30° and 80° . The two signals are uncorrelated and have ideal Gaussian white noise. The SNR is 50dB, and the element spacing is half the wavelength of the input signal. The number of elements in the array is 20 and the number of snapshots (input) is 1000.

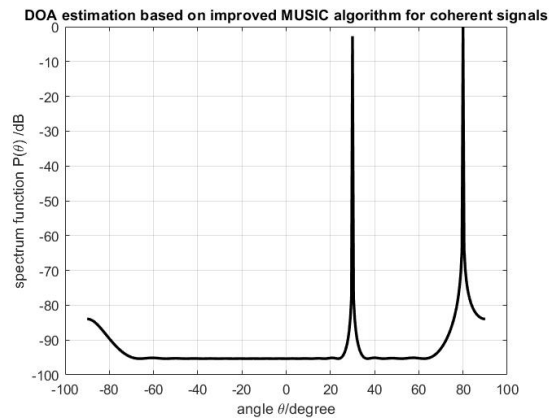


Fig.9 Simulation for the improved MUSIC algorithm when the signals are coherent

Fig.8 and Fig.9 show coherent signals. The improved MUSIC algorithm is better than the classic MUSIC algorithm, eliminating signal correlation, distinguishing coherent signals, and estimating angle of arrival more accurately. Using the MUSIC DOA algorithm you can achieve any high resolution, but the MUSIC algorithm focuses on uncorrelated signals. This deficiency is achieved by the improved MUSIC algorithm.

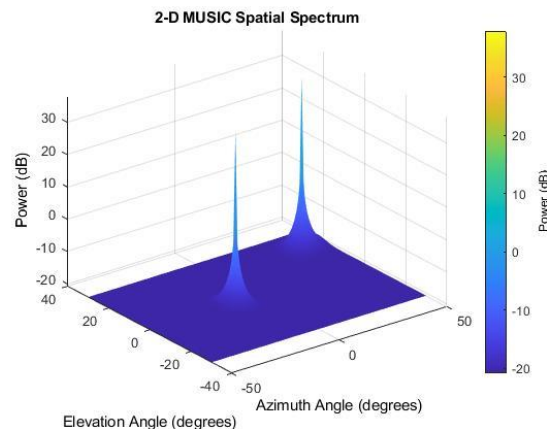


Fig 3.10 2-D Music Spatial spectrum

2-D MUSIC is used to estimate the directions of arrival of the two signals two sinusoidal waves strike a URA from two different directions. Signals arrive from -10° and 40° as shown in figure 3.10

III. CONCLUSION

The greater the number of elements in the array, the greater the number of snapshots; the greater the difference between the incident angles, the higher the resolution of the MUSIC algorithm. When the element spacing is not greater than half the wavelength, the resolution of the MUSIC algorithm increases with the increase of the element spacing; however, if the spacing of the matrix elements is greater than half the wavelength, the spatial spectrum will be in the direction other than the signal source False peaks are generated in the other direction. When moving a low SNR and a small incident angle difference, the performance of the MUSIC algorithm will degrade. Some scholars have proposed some improvements to the algorithm, but these issues are still hotspots of research. Therefore, the MUSIC algorithm still has a lot of room for development, and it is worthy of further study.



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