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Design of Fuzzy LQR

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Abstract: Fuzzy Logic Controller based Optimal Linear Quadratic Regulator (FC-LQR) for the control of a nonlinear CSTR system. The primary thought is to outline a supervisory fuzzy controller proficient to modify the controller parameters keeping in mind the end goal to get the coveted axes positions under varieties of the system parameters and load varieties. In the event that speed and precision are required, the control utilizing traditional techniques is hard to acknowledge in view of the high nonlinearity of the CSTR system. In control plan, it is regularly important to outline a controller to satisfy, in an optimal shape, certain performance criteria and requirements notwithstanding stability
Keywords: Fuzzy Logic Controller, FC-LQR, LMI, Stability.

I. INTRODUCTION

Fuzzy control has been pulling in expanding attention to the stabilization of nonlinear systems. The reason for an attraction of fuzzy control is that it empowers one to work in uncertain and equivocal arrangements and take care of not well postured problems or problems with deficient data. Fuzzy Logic Controller based Optimal Linear Quadratic Regulator (FC-LQR) for the control of a nonlinear CSTR system. The fundamental thought is to design a supervisory fuzzy controller fit to modify the controller parameters with a specific end goal to acquire the desired axes positions under variations of the system parameters and load variations. On the off chance that speed and precision are required, the control utilizing conventional strategies is hard to acknowledge due to the high nonlinearity of the CSTR system. In control design, it is frequently important to design a controller to satisfy, in an ideal shape, certain performance criteria and limitations notwithstanding soundness.

However, fuzzy controllers are essentially non-linear and sufficiently effective to provide the desired non-linear control activities via deliberately modifying their parameters. In this section, we propose an effective strategy to nonlinear ideal control in light of fuzzy control. The ideal fuzzy controller is designed by solving a minimization issue that restrains a given quadratic performance function. Both the controlled system and the fuzzy controller are represented to by the relative Takagi-Sugeno (T-S) fuzzy model contemplating the impact of the consistent term. The majority of the examination works broke down the T-S show expecting that the non-linear system is linearized regarding the cause in each IF-THEN rule, which implies that the consequent piece of each rule is a linear function with zero consistent terms. This will, thusly, diminish the precision of approximating non-linear systems. LQR is utilized to decide best values for parameters in fuzzy control rules in which the robustness is natural in the LQR in this manner robustness in fuzzy control can be improved. With the guide of LQR, it provides an effective design strategy for fuzzy control to guarantee robustness. In this section, we will indicate how the LQR, the structure of which depends on numerical investigation, can be made more proper for genuine usage by the presentation of fuzzy rules. The motivation behind this plan is to join the best highlights of fuzzy control and LQR to achieve quick and exact tracking control of a class of nonlinear systems. LMI-based conditions derived from the Lyapunov hypothesis, the solidness of a T-S fuzzy control system could be guaranteed. The T-S FLCS has been connected to manage plentiful control issues since it could simply be related with different calculations.

A large portion of the research works examined the T-S show expecting that the non-linear system is linearized regarding the inception in each IF-THEN run (Tanaka and Sano 1994), (Tanaka et al. 1996), which implies that the ensuing piece of each control is a linear function with zero consistent terms. This will, thusly, diminish the exactness of approximating non-linear systems. LQR is utilized to decide best esteems for parameters in fuzzy control runs in which the robustness is inalienable in the LQR it gives a viable plan strategy for fuzzy control to guarantee robustness. In this part, we will indicate how the LQR, the structure of which is based on mathematical analysis, can be made more fitting for genuine usage by a presentation of fuzzy rules. The inspiration driving this plan is to consolidate the best highlights of fuzzy control and LQR to accomplish quick and precise following control of a class of nonlinear systems.

II. FUZZY LQR

Fuzzy Logic Controller based Optimal Linear Quadratic Regulator (FC-LQR) for the control of a nonlinear CSTR system. The primary thought is to outline a supervisory fuzzy controller proficient to modify the controller parameters keeping in mind the end goal to get the coveted axes positions under varieties of the system parameters and

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load varieties. In the event that speed and precision are required, the control utilizing traditional techniques is hard to acknowledge in view of the high nonlinearity of the CSTR system. In control plan, it is regularly important to outline a controller to satisfy, in an optimal shape, certain performance criteria and requirements notwithstanding stability.

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III. ADAPTIVE LQ

A. *Determination characteristic equation*

System equation in the form of Laplace transform is given by,

$$Y(s)=\frac{b(s)q(s)}{a(s)p(s)+b(s)q(s)}W(s)+\frac{a(s)p(s)}{a(s)p(s)+b(s)q(s)}V(s) \quad (1)$$

Characteristic equation of overall closed loop system,

$$\text{Characteristic equation} = a(s)\cdot p(s) + b(s)\cdot q(s) \quad (2)$$

B. *Formation Diophantine equation*

This characteristic polynomial can be rewritten in the form,

$$a(s).p(s) + b(s).q(s) = d(s)$$

Where d (s) is a stable optional polynomial. The whole equation is called Diophantine equation. The stability of the control system is fulfilled for the stable polynomial d (s) on the left side of the Diophantine equation. The polynomial d (s) is in this case is

$$d(s) = n(s)\cdot g(s)$$

Where parameters of the polynomial n (s) are computed from the spectral factorization of the polynomial a (s), i.e.

$$n^{*}(s).\,n(s) = a^{*}(s).\,a(s)$$

The parameters of the polynomial g (s) are computed from the spectral factorization,

$$(a(s).f(s))^{*}\ \phi LQ\ (a(s).f(s)) + b^{*}(s)\ \mu LQ\ b(s) = g(s)^{*}.g(s) \quad (3)$$

Where $*$ again denotes stable mirror from the spectral factorization.

C. *Solution of Diophantine equation*

In discrete form we get the following Diophantine equation for CSTR T.F

$$(z^2+a_1z+a_2)\tilde{p}(s) + (b_0z^2+b_1z+b_2)\,q(s) = d_0z^3+d_1z^2+d_2z+d_3 \quad (4)$$

Compared with,

$$a(s)\cdot s\cdot \tilde{p}(s) + b(s)\cdot q(s) = d(s)$$

This Diophantine equation solved by using Silvester matrix method. Matrix E and D is given by,

$$\mathbf{E}=\begin{bmatrix} a_2 & 0 & b_2 & 0 \\ a_1 & a_2 & b_1 & b_2 \\ 1 & a_1 & b_0 & b_1 \\ 0 & 1 & 0 & b_0 \end{bmatrix}; \quad \text{and} \quad \mathbf{D}=\begin{bmatrix} d_3 \\ d_2 \\ d_1 \\ d_0 \end{bmatrix}$$

Matrix M gives parameters of controller and this matrix M is obtained as,

$$M = E^{-1} * D = \begin{bmatrix} \alpha_1 \\ \alpha_0 \\ \beta_1 \\ \beta_0 \end{bmatrix}$$

Controller Polynomial $\tilde{p}(s)$ and q(s) is obtained from matrix M,

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$\tilde{p}(s) = \alpha_0 z + \alpha_1$ and $q(s) = \beta_0 z + \beta_1$

$Q_1(z)$ is given by, $Q_1(z) = \frac{\beta_0 z + \beta_1}{\alpha_0 z + \alpha_1}$

Following this procedure we obtained controller transfer function for different weighting factor, which are written in the following table. This adaptive controller has two tuning parameters weighting factors which give attention to the output error or the change of the input variable.In first set we design the controller for constant μ_{LQ} and change in φ_{LQ}.In this set we take $\mu_{LQ} = 2$ and $\varphi_{LQ} = 0.2, 0.05, 0.005$. For this weighting factors we obtain the controller transfer function and observe the simulation results with this controller. From these results it is clear that there are not important values of these weighting factors separately, but ratio of them, e.g. $\varphi_{LQ}:\mu_{LQ}$. Decreasing value of this ratio produces smoother course of both the input and the output variables. Therefore for $\varphi_{LQ} = 0.005$ and $\mu_{LQ} = 2$ we get better result, now further decreasing this ratio and observing the results we get optimal results at $\mu_{LQ} = 2.9$ and $\varphi_{LQ} = 0.006$.Different weighting factors with their ratio and respective transfer function are written in following table.

TABLE 1: CONTROLLER TRANSFER FUNCTION FOR DIFFERENT WEIGHTING OF SET 1

No	Weighting factor	$\varphi_{LQ}:\mu_{LQ}$	Controller T.F.
1	$\mu_{LQ} = 2$ and $\varphi_{LQ} = 0.2$	1:10	$\frac{0.3822s^2 + 0.4608s + 0.173}{s^3 + 1.261s^2 + 0.4962s}$
2	$\mu_{LQ} = 2$ and $\varphi_{LQ} = 0.05$	1:40	$\frac{0.7362s^2 + 0.5448s + 0.1329}{s^3 + 0.9519s^2 + 0.3252s}$
3	$\mu_{LQ} = 2$ and $\varphi_{LQ} = 0.005$	1:400	$\frac{2.23s + 0.5638}{s^2 + 0.2472s}$
4	$\mu_{LQ} = 2.9$ and $\varphi_{LQ} = 0.006$	1:483.33	$\frac{2.634s + 0.7511}{s^2 + 0.452s}$

Similarly we obtain the controller transfer function for set 2. In this set we take $\varphi_{LQ} = 0.1$ and $\mu_{LQ} = 1,4,40$. For this weighting factors we obtain the controller transfer function and observe the simulation results with this controller. From these results it is clear that there are not important values of these weighting factors separately, but ratio of them, e.g. $\varphi_{LQ}:\mu_{LQ}$. Decreasing value of this ratio produces smoother course of both the input and the output variables. Therefore for $\varphi_{LQ} = 0.1$ and $\mu_{LQ} = 40$ we get better result, now further decreasing this ratio and observing the results we get optimal results at $\mu_{LQ} = 0.13$ and $\varphi_{LQ} = 62$.Different weighting factors with their ratio and respective transfer function are written in following table.

TABLE 2: CONTROLLER TRANSFER FUNCTION FOR DIFFERENT WEIGHTING OF SET 1

No.	Weighting factor	$\varphi_{LQ}:\mu_{LQ}$	Controller T.F.
1	$\mu_{LQ} = 1$ and $\varphi_{LQ} = 0.1$	1:10	$\frac{1.03s + 0.2365}{s^2 + 0.2238s}$
2	$\mu_{LQ} = 4$ and $\varphi_{LQ} = 0.1$	1:40	$\frac{0.2951s + 0.0588}{s^2 + 0.8683s}$
3	$\mu_{LQ} = 40$ and $\varphi_{LQ} = 0.1$	1:400	$\frac{5.548s^2 + 5.397s + 1.865}{s^3 + 0.9682s^2 + 0.333s}$
4	$\mu_{LQ} = 62$ and $\varphi_{LQ} = 0.1$	1:476.9	$\frac{5.058s^2 + 4.097s + 0.859}{s^3 + 0.959s^2 + 0.34s}$

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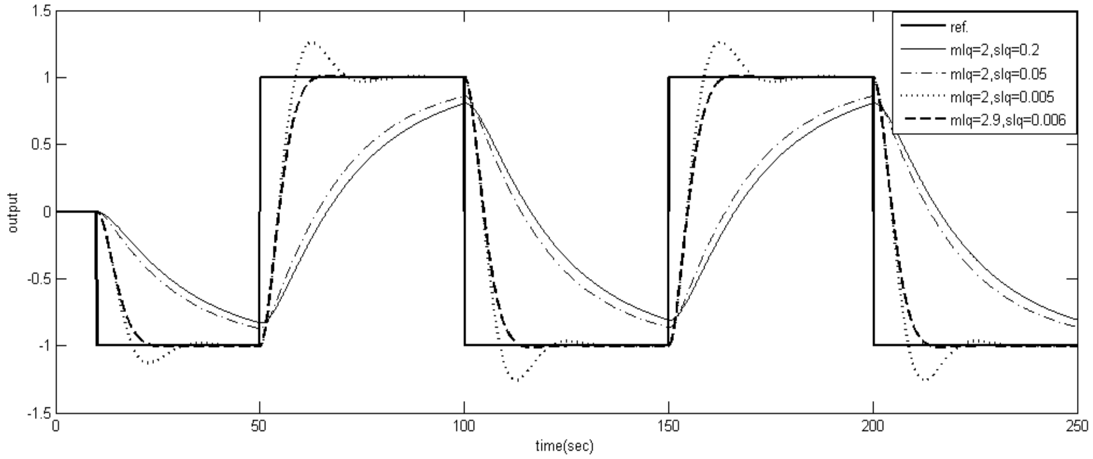

Fig. 1: ref. tracking results for set (1)

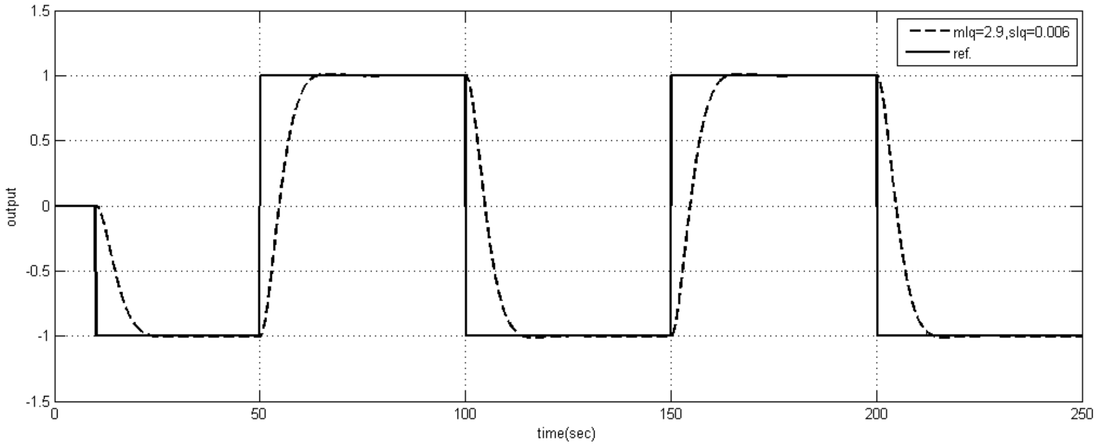

Fig. 2: results for optimal weighting factor of set 1 i.e. $\mu_{LQ} = 2.9$ and $\varphi_{LQ} = 0.006$

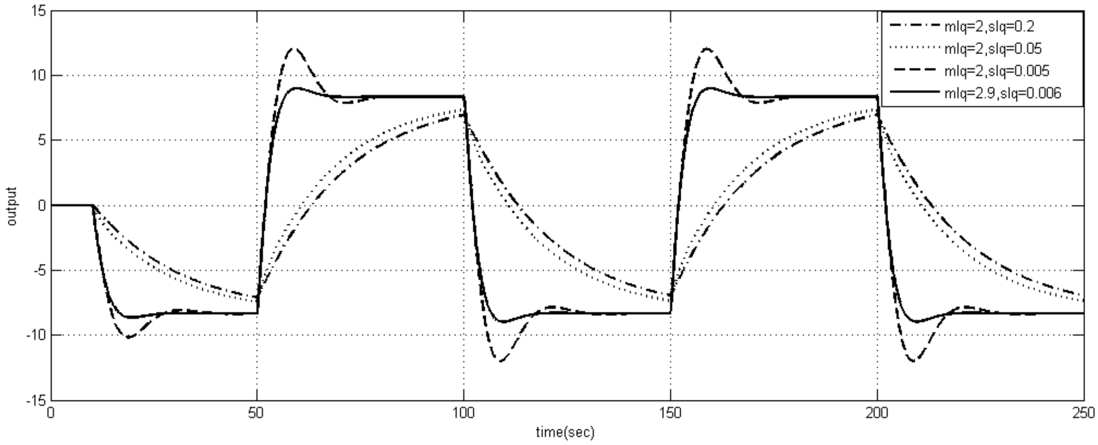

Fig. 3 simulation results for u(t)

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For second set

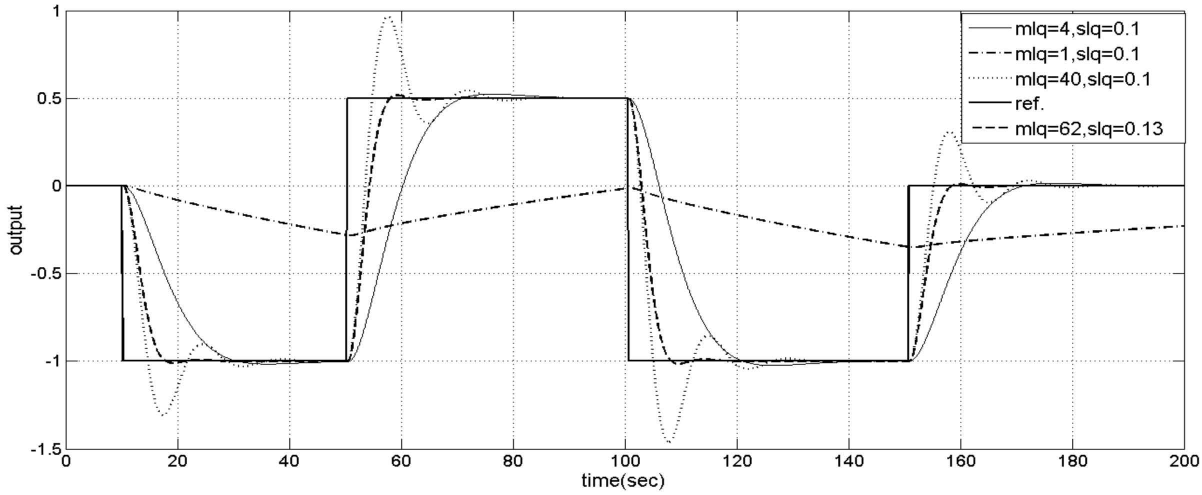

Fig. 4: ref. tracking results for set (2)

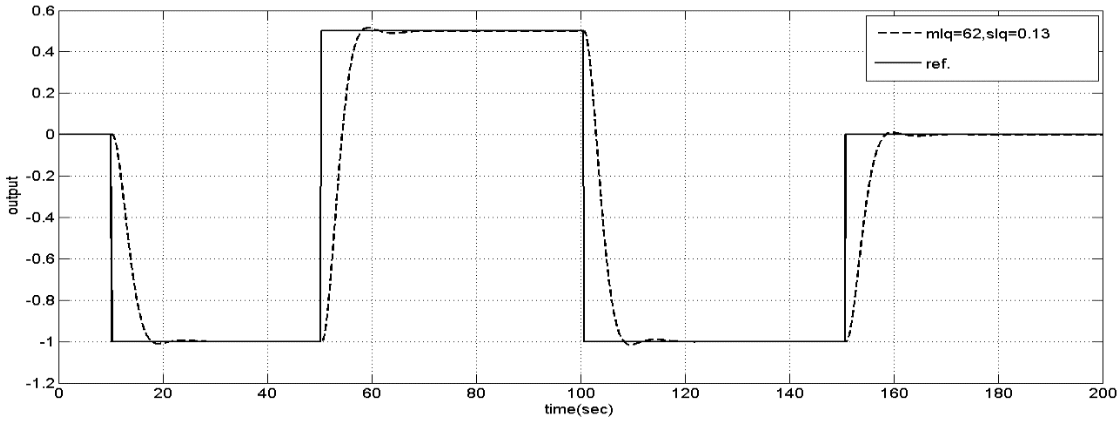

Fig. 5: results for optimal weighting factor of set 1 i.e. $\mu_{LQ} = 62$ and $\varphi_{LQ} = 0.13$

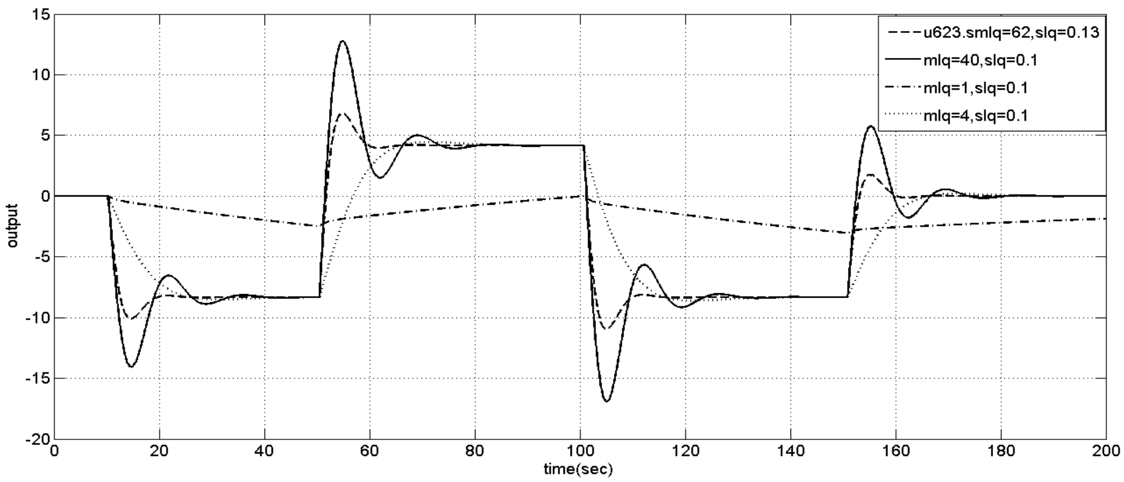

Fig. 6: simulation results for u(t)

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TABLE 3: COMPARISON WITH RISE TIME AND PEAK OVERSHOOT

No.	μ_{LQ}	φ_{LQ}	φ_{LQ}: μ_{LQ}	Rise time	Peak overshoot
1	2	0.2	1:10	36.36	--
2	2	0.05	1:40	34.556	--
3	2	0.005	1:400	7.253	12.56%
4	2.9	0.006	1:483.33	7.620	0.46%

TABLE 4: COMPARISON WITH RISE TIME AND PEAK OVERSHOOT

No.	μ_{LQ}	φ_{LQ}	φ_{LQ}: μ_{LQ}	Rise time	Peak overshoot
1	1	0.1	1:10	36.36	--
2	4	0.1	1:40	34.556	--
3	40	0.1	1:400	7.253	12.56%
4	62	0.13	1:483.33	7.620	0.46%

IV. STABILITY ANALYSIS USING LMI

The aim of this method is to show that we can reduce a very wide variety of problems arising in system and control theory to a few standard convex or quasiconvex optimization problems involving linear matrix inequalities (LMIs). Since these resulting optimization problems can be solved numerically very efficiently using recently developed interior-point methods, our reduction constitutes a solution to the original problem, certainly in a practical sense, but also in several other senses as well. In comparison, the more conventional approach is to seek an analytic or frequency-domain solution to the matrix inequalities. The types of problems we consider include: matrix scaling problems, e.g., minimizing condition number by diagonal scaling, construction of quadratic Lyapunov functions for stability and performance analysis of linear differential inclusions , joint synthesis of state-feedback and quadratic Lyapunov functions for linear differential inclusions , multicriterion LQG/LQR ,inverse problem of optimal control In some cases,

We are describing known, published results; in others, we are extending known results. In many cases, however, it seems that the results are new. By scanning the list above or the table of contents, the reader will see that Lyapunov's methods will be our main focus. Here we have a secondary goal, beyond showing that many problems from Lyapunov theory can be cast as convex or quasiconvex problems. This is to show that Lyapunov's methods.

Dong Hwan Lee, Young Hoon Joo*, and Myung Hwan Tak proposed the study of linear matrix inequality (LMI) formulations to analyze local stability and design controllers that locally stabilize continuous-time nonlinear systems represented by Takagi– Sugeno (T–S) fuzzy systems. In order to estimate the domain of attraction (DA), the so-called fuzzy Lyapunov function is used to characterize the subsets of the DA as sublevel sets of the Lyapunov function. Quadratic bounds on the time-derivative of the membership functions are employed to derive the main results. Finally, examples are given to illustrate the proposed methods. a disadvantage of the proposed method is that it can be more conservative than existing approaches since we regard the MFs as norm-bounded uncertain parameters.

The history of LMIs in the analysis of dynamical systems goes back more than 100 years. The story begins in about 1890, when Lyapunov published his seminal work introducing what we now call Lyapunov theory. He showed that the differential equation is stable (i.e., all trajectories converge to zero) if and only if there exists a positivedefinite matrix P such that

$$A^TP + PA < 0 \tag{5}$$

The requirement P > 0, A^T P +PA < 0 is what we now call a Lyapunov inequality on P, which is a special form of an LMI. Lyapunov also showed that this first LMI could be explicitly solved. we can pick any Q = QT > 0 and then solve the linear equation ATP +PA = −Q for the matrix P, which is guaranteed to be positive-definite if the system (1.1) is

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stable. In summary, the first LMI used to analyze stability of a dynamical system was the Lyapunov inequality, which can be solved analytically (by solving a set of linear equations).

V. CONCLUSION

The conventional controller gives a superior outcome to a linear system. In any case, for a non-linear system, the conventional controller does not ensure great performance. Settling time is additionally more in the event of a conventional controller. A fuzzy controller is based on administer base, which is user-defined. T-S Fuzzy Technique gives the advantages of to get nonlinear control systems, particularly within the sight of deficient healing of the plant or even of the exact control activity proper to a given circumstances. So, it is profoundly appropriate for non-linear and gives a superior outcome than a conventional controller.

The fuzzy LQR controller-based controller gives the best performance, yet the control build faces an alternate sort of difficulties to outlining such a controller. The key outline challenge is to produce an enhanced fuzzy rule base. This dissertation proposal LOT based optimization of existing fuzzy control base. These two controllers have a superior tuning performance than the other conventional control techniques.

Based on the Lyapunov functional approach, adequate stability conditions have been acquired in the LMI's. The feasibility and effectiveness of the proposed fuzzy model and the control outline text are illustrated.

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