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Improved Analytical Model for Load-Deflection Performance of SOI Pressure Sensor and Comparison with Simulation Results

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Abstract: Silicon-on-Insulator (SOI) pressure sensor diaphragm is a composite structure of the buried oxide and the SOI layer. This paper brings out the inadequacy of the existing analytical model in describing the deflection response of SOI square diaphragms and focuses on development of new improved analytical model that describes the load-deflection performance of SOI composite diaphragm. This improved model is able to predict the small scale as well as large scale deflection accurately when compared with deflection obtained by IntelliSuite FEA. Further, the new model has been demonstrated that the existing analytical model overestimates the deflection and hence inadequate for application to SOI pressure sensors.

Keywords: Silicon-on-insulator; Micro electro mechanical systems; Finite element analysis; Pressure sensor; IntelliSuite; Improved analytical model

I. INTRODUCTION

In the last decade significant research works have been carried out on micro machined diaphragm-type pressure sensors that are fabricated by using new technologies like bulk micromachining, surface micromachining or combination of both techniques. Most of them use silicon for diaphragm and piezoresistive property of silicon or polycrystalline silicon as sensing mechanism [1-7]. The interest of using Silicon-on-Insulator (SOI) material for CMOS production has considerably increased in the recent years because of the advantages of SOI-devices can offer in terms of reduced leakage currents and parasitic capacitances. The technology for producing high quality SOI wafers has also developed rapidly and SOI wafers are commercially available in a variety of sizes and film thicknesses. Thicker films are normally needed for micromechanical applications and SOI layers with thicker films are manufactured by fusion bonding where the device layer is bonded to handle wafer with an oxide between them, called Bond and Etch back SOI (BESOI). The inherent structure of an SOI material is attractive for Micro Electro Mechanical Systems (MEMS) manufacturing. SOI material provides features as thin silicon device layers which are interesting for bulk micromachining, inherent material etch stop due to the insulating buried oxide layer and adds functionality such as inherent overload protection, squeezed air film damping and packaging features to MEMS devices. The details of the fabrication and performance of piezoresistive and capacitive pressure sensors realized on SOI wafers can be found widely in the literature [7-11]. The combination of SOI material with wafer bonding can further allow building complex multi-material structures, thereby increasing the flexibility in forming microsystem structures such as sensors and actuators. Though SOI technology pressure sensor has been reported widely, the deflection performance is still defined by the Pressure - deflection equation available in the literature [10-12] for square and circular single layer silicon diaphragms. But, the diaphragm in SOI pressure sensors is heterogeneous since it is made of silicon and SiO₂ layers. In practical situations, thick layer of silicon along with very thin buried oxide layer is used as the diaphragm and therefore the oxide layer thickness can be neglected. However, the required silicon layer thickness in the SOI low pressure sensors is smaller and under such a circumstance the thickness of the oxide layer can not be neglected since it becomes comparable with silicon layer thickness. Therefore, the load-deflection performance has to be determined taking this oxide layer thickness also into account and it becomes necessary to develop analytical model for deflection in SOI composite diaphragms. In this paper, the authors present an improved analytical model for deflection of SOI diaphragm that incorporates the buried oxide thickness also in the modelling along with Si layer.

II. DEFLECTION RESPONSE OF CONVENTIONAL PRESSURE SENSORS

The pressure-deflection response of a flat square diaphragm of a silicon pressure sensor realized using bulk micromachining as shown in Fig. 1 is given as [11]



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$$\frac{Pa^4}{Eh_{si}^4} = \frac{4.2}{(1-\nu^2)} \left[\frac{y}{h_{si}}\right] + \frac{1.58}{(1-\nu)} \left[\frac{y}{h_{si}}\right]^3 \dots$$
(1)

where

P - Applied pressure in Pascal

y - Centre deflection of the diaphragm in µm

a - Half of side length of the diaphragm in µm

E - Young's modulus in GPa

 hs_i - Thickness of the S_i diaphragm in μm

U - Poisson's ratio of the diaphragm material



Fig.1. Structure of SOI MEMS pressure sensor.

The first term in the above equation represents the Small Scale Deflection (SSD) that is very small compared to the diaphragm thickness (deflection is less than 25% of the diaphragm thickness) where as the second term gives the Large Scale Deflection (LSD), the condition in which the deflection is larger than the diaphragm thickness.

III. ISSUES WITH APPLICATION OF EXISTING THEORY TO SOI STRUCTURE

However, there are issues with applying the above said theory to pressure sensors realized on SOI structure depicted in Fig.1. The direct application of this analytical model to SOI structure may not be accurate since the buried SiO₂ layer between the substrate and the diaphragm is not considered in this model. In such a situation, Finite Element Analysis (FEA) is an efficient design tool since it can estimate the deflection of the diaphragm and total stress developed accurately irrespective of the structure and dimensions of the various layers. In order to justify this point, the deflection at various pressure levels have been estimated using the conventional Small Scale Deflection Theory (SSDT) and electro thermo mechanical module of IntelliSuite for conventional and SOI pressure sensors with buried SiO₂ thicknesses (h_{ox}) from 0 to 2 µm for diaphragms with a side length of 500 µm and thicknesses of 15 µm and 3 µm. IntelliFab module has been used to build the SOI pressure sensor structure for analysis. The results presented in the form of a graph as shown in Fig.2(a) and Fig.2(b) respectively demonstrate that SiO₂ thickness has a role in deciding the deflection of the diaphragm and it clearly does not match with the values obtained using conventional SSDT [11].



Fig.2(a). Deflection of the diaphragm versus thickness of SiO₂.



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Fig.2(b). Deflection of the diaphragm versus thickness of SiO₂.

IV. MODIFIED ANALYTICAL MODEL FOR DEFLECTION OF SOI DIAPHRAGM

The pressure deflection response of a flat square diaphragm made of a single material whose edges are not free to move is given by equation (1) [11]. This equation has been obtained by solving the equation for the deflection curve of the elemental strip [13] represented in the following form:

$$D\frac{d^2y}{dX^2} = -M$$
 (2)

where D = Flexural rigidity, M = Bending moment in the elemental strip For a single layer diaphragm of thickness <math>h and side length 2a, D is derived to be

$$D = \frac{Eh^3}{12(1-\nu^2)}$$
(3)

where v is the Poisson's ratio and E is the Young's modulus of the diaphragm material. Hence, the equation (1) can be re-written as

for small scale deflection case. In the case of SOI composite diaphragm present in the SOI pressure sensor as shown in Fig.1, the boundary conditions remain the same as single material diaphragm and the difference lies only in the flexural rigidity since the composite diaphragm is heterogeneous. Therefore, the only change that is predicted in the load–deflection relationship described by equation (4) under small scale deflection condition is the flexural rigidity (D) since all other conditions remain the same as the single layer diaphragm. Therefore, the flexural rigidity has been obtained for SOI composite diaphragm (D_{SOI}) so that the applied load (P) can be related to the small scale deflection (y) by just replacing the D by D_{SOI} in equation (4). The flexural rigidity of SOI composite diaphragm with the cross-sectional view as shown in Fig.3 has been estimated from first principles as described in reference [13]. The bending moment in the elemental strip considered in the Z direction can be written as



Fig.3.Cross-sectional view of the SOI composite diaphragm.



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$$M = \begin{bmatrix} - \int_{0}^{h_{0x}} \sigma_{x1} . (Z - h_n) . dZ - \int_{h_{0x}}^{h_{0x} + h_{si}} \sigma_{x2} . (Z - h_n) . dZ \end{bmatrix} \dots \dots \dots (5)$$

for SOI composite diaphragm where h_n is the distance of centre of gravity from the base of the composite SOI diaphragm, h_{ox} , h_{si} are the thickness of the buried oxide and silicon layers of the composite SOI diaphragm respectively and σ_{x1} and σ_{x2} are the normal stresses acting on the element for the buried oxide layer and silicon layer respectively. σ_{x1} and σ_{x2} can be written as

 $\frac{d^2 y}{dX^2}$ is the curvature of the deflection curve where y is the deflection of the diaphragm in the Z direction, assumed to

be small compared to the side length (2*a*). E_{ox} and E_{si} are the Young's Moduli of the buried oxide and silicon layers of the composite SOI diaphragm respectively. Similarly the Poisson's ratios of these layers are denoted by v_{ox} and v_{si} respectively. Substituting the values of σ_{x1} and σ_{x2} given by equation (6) in equation (5) and performing the integration with appropriate limits results in equation (7) which gives M as,

$$M = \begin{bmatrix} -\frac{E_{ox}}{3.(1-v_{ox}^{2})} \left[(h_{ox} - h_{n})^{3} + h_{n}^{3} \right] \\ -\frac{E_{si}}{3.(1-v_{si}^{2})} \left[(h-h_{n})^{3} - (h_{ox} - h_{n})^{3} \right] \\ \frac{d^{2}y_{SOI}}{dX^{2}} \quad \dots (7)$$

Here, $h = h_{ox} + h_{si}$, the total thickness of the SOI composite diaphragm. This equation for bending moment (M) of the elemental strip can be written in the form of equation (2) as

where

$$D_{soi} = \begin{bmatrix} -\frac{E_{ox}}{3.(1-v_{ox}^{2})} \left[(h_{ox} - h_{n})^{3} + h_{n}^{3} \right] - \\ \frac{E_{si}}{3.(1-v_{si}^{2})} \left[(h - h_{n})^{3} - (h_{ox} - h_{n})^{3} \right] \end{bmatrix}$$
(9)

The distance of centre of gravity (h_n) from the base of the composite diaphragm is obtained as [30]

$$h_n = \left[\frac{(E_{ox}.h_{ox}.Y_1) + (E_{si}.h_{si}.Y_2)}{(E_{ox}.h_{ox}) + (E_{si}.h_{si})}\right] \dots (10) \text{ where } Y_1 = \left[\frac{h_{ox}}{2}\right] \text{ and } Y_2 = h_{ox} + \left[\frac{h_{si}}{2}\right]$$

If these heterogeneous layers in a SOI pressure sensor can be represented as a single equivalent silicon layer with a thickness of h_{EST} or single oxide layer with a thickness of h_{EoT} it is possible to apply the same equation (1) describing load-deflection response can be used to calculate the deflection of SOI pressure sensor. All that one has to do is to estimate the equivalent silicon thickness (h_{EST}) of the composite SOI diaphragm or to estimate the equivalent silicon-di-oxide thickness (h_{EoT}) of the composite SOI diaphragm. The following sections describe how this can be achieved.

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4.1 Equivalent oxide thickness (h_{EoT})

For a single layer diaphragm of thickness h and side length 2a, flexural rigidity (D) is given by equation (3). The flexural rigidity (D_{SOI}) of the SOI diaphragm has been obtained now and is given by equation (9). If this equation could be re-written in the format given by the following equation

then the composite SOI diaphragm can be considered as a single layer of silicon-di-oxide with a thickness of h_{EoT} .

Let
$$m = \left[\frac{E_{ox}}{E_{si}}\right]$$
 and $n = \left[\frac{v_{ox}}{v_{si}}\right]$ (12)

With this definition, Equation (9) that describes D_{SOI} can be rewritten as

$$D_{SOI} = \begin{bmatrix} \frac{E_{ox}}{3.(1-v_{ox}^{2})^{2}} [(h_{ox}-h_{n})^{3}+h_{n}^{3}] \\ + \frac{E_{ox}}{3m(1-\frac{v_{ox}^{2}}{n^{2}})} [(h-h_{n})^{3}-(h_{ox}-h_{n})^{3}] \end{bmatrix} .. (13)$$

This equation is now in the format as given by equation (11)

and one gets h_{EoT}

$$h_{EoT} = 4 \begin{bmatrix} (h_{ox} - h_n)^3 - h_n^3) \\ + \frac{1 - v_{ox}^2}{m(1 - \frac{v_{ox}^2}{n^2})} ((h - h_n)^3 - (h_{ox} - h_n)^3) \end{bmatrix}^{(1/3)} \dots (15)$$

The equivalent oxide thicknesses of the first SOI composite diaphragm with side length $(2a) = 500 \mu m$ and silicon layer thickness $(h_{si}) = 5 \mu m$ and the second SOI composite diaphragm with side length $(2a) = 1200 \mu m$ and silicon layer thickness $(h_{si}) = 14.65 \mu m$ [14] for various buried oxide thicknesses h_{ox} have been plotted and shown in Fig.4 and Fig.5 respectively. The calculated equivalent oxide thicknesses (h_{EoT}) for any buried oxide thickness is considerably larger than the silicon thickness (h_{si}) of the concerned device for the reason that $E_{ox} < E_{si}$.



Fig.4. Buried oxide thickness versus Equivalent oxide thickness for SOI pressure sensor [$2a = 500 \mu m$, $h_{si} = 5 \mu m$].



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SOI pressure sensor [$2a = 1200 \,\mu\text{m}$, $h_{si} = 14.65 \,\mu\text{m}$].

Now, the improved Pressure-deflection relationship for the SOI layer is simply represented as

$$\frac{Pa^{4}}{E_{ox}h_{EoT}^{4}} = \frac{4.2}{(1-v_{ox}^{2})} \left[\frac{y}{h_{EoT}}\right] + \frac{1.58}{(1-v_{ox})} \left[\frac{y}{h_{EoT}}\right]^{3} \qquad \dots \dots \dots (16)$$

4.2 Equivalent silicon thickness (h_{EST})

In a similar fashion, equation (9) can be re-written as

$$D_{SOI} = \begin{bmatrix} \frac{mE_{si}}{3(1 - (n\nu_{Si})^2)} \left[(h_{ox} - h_n)^3 + h_n^3 \right] \\ + \frac{E_{si}}{3(1 - \nu_{si}^2)} \left[(h - h_n)^3 - (h_{ox} - h_n)^3 \right] \end{bmatrix}$$

This equation is now re-arranged as

where
$$h_{EST} = 4 \begin{bmatrix} (\frac{m(1-v_{si}^2)}{(1-n^2v_{si}^2)})(h_{ox}-h_n)^3 + h_n^3) \\ + ((h-h_n)^3 - (h_{ox}-h_n)^3) \end{bmatrix}^{(1/3)} \dots (18)$$

The equivalent silicon thicknesses of the first SOI composite diaphragm with side length $(2a) = 500 \,\mu\text{m}$ and silicon layer thickness $(h_{si}) = 5 \,\mu\text{m}$ and the second SOI composite diaphragm with side length $(2a) = 1200 \,\mu\text{m}$ and silicon layer thickness $(h_{si}) = 14.65 \,\mu\text{m}$ [14] for various buried oxide thicknesses (h_{ox}) have been plotted and shown in



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Fig.6 and Fig.7 respectively. The calculated equivalent silicon thicknesses (h_{EST}) for any buried oxide thickness is slightly more than the silicon thickness (h_{si}) of the concerned device for the reason that $E_{ox} < E_{si}$.

The modified Pressure-deflection relationship for the SOI layer is simply represented as

$$\frac{Pa^4}{E_{si}h_{EST}^4} = \frac{4.2}{(1-\nu_{si}^2)} \left[\frac{y}{h_{EST}}\right] + \frac{1.58}{(1-\nu_{si})} \left[\frac{y}{h_{EST}}\right]^3 \dots \dots (19)$$



Fig.6. Buried oxide thickness versus Equivalent silicon thickness for SOI pressure sensor [$2a = 500 \mu m$, $h_{si} = 5 \mu m$].



Fig.7. Buried oxide thickness versus Equivalent silicon thickness for SOI pressure sensor [$2a = 1200 \ \mu m$, $h_{si} = 14.65 \ \mu m$].

V. SMALL SCALE DEFLECTION ANALYSIS

This load-deflection relation has now been used to estimate the small scale deflection analytically and compared with the IntelliSuite simulation results. The same Young's moduli and Poisson's ratios of the oxide and Silicon layers of the composite SOI diaphragm assumed in the IntelliSuite simulation studies have been used in the analytical calculations also. They are as follows: E_{ox} = 57 GPa, υ_{ox} = 0.175 and E_{si} = 160 GPa, υ_{si} = 0.3. The small scale analytical deflection values of two SOI pressure sensors obtained at different pressure using the improved analytical model reported in this paper have been plotted along with the deflection obtained through IntelliSuite simulation studies and



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the deflection calculated using the load-deflection relation described by existing analytical model [11] for comparison purpose. The load - small scale deflection response of the first SOI composite diaphragm with side length (2a) = 500µm and silicon layer thickness $(h_{si}) = 5$ µm and the second SOI composite diaphragm with side length (2a) = 1200µm and silicon layer thickness $(h_{si}) = 14.65$ µm [14] for various pressure have been plotted and shown in Fig.8 and Fig.9 respectively. The comparison results clearly show that the central small scale deflection (y_{SOI}) predicted by modified analytical model matches very closely with the FEA simulation results obtained with IntelliSuite. At the same time, it can be seen from Fig.8 and Fig.9 that the existing model for load-deflection response overestimates the deflection and the error is unacceptably larger when the buried oxide layer thickness (h_{ox}) is comparable with silicon layer thickness (h_{si}) while compared with the responses obtained with FEA simulations and the present improved analytical model described in this paper. These observations clearly demonstrates the validity of the improved analytical model presented in this paper in addition to explicitly bringing out the inadequacy of the existing analytical model in describing the load-deflection performance of SOI pressure sensor.



Fig.8. Comparison of small scale load-deflection response obtained by existing analytical model, modified analytical model and FEA by IntelliSuite for SOI Pressure sensor at various BOX thicknesses (h_{ox}) [$(2a) = 500 \text{ }\mu\text{m}, h_{si} = 5\mu\text{m}$].



Fig.9. Comparison of small scale load-deflection response obtained by existing analytical model, modified analyticalmodel and FEA by IntelliSuite for SOI Pressure sensor at various BOX thicknesses (h_{ox}) [$(2a) = 1200 \mu m$, $h_{si} = 14.65 \mu m$].



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VI. LARGE SCALE DEFLECTION ANALYSIS

The improved analytical model that describes load–deflection characteristics of SOI composite diaphragm has now been used to estimate the large scale deflection analytically and compared with the IntelliSuite simulation results obtained in large scale analysis. The load – large scale deflection response of the first SOI composite diaphragm with side length $(2a) = 500 \mu m$ and silicon layer thickness $(h_{si}) = 5 \mu m$ and the second SOI composite diaphragm with side length $(2a) = 1200 \mu m$ and silicon layer thickness $(h_{si}) = 14.65 \mu m$ [14] for various pressure have been plotted and shown in Fig.10 and Fig.11 respectively. The comparison results clearly show that the central small scale deflection response overestimates the deflection and the error is unacceptably larger when the buried oxide layer thickness (h_{ox}) is comparable with silicon layer thickness (h_{si}) when compared with the responses obtained with FEA simulations and the present improved analytical model described in this paper. However, the large scale deflection obtained using IntelliSuite is smaller than the deflection calculated by the present model especially when the oxide thickness is comparable with silicon thickness even though the small scale deflection is matching very closely. Further investigations are required to understand the reason for the large error in the large scale deflection.



Fig.10. Comparison of large scale load-deflection response obtained by existing analytical model, improved analytical model and FEA by IntelliSuite for SOI Pressure sensor at various BOX thicknesses (h_{ox}) [(2*a*) = 500 µm, h_{si} = 5 µm].



Fig11 Comparison of large scale load-deflection response obtain existing analytical model, improved analytical model & FEA by IntelliSuite for SOI Pressure sensor at various BOX thicknesses (h_{ox}) [(2a)=1200µm, h_{si} =14.65µm].





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VII. CONCLUSION

The inadequacy of the existing theory on square diaphragm [11, 14] for pressure sensors realized on SOI wafers has been explained. A new improved analytical model has been presented in this paper for load-deflection response of composite SOI diaphragm. The deflection obtained using this improved analytical model matches the FEA simulation results by IntelliSuite very closely. Further, comparison of these results with the deflection calculated using existing analytical model shows that the deflection is overestimated by the existing analytical model when directly applied to SOI pressure sensors and the error is considerably larger when the buried oxide thickness is comparable with the silicon layer thickness. Thus, it is established that the existing theory that describes the load-deflection response is inadequate for application to SOI pressure sensors.

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REFERENCES

- [1]. Bernd Folkmer, Peter Steiner and Walter Lang, A pressure sensor based on a nitride membrane using single-crystalline piezoresistors, Sensors and Actuators A 54 (1996) 488-492.
- [2]. A. Berns, U. Buder, E. Obermeier, A. Wolter and A. Leder, Aero MEMS sensor array for high-resolution wall pressure measurements, Sensors and Actuators A 132 (2006) 104–111.
- [3]. Milan M. Jevti and Milolju A. Smiljani, Diagnostic of silicon piezoresistive pressure sensors by low frequency noise measurements, Sensors and Actuators A 144 (2008) 267–274.
- [4]. Ingelin Clausen and Ola Sveen, Die separation and packaging of a surface micromachined piezoresistive pressure sensor, Sensors and Actuators A 133 (2007) 457–466.
- [5]. A. Wisitsoraat, V. Patthanasetakul, T. Lomas and A. Tuantranont, Low cost thin Actuators A 139 (2007) 17–22.
 film based piezoresistive MEMS tactile sensor, Sensors &
- [6]. Shyam Aravamudhan and Shekhar Bhansali, Reinforcedpiezoresistive pressure sensor for ocean depth measurements, Sensors and Actuators A 142 (2008) 111–117.
- [7]. K. Sivakumar, N. Dasgupta and K.N. Bhat, Sensitivity enhancement of polysilicon piezo-resistive pressure sensors with phosphorous diffused resistors, Journal of physics: Conference series 34 (2006) 216–221.
- [8]. C.S. Park, B.S. Kang, D.W. Lee, T.Y. Choi and Y.S. Choi, Fabrication and characterization of a pressure sensor using a pitch-based carbon fiber, Microelectronic Engineering 84 (2007) 1316-1319.
- [9]. P.D. Dimitropoulos, C. Kachris, D.P. Karampatzakisa and G.I. Stamoulis, A new SOI monolithic capacitive sensor for absolute and differential pressure measurements, Sensors and Actuators A 123-124 (2005) 36-43.
- [10]. K. N. Bhat, Silicon Micro machined Pressure Sensors, Journal of the Indian Institute of Science VOL 87:1 Jan–Mar 2007.
- [11]. Zhao Linlin, Xu Chen & Shen Guangdi, Analysis for load limitation of square-shaped silicon diaphragms, Solid-State Electronics 50 (2006) 1579-1583.
- [12]. M. Elwenspoke and R. Wiegerink, Mechanical Microsensors Springer Publications, NewYork, 2000.
- [13]. S.Timoshenko & S.Woinowsky-Krieger, Theory of Plates & Shells, 2nd Edi, McGraw-Hill Internat Edition, New York, 1959, pp. 4-32.
- [14] M.Narayanaswamy, R.Joseph Daniel, K.Sumangala, C.Antony Jeyasehar, Comp aided modelling & diaphragm design approach for high sensitivity silicon-on-insulator pressure sensors, Elsevier Measurement Journal, Measurement(2011), doi:10.1016/j. measurement, 2011.08.025.