

# A Simple Load Flow Technique for Unbalanced Distribution Networks

**R. Satish<sup>1</sup>, P. Kanta Rao<sup>2</sup>, K. Vaisakh<sup>3</sup>**

Asst.Prof, Dept. of EEE, Anil Neerukonda Institute of Technology & Sciences, Visakhapatnam, Andhra Pradesh, India<sup>1</sup>

Professor, Dept. of EEE, SRKR Engineering College, Bheemavaram, Andhra Pradesh<sup>2</sup>

Professor, Dept. of EE, Andhra University College of Engineering (A), Visakhapatnam, Andhra Pradesh<sup>3</sup>

**Abstract:** A simple algorithm is presented to solve three-phase unbalanced Radial Distribution Network (RDN). It solves a simple algebraic recursive expression of voltage, and all the data are stored in vector form. This algorithm requires Dynamic Data Structure (DDS) to store the details of the branches. The algorithm uses basic principles of circuit theory and can be easily understood. Mutual coupling between the phases has been included in the mathematical model. The proposed algorithm has been tested with several distribution networks and the result of an unbalanced RDS is presented in the article. IEEE 8 bus and IEEE 25 bus unbalanced radial distribution system results are in agreements with the literature and show that the proposed model is valid and reliable.

**Keywords:** Unbalance radial distribution networks, line model, load flow, 3-phase 4-wire line.

## I. INTRODUCTION

OAD flow technique is very important tool for analysis of power systems and used in operational as well as planning stages. Certain applications, particularly in distribution automation and optimization require repeated load flow solutions. As the power distribution networks become more and more complex, there is a higher demand for efficient and reliable system operation. Consequently, the most important system analysis tool, load flow studies, must have the capability to handle various system configurations with adequate accuracy and speed. Power distribution systems have different characteristics from transmission systems [1], [2]. They are characterized as Radial/weakly meshed structures, Unbalanced networks/loads: single, double and three phase loads, High resistance/reactance( $R/X$ ) ratio of the lines, Extremely large number of branches/nodes, Shunt capacitor banks and distribution transformers, Low voltage levels compared with those of transmission systems and distributed generators [3],[4]. Because of the inherent unbalanced nature of the power distribution system, each bus may be having loads that can be three phase grounded wye or ungrounded delta connected, two-phase grounded or single-phase grounded [5]. The unbalanced nature of power distribution systems requires special three phase component and system models [6]. Due to high  $R/X$  ratio, the conventional Newton Raphson method and Fast Decoupled Load Flow method fail to converge to a solution. Many other researchers [7-10] have suggested modified versions of conventional load flow methods for radial distribution networks with high  $R/X$  ratio.

Basu and Goswami [11] have also proposed a method for solving unbalanced radial distribution networks based on the Newton-Raphson method. Thukaram et al.[12] and Miu et al.[13] have also proposed methods for solving three-phase radial distribution networks.

In this paper a modification is done to the balanced power flow algorithm described in [14] to three-phase unbalanced algorithm. This method involves only recursive algebraic equations to be solved to get the following information:

1. Status of the feeder line, and overloading of the conductor and feeder line currents.
2. Whether the system can maintain adequate voltage remote loads.
3. The line losses in each segment.
4. It can also suggest the necessity of re-routing or network reconfiguration for the existing distribution network.

The algorithm has been developed considering that all loads draw constant power. However, the algorithm can easily accommodate composite load modeling, if the composition of load is known. The algorithm has good convergence property for practical radial distribution networks.

## II. THREE PHASE LINE MODEL

For the analysis of power transmission line, two fundamental assumptions are made, namely:

- Three-phase currents are balanced.
- Transposition of the conductors to achieve balanced line parameters.

However, distribution systems do not lend themselves to either of the two assumptions. Because of the dominance of single-phase loads, the assumption of balanced three-phase currents is not applicable. Distribution lines are seldom transposed, nor can it be assumed that the conductor configuration is an equilateral triangle. When these two assumptions are invalid, it is necessary to introduce a more accurate method of calculating the line impedance.

In this work, Carson's equations of a three-phase grounded four-wire system are used [15]. Carson's equations allow the computation of conductor self-impedance and the mutual impedance between any number of conductors above ground. A simple circuit model is shown in Fig. 1 for a three-phase, four-wire grounded star system. Line charging admittance is neglected at the distribution voltage level. For this four-wire system, Carson's equations lead to the development of an impedance matrix of 4x4 dimension. This matrix is used to calculate conductor voltage drop as shown below. Using Kirchhoff's voltage law, one may write:

$$\begin{bmatrix} V_i^{ag} - V_j^{ag} \\ V_i^{bg} - V_j^{bg} \\ V_i^{cg} - V_j^{cg} \\ V_i^{ng} - V_j^{ng} \end{bmatrix} = \begin{bmatrix} ze_{ij}^{aa} & ze_{ij}^{ab} & ze_{ij}^{ac} & ze_{ij}^{an} \\ ze_{ij}^{ba} & ze_{ij}^{bb} & ze_{ij}^{bc} & ze_{ij}^{bn} \\ ze_{ij}^{ca} & ze_{ij}^{cb} & ze_{ij}^{cc} & ze_{ij}^{cn} \\ ze_{ij}^{na} & ze_{ij}^{nb} & ze_{ij}^{nc} & ze_{ij}^{nn} \end{bmatrix} \begin{bmatrix} I_{ij}^a \\ I_{ij}^b \\ I_{ij}^c \\ I_{ij}^n \end{bmatrix} \quad (1)$$

In a grounded neutral system, the voltages at neutral and ground are the same, thus one gets:

$$V_i^{ng} - V_j^{ng} = 0. \quad (2)$$

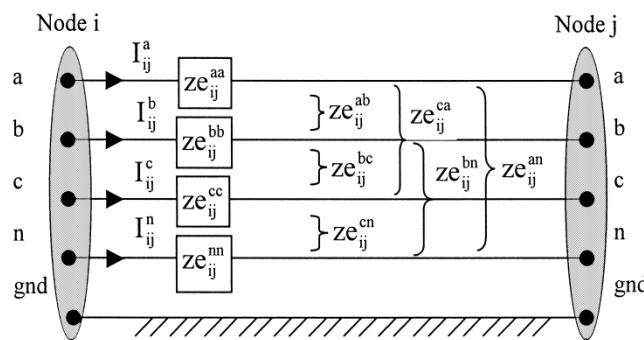


Fig.1. Three phase 4 wire line model.

By substituting equation (2) in (1)

$$\begin{bmatrix} V_i^{ag} - V_j^{ag} \\ V_i^{bg} - V_j^{bg} \\ V_i^{cg} - V_j^{cg} \end{bmatrix} = \begin{bmatrix} V_{ij}^a \\ V_{ij}^b \\ V_{ij}^c \end{bmatrix} = \begin{bmatrix} z_{ij}^{aa} & z_{ij}^{ab} & z_{ij}^{ac} \\ z_{ij}^{ba} & z_{ij}^{bb} & z_{ij}^{bc} \\ z_{ij}^{ca} & z_{ij}^{cb} & z_{ij}^{cc} \end{bmatrix} \begin{bmatrix} I_{ij}^a \\ I_{ij}^b \\ I_{ij}^c \end{bmatrix} \quad (3)$$

Where the values of the impedance elements are computed using the following expression based upon Carson's equation.

$$z_{ij}^{aa} = ze_{ij}^{aa} - \frac{ze_{ij}^{an} * ze_{ij}^{na}}{ze_{ij}^{nn}} \quad (4)$$

Other values of the impedance matrix in equation (3) may be similarly expressed. Fig. 2 represents the final model of three-phase four-wire grounded star system as defined by equation(3).

The same methodology is used to model double and single-phase line sections. For example, in the case of double-phase line involving phases *a* and *c*, equation (3) may be written as

$$\begin{bmatrix} V_i^{ag} - V_j^{ag} \\ 0 \\ V_i^{cg} - V_j^{cg} \end{bmatrix} = \begin{bmatrix} V_{ij}^a \\ 0 \\ V_{ij}^c \end{bmatrix} = \begin{bmatrix} z_{ij}^{aa} & 0 & z_{ij}^{ac} \\ 0 & 0 & 0 \\ z_{ij}^{ca} & 0 & z_{ij}^{cc} \end{bmatrix} \begin{bmatrix} I_{ij}^a \\ 0 \\ I_{ij}^c \end{bmatrix} \quad (5)$$

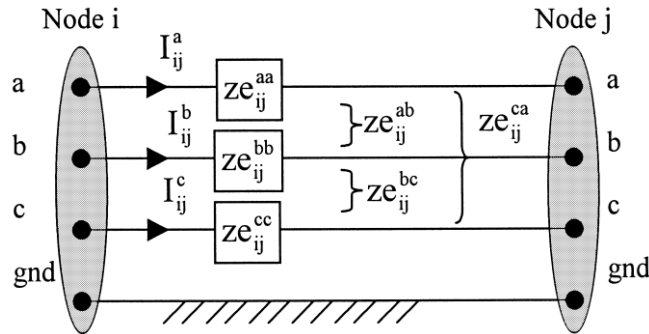


Fig.2. Three-phase three wire line model.

Similarly in the case of single-phase line involving phase *a* only, equation (3) may be written as

$$\begin{bmatrix} V_i^{ag} - V_j^{ag} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} V_{ij}^a \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} z_{ij}^{aa} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{ij}^a \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

However in many cases, in a single-phase line, current returns through the neutral conductor. A different procedure is followed when current returns through the neutral conductor. Fig. 3 shows a single-phase line consisting of phase *a* and neutral.

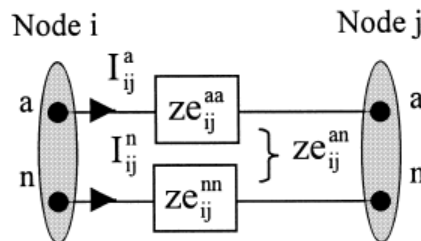


Fig. 3. single phase line section

From Fig. 3, the equation for voltage drop on phase *a* may be written as below:

$$\begin{aligned} V_{ij}^a &= V_i^{an} - V_j^{an} \\ &= I_{ij}^a * [z_{ij}^{aa}] + I_{ij}^n * [z_{ij}^{an}] - [I_{ij}^a * [z_{ij}^{an}] + I_{ij}^n * [z_{ij}^{nn}]] \\ &\text{As all of the phase current returns in the neutral conductor:} \\ &I_{ij}^a = -I_{ij}^n \\ V_{ij}^a &= I_{ij}^a * [z_{ij}^{aa}] - I_{ij}^a * [z_{ij}^{an}] - I_{ij}^a * [z_{ij}^{an}] + I_{ij}^a * [z_{ij}^{nn}] \\ &= I_{ij}^a * [z_{ij}^a] \end{aligned} \quad (8)$$

Therefore for phase *a*, one may write the equation as below:

$$\begin{bmatrix} V_{ij}^a \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} z_{ij}^a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{ij}^a \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

Similarly for phase  $b$  and  $c$ , one may write expressions as below:

$$\begin{bmatrix} 0 \\ V_{ij}^b \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & z_{ij}^b & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ I_{ij}^b \\ 0 \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} 0 \\ 0 \\ V_{ij}^c \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & z_{ij}^c \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ I_{ij}^c \end{bmatrix} \quad (11)$$

In general the voltage equations for unbalanced radial distribution networks may be written as below:

$$\begin{bmatrix} V_i^a \\ V_i^b \\ V_i^c \end{bmatrix} = \begin{bmatrix} V_j^a \\ V_j^b \\ V_j^c \end{bmatrix} + \begin{bmatrix} z_{ij}^{aa} & z_{ij}^{ab} & z_{ij}^{ac} \\ z_{ij}^{ba} & z_{ij}^{bb} & z_{ij}^{bc} \\ z_{ij}^{ca} & z_{ij}^{cb} & z_{ij}^{cc} \end{bmatrix} \begin{bmatrix} I_{ij}^a \\ I_{ij}^b \\ I_{ij}^c \end{bmatrix} \quad (12)$$

Where  $i$  is the sending end node and  $j$  is the receiving end node with phases  $a$ ,  $b$  and  $c$  considering the branch between the two nodes. Then, the voltages at the receiving end node  $j$  may be determined with the knowledge of the sending voltage at node  $i$  by rewriting (13):

$$\begin{bmatrix} V_j^a \\ V_j^b \\ V_j^c \end{bmatrix} = \begin{bmatrix} V_i^a \\ V_i^b \\ V_i^c \end{bmatrix} - \begin{bmatrix} z_{ij}^{aa} & z_{ij}^{ab} & z_{ij}^{ac} \\ z_{ij}^{ba} & z_{ij}^{bb} & z_{ij}^{bc} \\ z_{ij}^{ca} & z_{ij}^{cb} & z_{ij}^{cc} \end{bmatrix} \begin{bmatrix} I_{ij}^a \\ I_{ij}^b \\ I_{ij}^c \end{bmatrix} \quad (13)$$

### III. LOAD MODEL

All the loads are assumed to draw constant complex power ( $S = P + jQ$ ). It is further assumed that all three-phase loads are star connected and all double- and single-phase loads are connected between line and neutral. The three-phase load model is shown in Fig. 4. In Fig. 4 the three-phase load may not be balanced. That is considering node  $i$ ,  $S_i^a$ ,  $S_i^b$ , and  $S_i^c$  can be of different values or even zeroes. In fact, double-phase and single-phase loads are modeled by setting the complex power of the non-existing phases to zero.

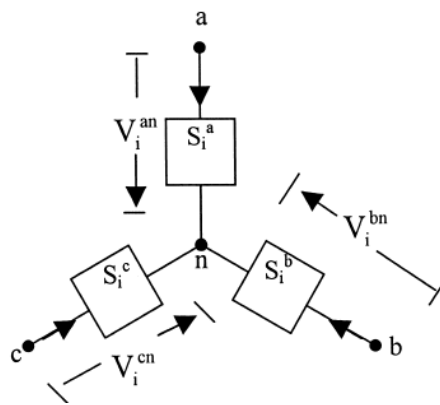


Fig. 4. load model

Fig. 5 shows the single phase line section with load connected at node  $j$  between phase  $a$  and neutral  $n$ , also at node  $k$  between phase  $a$  and neutral  $n$ .

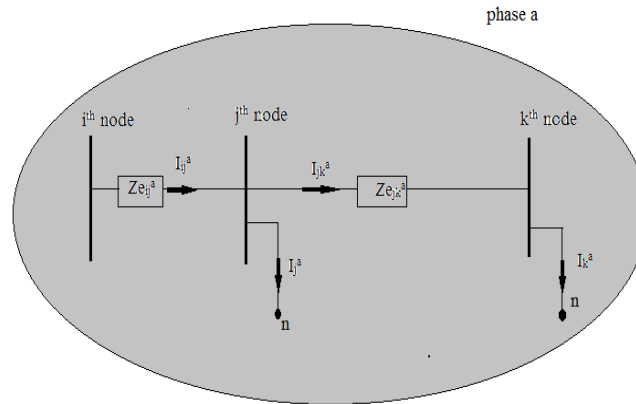


Fig.5. Single phase line section with load connected at node  $j$  between to phase  $a$  and neutral  $n$

In Fig. 4 it is assumed that the complex powers of each phase are known and that the line-to-neutral voltages have been specified. With this known, the load currents are determined by:

$$I_i^a = (S_i^a / V_i^{an})^* \quad (14)$$

$$I_i^b = (S_i^b / V_i^{bn})^* \quad (15)$$

$$I_i^c = (S_i^c / V_i^{cn})^* \quad (16)$$

Also from Fig. 5, the branch current ( $I_{ij}^a$ ) for phase  $a$  can be written as follows

$$I_{ij}^a = I_j^a + I_{jk}^a \quad (17)$$

Similarly, one can write the branch currents for other phases.

#### IV. DATA STRUCTURE AND PSEUDOCODE FOR ITS CREATION

This section presents details of the novel DDS [14] that is used to store details of a branch. One may visualize several attributes for the best data structure. The following attributes were envisioned while developing the DDS.

- (1) The proposed data structure must be dynamic in nature such that it may be created and altered in the execution stage.
- (2) DDS required for holding the information of a branch must be compact and addressable from any function.
- (3) It must be flexible to accommodate any number of buses within a branch.
- (4) It must also be flexible to address (point to) however many data Structures of branches emanate from the end of a branch.

The proposed data structure of a branch needs to hold the details of:

- (a) Parent bus to which the branch is connected;
- (b) Number of buses in the branch;
- (c) An array to store IDS of all the buses in the order of their location from the head of the branch;
- (d) Number of branches emanating from the last bus of this branch;
- (e) A list of pointers pointing to the data structures that store information of emanating branches; and
- (f) An array to store IDS of buses at the head of emanating branches.

The definition of the data structure is given below:

```

Typedef struct {
Integer parent-bus-id, number-of-buses-in-branch, array-of-bus-
ids. Number-of-branches, array-of-head-bus-id-of-emanating-
branches;
Void pointers-to-structure-of-emanating-branches;
} branch
    
```

The pictorial representation of the proposed DDS is shown in Fig. 6. Starting from the main substation, branches are sequentially stored in the data structure using a recursive function that is outlined in the following pseudocode.

```

Void create-structure (first bus of the branch. Address pointer to) of the data structure, parent bus ID number from
which this branch emanates)
}
temp = temporary array to store the IDS of buses in this
branch;
number-of-buses-in-branch = I
temp [first location]= bus-ID= head bus ID
While (number of branches emanating from bus-ID = 2)
{
increase number-of-buses count by 1;
store this bus-ID in temp array;
bus Id changes = next connected bus ID;
}
Dynamically define array-of-bus-IDs to hold bus IDS of buses in the branch.
Store the content of temp in array-of-bus-IDs
Determine number-of-branches emanating from the last bus
this branch - > nbr
If nbr>0
{
Dynamically define array-of-head-bus-id-of-emanating branches
and fillup;
dynamically define pointers-to-structure-of- emanating branches;
Allocate space for nbr data structure and store their addresses in above;
for each of the nbr emanating branches -> call function
Create-structure ();
}
}

```

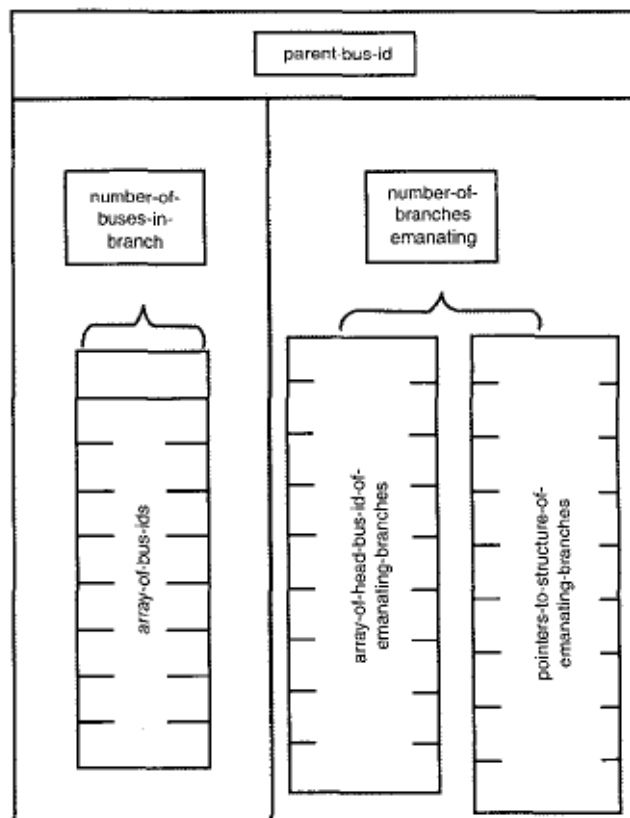


Fig. 6. Schematic diagram of dynamic data structure

The pseudocode presented above creates a structure for a branch. Starting at the main substation bus, the pseudocode forms the first branch by sequentially considering buses until it finds one that has several branches emanating from it. Referring to Fig. 7, which presents a sample radial system. It builds the first branch from bus 1 up to bus 3 where two branches emanate. The data structure created for branch labeled I, starts with bus 01 and ends with bus 03, comprising three buses. It has two branches emanating from it namely II and III. This structure stores pointers that point towards the structures storing details of branches II and III. The pseudocode calls itself twice, once each for branches II and III. This process proceeds until data structures for all the branches are built. A pictorial representation of the data structure storing details of branch II is shown in Fig. 8.

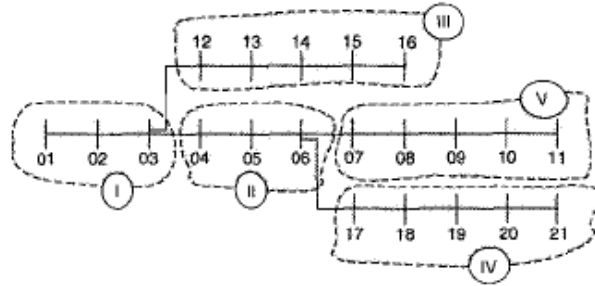


Fig. 7. A simple radial network with several branches

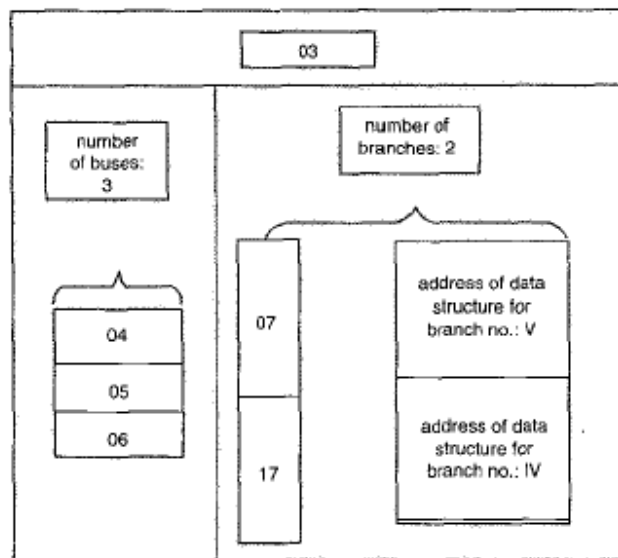


Fig. 8. Data structure for branch II

## V. THE LOAD FLOW ALGORITHM

The first step in the load flow algorithm is to organize the RDS information into a suitable structure. The RDS data must be organized in an appropriate manner so that the load flow program can begin at the right RDS bus and iterate through the rest of the RDS buses in the proper order. The load flow program must start its calculations from the last bus of a terminating RDS branch (the “Start Bus”).

Once the load flow has started from the “Start Bus” (an end bus), it can continue to solve the currents in branches (equation 17) upwards (towards the substation) until the head bus of that particular branch has been reached. When the branch head bus has been reached, the load flow program must look at the branch parent bus and see if the branch currents have been done for all of the buses below that parent bus. Then continue to solve the node voltages (equation 13) downwards (from the substation). The algorithm stops if the changes in the computed bus voltage magnitudes are the same in two successive iterations

The real and reactive power losses in the line between buses  $i$  and  $j$  may be written as:

$$SL_{ij}^a = PL_{ij}^a + jQL_{ij}^a = V_i^a * (I_{ij}^a)^* - V_j^a * (I_{ji}^a)^* \quad (18)$$

$$SL_{ij}^b = PL_{ij}^b + jQL_{ij}^b = V_i^b * (I_{ij}^b)^* - V_j^b * (I_{ji}^b)^* \quad (19)$$

$$SL_{ij}^c = PL_{ij}^c + jQL_{ij}^c = V_i^c * (I_{ij}^c)^* - V_j^c * (I_{ji}^c)^* \quad (20)$$

The overall algorithm for load flow analysis of unbalanced radial distribution network is presented in Fig. 9

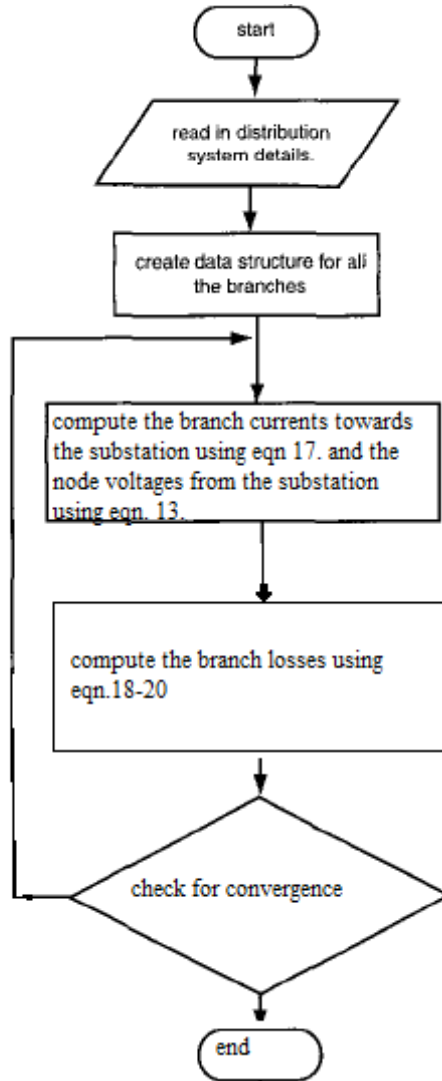


Fig.9. Flowchart for the overall algorithm

## VI. RESULTS

The effectiveness of the proposed method has been explained with two unbalanced radial distribution system.

### A. Example 1

A sample system 14.4 kV of 8 buses shown in Fig. 10 has been taken from the Taiwan Power Corporation [16]. The base values of the system are 14.4 kV and 100 kVA. The convergence tolerance specified is 0.001 p.u. The converged solutions (voltage magnitudes and phase angles) are given in Table III. Method [17] is used for comparison of test results.

Method-1: method reported in [17]

Method-2: proposed method



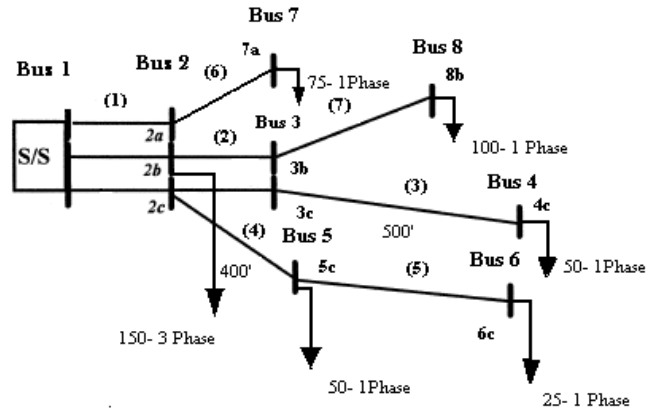


Fig. 10. IEEE 8 Bus unbalanced radial distribution network.

TABLE I IEEE 8 Bus System Data.

Sending end Node	Sending end node phase	Receiv end Node	Receiv end Node Phase	Br. no.	Conf ID.	Br. lngth. (ft.)
1	3	2	3	1	1	1000
2	3	3	2	2	7	400
3	2	4	1	3	5	500
2	3	5	1	4	5	400
5	1	6	1	5	5	400
2	3	7	1	6	3	500
3	2	8	1	7	4	500

TABLE II Configuration IDs

Conf. ID	Phasing	Phase Cond.	Neutral Cond.	Spacing ID	Type
1	ABCN	336,400	336,400	500	3- $\Phi$ 4wir
2	ABCN	1/0	1/0	500	3- $\Phi$ 4wir
3	AN	1/0	1/0	510	1- $\Phi$ 2wir
4	BN	1/0	1/0	510	1- $\Phi$ 2wir
5	CN	1/0	1/0	510	1- $\Phi$ 2wir
6	ABN	1/0	1/0	505	2- $\Phi$ 3wir
7	BCN	1/0	1/0	505	2- $\Phi$ 3wir
8	ACN	1/0	1/0	505	2- $\Phi$ 3wir

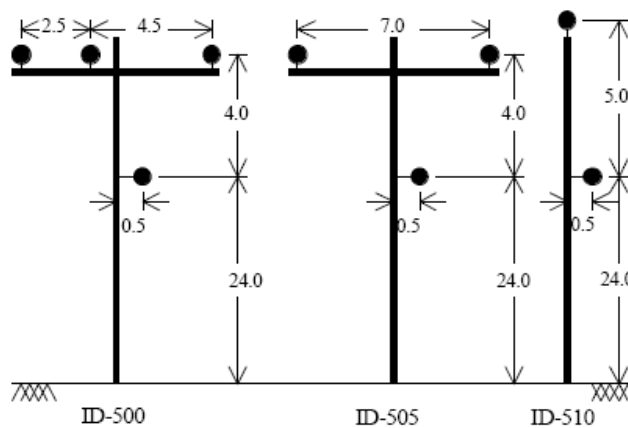


Fig. 11. Overhead Line Spacings.

TABLE III Load flow solution for IEEE 8 bus RDS

Node no.	Method-1		Method-2	
	$ V $ in P.U	$\angle V$ in (rad)	$ V $ in P.U	$\angle V$ in (rad)
2a	0.9974	-0.0007	0.9975	-0.0009
2b	0.9967	-2.094	0.9969	-2.095
2c	0.9968	2.0927	0.9961	2.0918
3b	0.9942	-2.096	0.9944	-2.095
3c	0.9957	2.0921	0.9955	2.0913
4c	0.9937	2.0921	0.9932	2.0913
5c	0.9944	2.0928	0.9934	2.0918
6c	0.9936	2.0928	0.9924	2.0918
7a	0.9945	-0.0007	0.9940	-0.0009
8b	0.9902	-2.096	0.9897	-2.095

### B. Example 2

In this paper, the standard 4.16 kV IEEE 25 Bus Radial Distribution System is used to evaluate the performance of the proposed load flow algorithm. The line data and load data of the system are given in [1]. It is assumed that the transformer at the substation is balanced, voltage regulators and capacitors at various buses is neglected. For the load flow, base voltage and base MVA are chosen as 4.16 kV and 100 MVA respectively. The results are presented in Table IV.

The total system losses were found to be the following in each phase of the radial system:

- Phase A: 14.179 kW, 17.91 kVAr
- Phase B: 8.872 kW, 11.586 kVAr
- Phase C: 10.846 kW, 14.286 kVAr

TABLE IV

load flow solution for IEEE 25 RDS

No-de no.	$ V_a $ in P.U	$\angle V_a$ in rad.	$ V_b $ in P.U	$\angle V_b$ in rad.	$ V_c $ in P.U	$\angle V_c$ in rad.
1	1	0	1	-2.094	1	2.094
2	0.9859	-0.0047	0.9878	-2.097	0.9887	2.089
3	0.9836	-0.0054	0.9849	-2.097	0.9863	2.088
4	0.9822	-0.0058	0.9833	-2.098	0.9856	2.087
5	0.9811	-0.0058	0.9823	-2.098	0.9847	2.087
6	0.9771	-0.0048	0.9816	-2.096	0.9821	2.089
7	0.9750	-0.0048	0.9796	-2.096	0.9803	2.089
8	0.9693	-0.0049	0.9764	-2.096	0.9765	2.089
9	0.9648	-0.0050	0.9748	-2.096	0.9768	2.089
10	0.9619	-0.0050	0.9729	-2.097	0.9771	2.089
11	0.9594	-0.0061	0.9724	-2.097	--	--
12	0.9582	-0.0061	--	--	--	--
13	--	--	0.9716	-2.097	--	--
14	0.9661	-0.0049	0.9729	-2.095	0.9706	2.089
15	0.9640	-0.0049	0.9708	-2.095	0.9687	2.089
16	--	--	--	--	0.9694	2.089
17	0.9693	-0.0049	0.9764	-2.096	0.9765	2.089
18	0.9816	-0.0054	0.9822	-2.097	0.9829	2.087
19	0.9805	-0.0052	0.9807	-2.097	0.9832	2.087
20	--	--	0.9783	-2.097	--	--
21	--	--	--	--	0.9789	2.087
22	--	--	--	--	0.9765	2.087
23	0.9809	-0.0057	0.9817	-2.098	0.9850	2.087
24	--	--	0.9791	-2.099	0.9843	2.087
25	--	--	0.9759	-2.099	--	--

## V. CONCLUSION

In this paper, a simple algorithm for unbalanced distributions system load flow analysis was proposed, which is basically a Forward Backward Sweep based method which requires Dynamic Data Structure (DDS) to store the details of a branches of RDS .The proposed method has good convergence property for any practical distribution networks with practical  $R/X$  ratio. Computationally, this method is extremely efficient, as it solves simple algebraic recursive equations for voltage phasors. Another advantage of this method is all the data is stored in vector form, thus saving enormous amount of computer memory. The proposed algorithm can be used effectively with Supervisory Control and Data Acquisition (SCADA) and Distribution Automation and Control (DAC) as the algorithm quickly gets the voltage solution and can be used to suggest rerouting or network reconfiguration for efficient operation of the system memory.

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