

# A Complete Analysis of the Performance of Extended Kalman Filter for the State Estimation of Three Phase Induction Motor.

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Abstract: In this paper the author have designed and implemented an Extended Kalman filter (EKF) on the simulated model of the three-phase induction motor. An extensive simulation study has been carried out to asses the performances of the filter under various machine operating conditions and model uncertainties. In this work, it has been shown that the performance of EKF is found to be better under normal operating conditions and Initial condition mismatch, loading the machine as well as speed reversal. Here performance of the filter is analyzed as a measure of Mean Square Error.

Keywords: EKF, Induction Motor, State Estimation. Mean square error

#### **INTRODUCTION** L

motor without speed and flux sensors for field oriented (FOC) and direct torque (DTC) control schemes have become popular in the recent years. This is mainly because these methods are able to achieve better dynamic Rotor flux based differential equations are used to develop performance and improved reliability of operation. Use of a speed encoder for velocity control is generally considered undesirable because it adds to overall cost and gives raise to shaft mounting related problems and hence poor reliability [2]. On the other hand, it is possible to estimate rotor speed from machine terminal voltages and currents. Even though several deterministic methods like model reference adaptive system [9] speed adaptive observers [2], neural network based estimation [11] are available they generally suffer from inaccuracy due to noise in measurements and parameter uncertainties. Moreover obtaining accurate measurement of winding parameters like rotor resistance and mutual inductance are difficult [10]. Hence, the actual machine parameters and that of the model will never be the same. Also, parameters such as stator and rotor resistances and inductances change under different operating conditions mainly due to temperature, magnetic saturation, and change in supply frequency [3].

A technique for accurate state estimation under these conditions with less complexity is still quite a challenge [2],[8].State estimation using Extended Kalman filter and their implementation for induction motor has been quite extensive [4]-[7] both in extended and reduced order form due to its inherent ability to handle measurement and model uncertainties. EKF which is used for nonlinear processes basically uses linearization approach to determine the current mean and covariance of the states. But EKF has well known drawbacks such as filter instability due to linearization if sampling time is not In the above equation, x(k) represent the unobserved state small, biasness in its estimate and complex calculation of Jacobian matrices [13].

Estimation of speed and rotor flux position of an induction The main contributions of this work are as follows. A model of three phase induction motor has been developed which is as realistic as actual machine. Here the model is represented as first principle model in state space form. the model.

> It is shown that under different model uncertainties and operating conditions EKF give fairly accurate estimate of states. in certain condition like certain initial state vector differences and model and machine parameter mismatch EKF performance is not satisfactory.

> The model of the three-phase induction motor used for simulation is the rotor flux based state space model [1] represented in stationary reference frame.

#### **EXTENDED KALMAN FILTER** II.

The well knownKalman filter [12] solves the state estimation problem in a stochastic linear system. The extended Kalman Filter (EKF) is probably the most widely used nonlinear filter. For nonlinear problems, the Kalman Filter is not strictly applicable since linearity plays an important role in its derivation and performance as an optimal filter. The EKF attempts to overcome this difficulty by using a linearized approximation where the linearization is performed about the current state estimate. The basic framework for the EKF (and the UKF) involves estimation of the state of anonlinear dynamic system,

$$\mathbf{x}(\mathbf{k}) = \left[ \mathbf{x}(\mathbf{k}-1) + \int_{t_{k-1}}^{t_k} \mathbf{F}[\mathbf{x}(\tau), \mathbf{u}(\mathbf{k})] d\tau \right] + \mathbf{w}(\mathbf{k}) \quad (1)$$
$$\mathbf{y}(\mathbf{k}) = \mathbf{H}[\mathbf{x}(\mathbf{k})] + \mathbf{v}(\mathbf{k}) \quad (2)$$

of the system, u(k) is a known exogenous input and



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y(k) is the only observed signal. We have assumed w(k) and v(k) as a zero mean Gaussian white noise sequences with covariance matrices Q and R respectively.

The symbols F and H represent an n-dimensional function vector and are assumed known. EKF involves the recursive estimation of the mean and covariance of the state under maximum likelihood condition. The function Fcan be used to compute the predicted state from the previous estimate and similarly the function H can be used to compute the predicted measurement from the predicted state.

However, F and H cannot be applied to the covariance directly. Instead a matrix of partial derivatives (Jacobian) is computed at each time step with current predicted state and evaluated. This process essentially linearizes the nonlinear function around the current estimate.

The predicted state estimates are obtained as

$$\hat{\mathbf{x}}(k | k-1) = \hat{\mathbf{x}}(k-1 | k-1) + \int_{t_{k-1}}^{t_k} F[\mathbf{x}(\tau), \mathbf{u}(k-1)] d\tau$$
<sup>(3)</sup>

The covariance matrix of estimation errors in the predicted estimates is obtained as

$$P(k | k-1) = \Phi(k)P(k-1 | k-1)\Phi(k)^{T} + Q$$
 (4)

Where  $\Phi(k)$ , is nothing but jacobian matrices of partial derivatives of F with respect to x

$$\Phi(\mathbf{k}) = \left[\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right]_{[\hat{\mathbf{x}}(\mathbf{k}-1|\mathbf{k}-1),\mathbf{u}(\mathbf{k}-1)]}$$
(5)

Note that the extended Kalman filter (EKF) computes covariances using the linear propagation (Equation 5). The measurement prediction, computation of innovation and covariance matrix of innovation are as follows

$$\hat{y}(k | k-1) = H[\hat{x}(k | k-1)]$$
 (6)

 $\gamma(k | k-1) = \gamma(k) - \hat{\gamma}(k | k-1)$ (7)

$$V(k) = C(k)P(k|k-1)C(k)^{T} + R$$
 (8)

Where C(k) is the jacobian matrix of partial derivatives of H with respect to x.

$$\mathbf{C}(\mathbf{k}) = \left[\frac{\partial \mathbf{H}}{\partial \mathbf{x}}\right]_{\left[\hat{\mathbf{x}}(\mathbf{k}-\mathbf{l}|\mathbf{k}-\mathbf{l}),\mathbf{u}(\mathbf{k}-\mathbf{l})\right]}$$
(9)

The Kalman gain is computed using the following equation

$$K(k) = P(k | k-1)C(k)^{T}V^{-1}(k)$$
(10)

following equation

$$\hat{x}(k \mid k) = \hat{x}(k \mid k-1) + K(k)\gamma(k \mid k-1)$$
(11)

The covariance matrix of estimation errors in the updated state estimates is obtained as

$$P(k | k) = [I - K(k)C(k)]P(k | k - 1)$$
(12)

#### III. SIMULATION RESULTS AND ANALYSIS

Simulations of Induction motor and state estimation with EKFhave been carried out using MATLAB programming in open loop condition. For process, the load is applied as an external input. Simulation result shows that EKF is able to track the speed as well as load disturbance. The induction motor specifications and parameters used in this paper are given in the table. 1.

TABLE 1
MACHINE SPECIFICATION AND PARAMETERS
USED

P(Kw)	3	V(volt)	380
f(Hz)	50	Poles	4
I(amp)	6.9	Nm(rpm)	1430
$J_L(kgm^2)$	0.05	$B_L$	0.01
T <sub>L</sub> (Nm)	20	Ls(H)	0.23
$R_s(\Omega)$	2.283	Lr(H)	0.23
Rr'(Ω)	2.133	Lm(H)	0.22

In this work, it is assumed that only the stator currents are measurable and the load disturbances were treated as additional state to be estimated to avoid generation of biased state estimates in EKF. The sample time chosen was 1ms and simulations were conducted for 1000 sampling instances.

A limiting condition forced on filter algorithm is that a negative estimate value is made equal to zero when the motor voltage sequence is such a way that it will be in the positive direction of rotation and vice versa. It is assumed that the machine was supplied with sinusoidal voltage for simulation. The tuning parameter values and other specifications of the filter are listed in table 2.

TABLE 2 SPECIFICATIONS USED IN EKF ALGORITHM

Process Noise covariance matrix	Q= Diag{ $1.81e^{-5}$ , $1.81e^{-5}$ , $2.5e^{-7}$ , $2.5e^{-7}$ , $2.5e^{-7}$ , $1.62e^{-3}$ , $1e^{-4}$ }
Measurement Noise covariance matrix	R= Diag{7.225 $e^{-3}$ , 7.225 $e^{-3}$ }
Initial state error covariancematrix	$P = 1000 I_{(6x6)}$

The updated state estimates are obtained using the The performance of the filter with motor mechanical speed  $(\omega_r)$  as reference state were studied under following conditions and mean square error (MSE) was used as performance index.

> Throughout the analysis the measurement noise was fixed as constant known value with zero mean and variance: Diag  $\{7.225e^{-3}, 7.225e^{-3}\}$ . Simulations were carried out in



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a pentium 4,1GB internal RAM based system. The EKF algorithm took around 20-30 mS duration for one step prediction of states..

#### A. Initial state mismatch.

Simulation studies were conducted with different initial state values and was observed that the performance of the filter is affected mainly due to difference in speed initial values between process and model. Accordingly simulations were carried out for different initial settings at low speed and near rated speed levels. Moreover for realistic realization the filter estimation were started at different instances after the machine has been started. The initial values of currents are assumed to be the previous measured values and the initial flux values are randomly chosen within their limit. It should be stated that identical realization of state noiseand measurement noise has been introduced in all the simulation trials. EKF estimate is slightly more accurate at high speed initial value mismatch than lower speed initial value mismatch. The MSE value and estimation time for different initial state differences are indicated in Table 3.

TABLE 3 MEAN SQUARE ERRORS AND ESTIMATION TIME AT DIFFERENT INITIAL STATE VECTOR MISMATCH

Initial conditions	MSE	Tim e
At $150^{\text{th}}$ sampling instant $\hat{\mathbf{x}} = [\text{Id}(149), \text{Iq}(149), 0.4, 0.4, 500, 0]$	1.952	0.02 46
At $150^{\text{th}}$ sampling instant	2.182	0.02
$\hat{\mathbf{x}} = [Id(149), Iq(149), 0.4, 0.4, 1500, 0]$	4	05
At 50 <sup>th</sup> sampling instant	4.749	0.02
$\hat{x} = [Id(49), Iq(49), 0.4, 0.4, 200, 0]$	7	90
At 50 <sup>th</sup> sampling instant	4.097	0.02
$\hat{x} = [Id(49), Iq(49), 0.4, 0.4, 500, 0]$	1	78

#### B. Model parameter mismatch:

As mentioned earlier due to inaccuracy in measurement methods and variation in operating conditions, parameters of machine and model will never be identical which may lead to poor estimation and hence control. Here, simulation studies were conducted considering both scenario mentioned.

#### Case-1:

In first case, initially both machine and model are assumed to have the same value and at a certain instant, a step change in the machine parameters were introduced. Assuming an extreme condition fig. 2.a shows expanded view of filter response for a case where the machine parameters were given a step change of 70% increase in resistances and 30% decrease in inductances at 300<sup>th</sup> instant. Table 4 shows the corresponding MSE and estimation time.

TABLE 4. MEAN SQUARE ERROR FOR SUDDEN CHANGE IN PROCESS PARAMETER

Parameter mismatch condition	MSE	Time
Machine parameters at 300 <sup>th</sup>		
instant: Resistances 70%	1.7034	0.0281
increase; Inductances 30%		
decrease		

Case 2:

In the second case, it was assumed that model parameters excluding machine-load inertia and frictional coefficient are either higher or lower than that of actual machine and studies were conducted. Fig.2.b shows the filter response for model parameters 1.1 times that of machine's nominal value. It was observed that for parameter mismatch conditions, EKF failed to converge. Error index and estimation time for some cases are given in table 5.

TABLE 5: MEAN SQUARE ERROR FOR PROCESS- MODEL PARAMETER MISMATCH

Parameter mismatch condition	MSE	Time
Model: 0.9 times nominal	34.74	0.0285
Model: 0.9 times nominal	Failed	
Model: 1.1 times nominal	Failed	
Model: Rsistances1.2 times; Inductances 0.9 times nominal	Failed	

# C. Different Tuning conditions:

Studies were conducted by assuming different values of Q under the assumptions the process noise is unknown. However it is assumed that measurement noise covariance is known and kept constant. Q values were gradually increased and the corresponding mean square error and prediction time were shown in Table 6. The value of Q which gives minimum MSE is assumed as the process error covariance for the algorithm throughout the simulation.

TABLE 6. MEAN SQUARE ERROR FOR DIFFERENT STATE NOISE VARIANCE IN MACHINE.

Process error added (standard deviation in % of nominal value)	MSE	Time
0.05	0.0478	0.0251
0.1	0.0543	0.0248
0.5	0.1907	0.0300
1	0.3591	0.0271

# D. Application of load and Speed reversal:

In this test initially machine was started with no load and full load was applied at 500<sup>th</sup> instant. In another test again the machine was started at no load and the speed was



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reversed by changing the voltage phase sequence at 500<sup>th</sup> instant. Figure 3 shows the speed, direct axis current and torque estimations under load application. Figure 4 depicts the speed estimations under speed reversal condition. In both cases EKF is able to track the states with less error as shown in table 7

TABLE 7. MSE FOR LOAD APPLICATION AND SPEED REVERSAL.

Conditions	MSE	Time
Loading Condition	1.59	0.0292
Speed reversal	0.117	0.0281

#### IV. CONCLUSION

In this paper a three phase induction motor is developed and different states are estimated using extended Kalman filter based state estimators. A comprehensive simulation study was carried out for state estimation specifically speed of a three-phase induction motor. Simulation studies were conducted at different operating conditions. The result shows that for most of the operating conditions, where the change is not large enough, EKF is found to be satisfactory and converges quickly. However this simulation study also shows that under model-process parameter mismatch condition, EKF failed to converge in most cases.



Fig 1 Evolution of true and estimated state (speed) for a moderate initial state mismatch condition at lower,  $\hat{\mathbf{x}} = [Id(49), Iq(49), 0.4, 0.4, 200, 0]$ 



Fig.2.aExpanded view of true and estimated state (speed) for a step increase of 70% in resistances and decrease of 15% in inductances of machine from nominal value at 300<sup>th</sup> instant.







Fig.3.a Evolution of true and estimated speeds under loaded Condition









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Fig.4. Evolution of true and estimated state (speed) for reversal of machine rotation at 500<sup>th</sup> instant

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