



# Comparison of CORDIC and Modified CORDIC II Algorithm

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**Abstract:** CORDIC algorithm is also known as digit by digit method. It is a special purpose digital computer for real time application. This is a unique computational technique which is suitable for trigonometric, square root, Logarithm, Exponential function. CORDIC algorithm calculates by breaking down the angle into number of rotation whereas the modified CORDIC calculates by breaking the rotation into nanorotation and makes use of novel set of angles which reduces the number of iterations, thus accuracy is increased and latency is reduced. In this paper we have carried out MATLAB simulation to determine the reduced number of iterations, increased accuracy and reduction in latency between existing CORDIC and modified CORDIC II algorithm.

**Keyword:** CORDIC, trigonometric, logarithm, exponential function.

## I. INTRODUCTION

CORDIC algorithm is an effective algorithm which is applicable for square root, Logarithmic, Exponential function and for digital computer. In computation unit trigonometric functions are very crucial; at present many mathematical functions require Sine, Cosine, Tangent etc and by using this algorithm it is easy to compute.

With on going technology and limitations on power, operating frequency and energy consumption, if we are generating any trigonometry functions by using of multiplier, adder, divider, those architectures consumes more hardware and computational time increase for reduction of this problem. CORDIC algorithm is converted into hardware form which is known as CORDIC processor. This processor reduces the problem of division and multiplication. In CORDIC processor we can compute the functions by using of shifter, adder and subtractor. In present era CORDIC algorithm is used in many applications like Multimedia, Digital Signal Processing. First CORDIC algorithm is converted into hardware so it was facing some problems like scale factor, time consuming etc. In CORDIC algorithm many modification is done but still it facing many problems so as for future scope this CORDIC processor require many modification.

As we know in present era every multimedia and aerospace based application is require fast processing unit. We also know if any system is complete based on General purpose processor so efficiency of the complete system will be reduce. For application specification there is need of dedicated application specific processor.

So here for the aerospace and multimedia application this is based on trigonometric function. So for those kind of application there is need of specific process which is known as CORDIC processor.

## II. LITERATURE SURVEY

THE CORDIC algorithm [1] to calculate rotations in digital systems. Its main principle is it breaks down the rotation angle in a sum of angles, and carries out the rotation by a series of the rotation by these angles. The benefit of the CORDIC algorithm is that the rotations are calculated by simple shift-and-add operations, which is very efficient in hardware.

Many changes of the CORDIC algorithm have been proposed. Different approach which is made previously to calculate general rotations, this means that they rotate by any angles provided as an input of the rotator. Constant rotators used for specific sets of rotation angles are studied in [2].

Among rotators we find numerous variations of the CORDIC algorithm. Some works combine several rotation stages into a single stage [3], [4] in order to reduce the number of iterations of the CORDIC. The work in [5] is based on



skipping rotations. Some approaches focus on representing the rotation using a Taylor series approximation [6]–[8]. Other approaches divide the rotations into a fine part [9]. There are papers which focus on reducing the rotation memory [9]–[11]. Scaling-free CORDIC approaches pursue to compensate the scale factor of the CORDIC [6], [7]. In this paper [12] a pipelined CORDIC design with the minimum number of adders. It is the modified CORDIC II algorithm. It differs from previous approaches, in this set of micro-rotations, called angle set is followed. The new set of micro-rotations provides a fast convergence of the rotation angle. This leads to a reduced latency and a smaller number of adders than in previous CORDIC algorithms.

### III. CORDIC ALGORITHM

#### A. Rotations in Digital Systems

This section reviews key concepts related to rotations in digital systems. Further information can be found in [2]. In a digital system, a rotation by an angle can be described as a multiplication by a complex coefficient  $C=A+jB$

$$\begin{bmatrix} Y_D \\ Z_D \end{bmatrix} = \begin{bmatrix} A & -B \\ B & A \end{bmatrix} \begin{bmatrix} Y_D \\ Z_D \end{bmatrix} \quad (1)$$

Where  $y+jz$  is the input and  $Y_D+jZ_D$  is the result of the rotation.  $A$  and  $B$  are bit integer numbers in 2's complement in the range  $[-2^{b-1}, 2^{b-1}-1]$ . They are obtained from the rotation angle as [14]

$$A = R(\cos\alpha + \varepsilon_c) \quad (2)$$

$$B = R(\sin\alpha + \varepsilon_s)$$

Where  $\varepsilon_c + \varepsilon_s$  are the quantization errors of the cosine and sine components and  $R$  is the scaling factor. The output  $Y_D+jZ_D$  is also scaled by  $R$ .

The rotation error is the distance between the exact rotation and the actual rotation due to the quantization. If the rotator has multiple rotation the corresponding co-efficient  $C_i=A_i+jB_i$  the rotation error is calculated

$$\varepsilon = \max_i(\varepsilon(i)) = \max_i(\sqrt{\varepsilon_c^2(i) + \varepsilon_s^2(i)}) \quad (3)$$

Finally, the effective word length is the number of bits of the output that are guaranteed to be accurate [2] and is calculated from the rotation error as

$$WL_E = -\log_2 \varepsilon = \varepsilon / (2\sqrt{2}) = -\log_2 \varepsilon + 3/2 \quad (4)$$

#### B. CORDIC algorithm

The CORDIC algorithm considers the coefficients  $C = A + jB = 2^k + j\delta^k$ , where  $\delta^k \in \{-1, 1\}$  and  $k = 0, \dots, M$  is the micro-rotation stage. The corresponding angles are  $\alpha^k = \tan^{-1}(B/A) = \delta^k \tan^{-1}(2^{-k})$ . This is shown in Fig. 1.

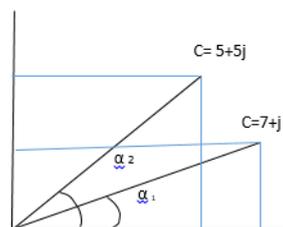


Figure 1 :Example of friend angles for  $C_1=7+j$  and  $C_2=5+j5$

The CORDIC algorithm breaks down the rotation, angle into a sum of micro rotation i.e.

$$\theta = \sum_{k=0}^M \alpha_k + \varepsilon_k \quad (5)$$

where  $\varepsilon$  is the remaining phase error. Each rotation stage calculates

$$\begin{bmatrix} Y_D \\ Z_D \end{bmatrix} = \begin{bmatrix} 2^k & -\delta^k \\ \delta^k & 2^k \end{bmatrix} \begin{bmatrix} Y \\ Z \end{bmatrix} \quad (6)$$

The rotation error at each rotation stage and the word length is  $WL \varepsilon = \infty$ . This means that the coefficient  $C_k$  rotates exactly  $\alpha_k$  degrees and the scaling factor for both angles in each micro-rotation is the same. The latter is always true, as the coefficients are conjugated.



IV. BASIC ANGLE SETS

There are three new types of angle sets proposed for modified CORDIC II, which are described in the following.

A. Friend Angles

We define friend angles as a set of angles  $\alpha_i$  for which there exists a set of coefficients  $C_i = A_i + jB_i$  with angles  $\alpha_i$  i.e.,  $\alpha_i = \tan^{-1}(B_i/A_i)$  whose magnitude is the same, i.e.,  $\forall i, j, |C_i| = |C_j|$ . As all the coefficients have the same magnitude, a kernel composed by friend angles  $\alpha_i$  does not have any rotation error. This is equivalent to say that  $WL_E = \infty$ . The angles  $\alpha_1 = 8.13^\circ$  and  $\alpha_2 = 45^\circ$  are an example of friend angles. For these angles there exist the coefficients  $C_1 = 7+j$  and  $C_2 = 5+j5$  whose angles are  $\alpha_1$  and  $\alpha_2$ , respectively, and  $|C_1| = |C_2| = \sqrt{50}$ . This example is shown in Figure 2.

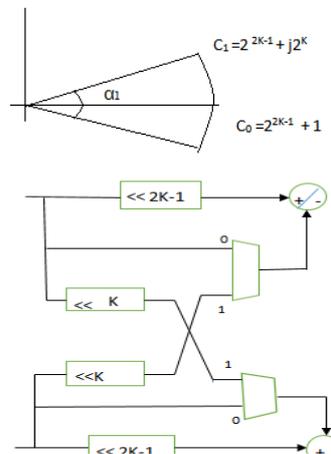


Figure 2: CORDIC. (a) Graphical representation of the coefficients. (b) Hardware circuit.

A property that can be extracted from the definition of friend angles is that any angle  $\alpha$  is friend to itself and also to  $-\alpha + n\pi/2$  and  $\alpha + n\pi/2$  for any value of  $n$ . According to this property, the angles used in the CORDIC for each microrotation stage are friend angles. This happens because each micro-rotation only considers the pair of angles  $\pm\alpha_k$ .

B. Nano-Rotations

Nano-rotations refer to the kernel formed by the coefficient set

$$C_k = A + jk, k = 0, \dots, N, \quad (4)$$

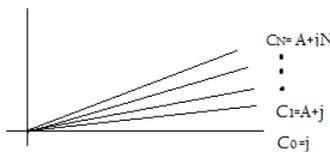


Figure 3: Kernel to calculate nano-rotations.

Where  $A$  is constant and the corresponding angles are

$$\alpha_k = \tan^{-1}(k/A) \quad (5)$$

In (4),  $N$  is considered to be much smaller than  $A$ . This makes  $\alpha_k$  small and fulfills  $\alpha_k \approx \tan(\alpha_k)$ . This leads to  $\alpha_k \approx k/A$ , which is a kernel with equally distributed angles. The fact that  $N \ll C$  also makes the scaling of the coefficients very similar. Figure 3 shows the kernel to calculate nano-rotations. It can be observed that the angle changes by simply changing the value of the imaginary part. Figure 5 shows the  $WL_E$  as a function of the largest angle of the kernel,  $\alpha_N$ , where

$$\alpha_N(\text{rad}) = \tan^{-1}(N/C) \approx N/C \quad (6)$$

Note that  $WL_E$  only depends on  $\alpha_N$  independently of the number of angles,  $N$ . Figure 5 shows that  $WL_E$  larger than 15 bits is achieved for angles smaller than  $\alpha_N = 1^\circ$ . Thus, to design the rotator, the angle  $\alpha_N$  must be selected first, according to the range of input angles. Then,  $N$  is selected. Finally, the constant  $C$  is obtained from (6).



### V .THE MODIFIED CORDIC II ALGORITHM

Figure 4 shows the architecture of the modified CORDIC II rotator includes detailed information about each rotation stage. The modified CORDIC II algorithm consists of six rotation stages in pipeline that use the angle sets describes in previous sections.

Stage 1: The first stage calculates trivial rotations by  $\pm 180^\circ$  and  $\pm 90^\circ$  to set the remaining angle in the range of  $\pm 45^\circ$ . The hardware architecture for the trivial rotator is shown in Fig. 4.

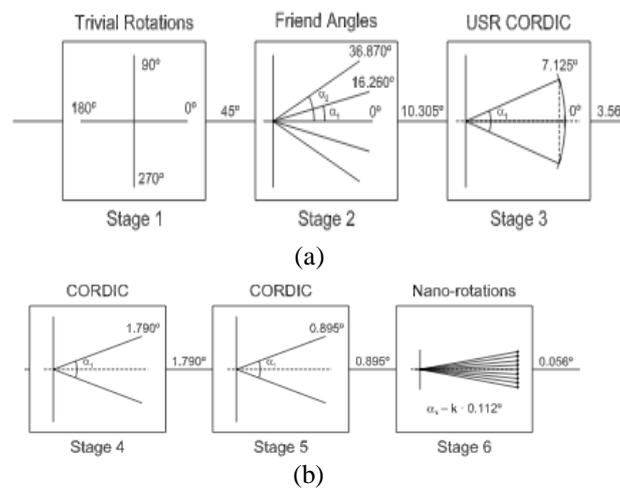


Figure 4: Architecture of the modified CORDIC II rotator.

It uses two negators, which are approximately equivalent to half an adder each, and four 2:1 multiplexers.

Stage 2: The second stage of the modified CORDIC II algorithm uses friend angles. It consists of the kernel  $[25, 24 + j7, 20 + j15]$ . The scale factor for all the coefficients is  $R=25$ , as  $625=25^2=24^2+7^2=20^2+15^2$ . Thus, there is no rotation error and  $WL_E = \infty$ . The friend angles that correspond to the coefficients are  $0^\circ, 16.260^\circ, 36.870^\circ$ , with normalized scaling  $R_{norm}=1.563$ , according to

$$R_{norm} = R/2^{\log_2 R} \quad (7)$$

The hardware architecture for the friend angle stage is shown in Fig 4. It consists of five adders and seven 2:1 multiplexers, and can calculate all the rotations of the kernel depending on the configuration of the multiplexers. In Table II, two additional (+2) multiplexers are needed between stages in order to rotate the entire kernel (positive and negative rotations), as in [11].

Stage 3: The third stage of the modified CORDIC II algorithm uses the USR CORDIC. It consists of the kernel  $[129, 128 + j16]$ . This stage reduces the remaining angle to  $\pm 3.563^\circ$ . The hardware architecture for the USR CORDIC stage is as shown in Fig. 4(b) for  $k=4$ . It consists of two adders and two 2:1 multiplexers.

Stages 4 and 5: The fourth and fifth stages of the modified CORDIC use conventional CORDIC rotations by  $1.790^\circ$  and  $0.895^\circ$ .

Stage 6: The sixth stage uses nano-rotations. The kernel used is

$$C_k = 512 + jk, \quad k=0, \dots, 8.$$

The rotation angles of the kernel are  $\alpha_k = k \cdot 0.112^\circ$ . The remaining angle of the modified CORDIC is  $\pm 0.056$ . The hardware circuit for the nanorotation stage shows how the multiplication by  $k$  is implemented. The decoder in Fig. 4(b) consists of a few logic gates.

Stages 6 b is and 7 b :An alternative to the sixth stage of the modified CORDIC is to add one more CORDIC rotation (stage 6 b) followed by a nano-rotator. This increases the WL of the nano-rotator and reduces the remaining angle of the modified CORDIC II bis to  $\pm 0.028^\circ$ .

Note also that the modified CORDIC II provides convergence for the entire circumference, as  $\alpha_{in}$  for each stage is larger than  $\alpha_{out}$  of the previous stage. Finally, the control logic is similar to [11]: By representing the angle in the range  $[0, 1]$  the first three bits determine the trivial rotations, stages 2 and 3 use comparators, and the control for the rest of stages is obtained directly by representing the angle proportionally to the minimum rotation angle.



VI. COMPARISON AND RESULT

Figure 5 &6 compares the Approximation error and reduced number of iterations carried out by function of  $\cos(\pi/5)$  for both CORDIC and modified CORDIC II algorithm. Here there is the reduction of error due to the changes in the rotation and angles which has been taken for the calculation of CORDIC and modified CORDIC.

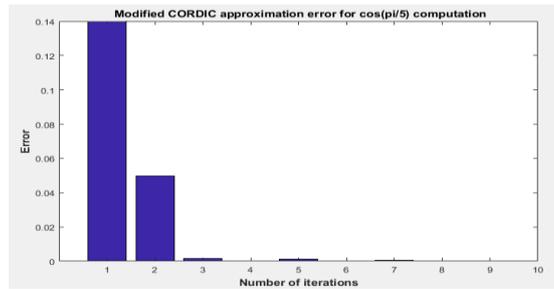


Figure 5 : Results of the Modified CORDIC II algorithm interms of reduced number of iteration and error

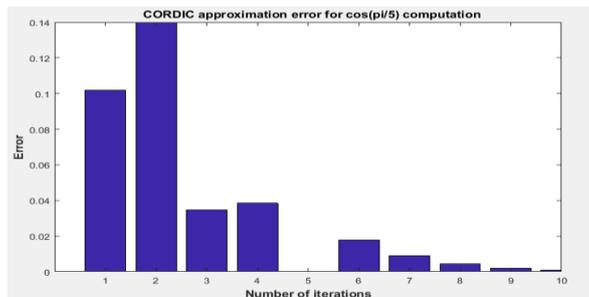


Figure 6: Results of the CORDIC algorithm interms of reduced number of iteration and error

CORDIC close 50% and the reduced number of iteration reduces the adders in the Modified CORDIC II at the cost of number of adders, as shown in Figure 7.

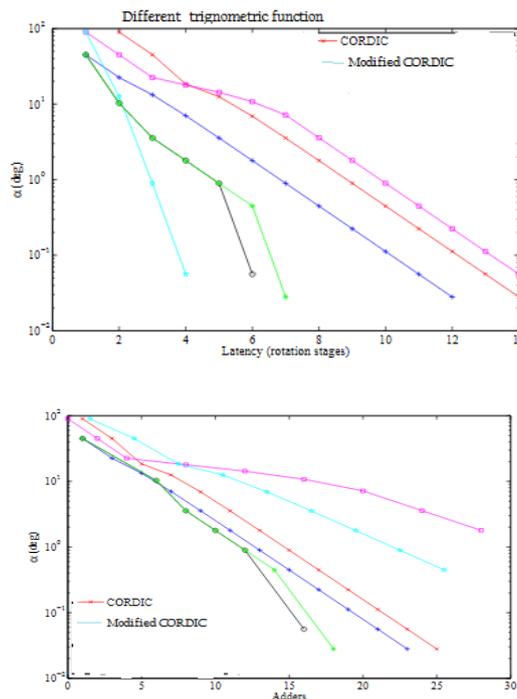


Figure 7 : Results of CORDIC and Modified CORDIC II for latency and reduced adders



### VII. CONCLUSION

The Modified CORDIC II is a new algorithm that substitutes the CORDIC rotation by a new angle set. This involves two new types of rotators: friend angles and nanorotations. By using the proposed nanorotations, the modified CORDIC II requires to reduce number of adders and decrease latency.

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