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Model Reference Adaptive Fault Tolerant Flight Control Framework for Structural Damage in Aircraft

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Abstract: This paper examines an Adaptive fault tolerant flight control for an aircraft that experiences structural damage. A damage degree is introduced for parameterization of the damaged aircraft dynamics. Demonstrating of the damaged flight dynamics and control configuration is presented by following numerical examination of a Boeing 747-100 aircraft model. Here a damage tolerant control framework is introduced for aircraft suffering with vertical tail damage. Model Reference Adaptive control (MARC) procedure is used to address the stability recuperation and robustness of performance in keeping up safe flight operation of the aircraft. The adequacy of the proposed control approach is approved in examination with a standard Linear Quadratic Regulator (LQR) outline through numerical recreations.

Keywords: Adaptive control, Fault tolerant flight control (FTFC), Model reference adaptive control (MRAC), Linear quadratic regulator (LQR), Structural damage.

1. INTRODUCTION

Air transportation is irrefutably one of the most secure methods for transportation as of now, on account of the improvement and development. In any case, on a few events, episodes that include various setbacks did happen. Mechanical fault or failure to parts of the aircraft is the second most reason for plane crashes after pilot blunder, representing around 22% of all aeronautical accidents. November 12, 2001, an episode has happened in which the vertical tail isolated from a passenger carrier. An A300 slammed without its 27-foot vertical stabilizer because of Rudder deflections amid the last 8 seconds of flight which developed lateral forces resulting in the tail failure.

In the aeronautical community, the concentration of study is to discover improvement in technology that a portion of the revealed accidents might be possibly avoided from happening or endure less disastrous effect. One thought is the idea of aircraft fault tolerant control. In any case, so far there has been genuinely little examination and study on structural damage in aviation field contrasted to faults of actuator and sensor. As we as whole know, structural damage generally incorporates the partial loss of wing, vertical tail damage, horizontal tail loss, engine damage etc. On account of fault tolerant control for vertical tail damaged aircraft where our analysis interest lies, research work is found in both the aircraft dynamics oriented analysis and the control oriented examination. The damage on aircraft geometric shape significantly changes the aerodynamic attributes, regularly spoken by stability and control derivatives. Wind tunnel tests conducted by NASA affirm that changes of the stability derivatives are around nearly proportional corresponding to the percentage of loss in structure. [2]

Our principle commitment in this paper is to outline an adaptive fault tolerant controller for the aircraft with damaged vertical tail, in our case Model reference adaptive control which can keep up aircraft's stability and dependability under various damage degrees. The flight dynamics modelling of a damaged aircraft, control definition, and point by point design process are presented. The developed controllers are tried on the linear simulations utilizing a Boeing 747-100 aircraft model with vertical tail damage. The simulations results and comparative performance investigation of the developed controllers conclude the paper.

2. MATHEMATICAL MODELLING

The aircraft dynamics model has been developed using the given data. It is a standard practice to consider the model as linearized around a steady level flight operating point. The linearized state-space for the motion of aircraft is mentioned as: [10]

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$$\frac{d}{dt}\begin{bmatrix} u \\ w \\ q \\ \theta \\ v \\ p \\ r \\ \emptyset \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g\cos\theta & 0 & 0 & 0 & 0 \\ \bar{Z}_u & \bar{Z}_w & \bar{Z}_q & -g\sin\theta & 0 & 0 & 0 & 0 \\ \bar{M}_u & \bar{M}_w & \bar{M}_q & \bar{M}_\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_v & Y_p & (Y_r - V_{TAS}) & g\cos\theta \\ 0 & 0 & 0 & 0 & \bar{L}_v & \bar{L}_p & \bar{L}_r & 0 \\ 0 & 0 & 0 & 0 & \bar{N}_v & \bar{N}_p & \bar{N}_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.00 & \tan\theta & 0 \end{bmatrix} * \begin{bmatrix} u \\ w \\ q \\ \theta \\ v \\ p \\ r \\ \emptyset \end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_f} & 0 & 0 \\ \bar{Z}_{\delta_e} & \bar{Z}_{\delta_f} & 0 & 0 \\ \bar{M}_{\delta_e} & \bar{M}_{\delta_f} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{Y}_{\delta_a} & \bar{Y}_{\delta_r} \\ 0 & 0 & \bar{L}_{\delta_a} & \bar{L}_{\delta_r} \\ 0 & 0 & \bar{N}_{\delta_a} & \bar{N}_{\delta_r} \\ 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} \delta_e \\ \delta_f \\ \delta_a \\ \delta_r \end{bmatrix}$$

Where

$$X = [u \quad w \quad q \quad \theta \quad v \quad r \quad p \quad \emptyset\ ]^T;$$
$$U^T = [\delta_e\ \delta_f\ \delta_a\ \delta_r];$$

This is the generalized representation for a state space model.

$$\dot{X} = AX + BU;$$

Where A and B are the corresponding matrix coefficients as shown above.

2.1 Damaged Aircraft Modelling:

At the point when the aircraft experiences structural damage, for example vertical tail damage, the damage actuated aerodynamic attributes change is represented by comparing the stability and control derivatives. In flight dynamics, stability derivatives and control derivatives measure how much specific forces and moments acting on an aircraft change when there is a little change in flight condition or the diversion of control surfaces, for example velocity, height, angle of attack and so forth. They are extremely basic in flight control for investigation and design. Figure 1 demonstrates an aircraft with fractional loss of vertical tail. [10] Plainly the aerodynamic center of vertical tail moves alongside with the damage. [2]

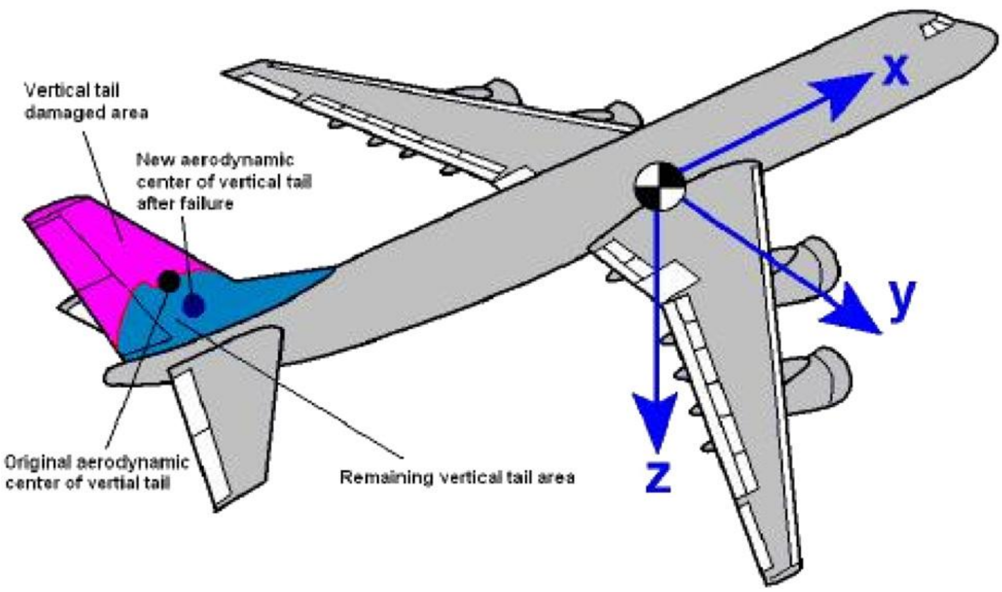

Fig. 1 An aircraft with partial vertical tail loss, resulting in ac shift of vertical tail.

In order to model the dynamics of aircraft experiencing vertical tail damage, the damage-induced derivatives are to be estimated. In general, the vertical tail damage/loss would have significant impact on lateral and directional dynamic behaviours. As a result, the following lateral stability derivatives are of our main concerns in the study case of vertical tail damage: [2]

$$C_{y_\beta} \quad C_{n_\beta} \quad C_{l_\beta} \quad C_{y_p} \quad C_{n_p} \quad C_{l_p} \quad C_{y_r} \quad C_{n_r} \quad C_{l_r}$$

For an aircraft suffering from vertical tail damage, the estimation of the considered stability and control derivatives, where the deviation in the derivative value such as C_{y_β} is represented by ΔC_{y_β}. Furthermore, the parameter μ, the

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damage degree such that the deviation ΔC_{y_β} is factored by μ, $0 \le \mu \le 1$, as $\Delta C_{y_\beta} = \mu\, \Delta C_{y_\beta}^{max}$, where $\mu\, \Delta C_{y_\beta}^{max}$ represents the maximum damage here total vertical tail loss. μ is the parameter representing the damage degree. Specifically, μ = 0 represents the conventional case, μ = 1 represents the most critical damage on the tail loss, i.e., the complete loss, and 0 < μ <1 represents partial vertical tail loss.

And Δ denote the change in the derivatives caused by the damage. In correspondence, the set of lateral derivative changes of our interest are represented by [2]

$$\Delta C_{y_\beta} \quad \Delta C_{n_\beta} \quad \Delta C_{l_\beta} \quad \Delta C_{y_p} \quad \Delta C_{n_p} \quad \Delta C_{l_p} \quad \Delta C_{y_r} \quad \Delta C_{n_r} \quad \Delta C_{l_r}$$

The linearized flight dynamic model of the damage induced aircraft in the state space representation is given as

$$\dot{X} = (A - \mu\bar{A})X + (B - \mu\bar{B})U$$

The specific expressions for the system coefficients $\bar{A}$ and $\bar{B}$ are given below. [2]

$$\bar{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta Y_v & \Delta Y_p & \Delta Y_r & 0 \\ 0 & 0 & 0 & 0 & \Delta L_v & \Delta L_p & \Delta L_r & 0 \\ 0 & 0 & 0 & 0 & \Delta N_v & \Delta N_v & \Delta N_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad \bar{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta Y_{\delta_r} \\ 0 & 0 & 0 & \Delta L_{\delta_r} \\ 0 & 0 & 0 & \Delta N_{\delta_r} \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Where

$$X = [u \quad w \quad q \quad \theta \quad v \quad r \quad p \quad \emptyset\ ]^T;$$
$$U^T = [\delta_e\ \delta_f\ \delta_a\ \delta_r];$$

Where the maximum changes in the dimensional aerodynamic derivatives are given below. [2]

Table 1: Maximum changes of aerodynamic derivatives

$$\Delta Y_v = \tfrac{1}{2}\rho u_0 S \Delta C_{y_\beta}^{max} \qquad \Delta Y_p = \tfrac{1}{4}\rho u_0 S b \Delta C_{y_p}^{max}$$
$$\Delta L_v = \tfrac{1}{2}\rho u_0 b S \Delta C_{l_\beta}^{max} \qquad \Delta L_p = \tfrac{1}{4}\rho u_0 S b^2 \Delta C_{l_p}^{max}$$
$$\Delta N_v = \tfrac{1}{2}\rho u_0 b S \Delta C_{n_\beta}^{max} \qquad \Delta N_p = \tfrac{1}{4}\rho u_0 S b^2 \Delta C_{n_p}^{max}$$
$$\Delta Y_r = \tfrac{1}{4}\rho u_0 S b \Delta C_{y_r}^{max} \qquad \Delta Y_{\delta_r} = \tfrac{1}{2}\rho u_0^2 S \Delta C_{y_{\delta_r}}^{max}$$
$$\Delta L_r = \tfrac{1}{4}\rho u_0 S b^2 \Delta C_{l_r}^{max} \qquad \Delta L_{\delta_r} = \tfrac{1}{2}\rho u_0^2 S \Delta C_{l_{\delta_r}}^{max}$$
$$\Delta N_r = \tfrac{1}{4}\rho u_0 S b^2 \Delta C_{n_r}^{max} \qquad \Delta N_{\delta_r} = \tfrac{1}{2}\rho u_0^2 S \Delta C_{n_{\delta_r}}^{max}$$

3. DAMAGE TOLERANT CONTROL DESIGN

The nominal control design is based on a linear quadratic regulator (LQR). This will be in the form of normal feedback control system with fixed feedback gain. But, this gain is chosen in a way such that it will provide optimum performance of the system. The general representation of feedback control law will be of the form [11]

$$v(t) = kx(t);$$

Where K ε R 4×8 is a feedback gain matrix chosen to minimize a quadratic performance index

$$J = \int_0^\infty (x^T Q x + u^T R u) dt;$$

Where $Q = Q^T > 0$ and $R = R^T > 0$ are the weighting matrices;
For $P = P^T > 0$ satisfying alzebric recatti equation which is given below in eq.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0;$$

K is a feedback gain matrix, obtained by the eq.

$$K = -R^{-1}B^T P.$$

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3.1 Adaptive Controller Design:
In Adaptive controller design the man objective is to minimize the error measured between the reference model and the damaged aircraft model.

The state feedback control law [11]

$$U(t) = \overline{K}x(t) + \overline{k} + \overline{\emptyset};$$

Where,

$$\overline{K} = [\overline{K}_1, \overline{K}_2, \overline{K}_3, \dots\dots \overline{K}_m]^T \in R^{m\times n};$$
$$\overline{k} = [\overline{k}_1, \overline{k}_2, \overline{k}_3, \dots\dots \overline{k}_m]^T \in R^{m\times m_r};$$
$$\overline{\emptyset} = [\overline{\emptyset}_1, \overline{\emptyset}_2, \overline{\emptyset}_3, \dots\dots \emptyset_m]^T \in R^{m\times 1}.$$

The above mentioned equations are the parameters updated from control law

$$\overline{K_i} = -\alpha_i x e^T PB, i = 1,2,3 \dots, m;$$
$$\overline{k_i} = -\beta_i r_d e^T PBf_1, i = 1,2,3 \dots, m.$$

Where e is the error.

4. SIMULATION RESULTS IN THE CASE OF B747-100/200

4.1 Aircraft Parameters and Flight Conditions:
Here the developed control system design along with the conventional LQR controller is verified and analysed for a given damage degree through numerical recreations. Boeing 747-100 numerical model used in the research is steady and cruise flight. The atmospheric conditions and the estimates of the stability and control derivatives are corresponding linearized model of the aircraft. [2]

Using the given atmospheric conditions and the stability and control derivatives the calculated A and B matrices are given below. [2]

$$A = \begin{bmatrix} -0.0119 & 0.0237 & -11.4761 & -32.1804 & 0 & 0 & 0 & 0 \\ -0.1086 & 0.5165 & 6544.8360 & -1.1238 & 0 & 0 & 0 & 0 \\ 0.000 & -0.0020 & -0.6444 & 0.0002 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1068 & 0 & -673.00 & 32.1804 \\ 0 & 0 & 0 & 0 & -3.5276 & -0.8442 & 0.3088 & 0 \\ 0 & 0 & 0 & 0 & 3.6534 & -0.0401 & -0.2479 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.000 & 0.0349 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -25.1385 & -98.7583 & 0 & 0 \\ -1.6895 & 0.0155 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9.5858 \\ 0 & 0 & 0.2219 & 0.1030 \\ 0 & 0 & 0.0155 & -0.6208 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Due to the damage introduced to the aircraft the changes in A and B matrices are given below mentioned as A_1 and B_1.

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -11.1668 & -1.3423 & 7.0467 & 0 \\ 0 & 0 & 0 & 0 & -0.1507 & -0.0446 & 0.2014 & 0 \\ 0 & 0 & 0 & 0 & 7.0513 & 0.0652 & -0.3431 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0. & 0 \end{bmatrix}; \quad B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9.5858 \\ 0 & 0 & 0 & 0.1030 \\ 0 & 0 & 0 & -0.6208 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Where the damage degree μ considered in this case is 30% (or) 0.30.

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4.2 Controller Design Results:
In order to compare the developed controller designs the gain of the LQR controller ($Q = I_8$ and $R = I_4$) is

$$K = \begin{bmatrix} 0.709 & -0.599 & -38.133 & -48.636 & 0.005 & -0.002 & 0.500 & -0.0062 \\ -0.670 & -0.812 & 16.862 & 41.531 & -0.010 & 0.004 & -0.264 & 0.0031 \\ -0.020 & 0.0002 & 2.231 & 2.085 & -0.002 & 0.428 & -0.593 & 0.988 \\ -0.002 & -0.0005 & -0.182 & 0.498 & 0.962 & -0.536 & -20.983 & 0.713 \end{bmatrix};$$

The MARC controller design gain for 30% loss of vertical tail is

$$G = \begin{bmatrix} 0.876 & -0.316 & -44.807 & -62.120 & 0.000 & 0.000 & 0.000 & 0.000 \\ -0.427 & -0.953 & 11.422 & 25.830 & -0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & -0.002 & 0.428 & -0.593 & 0.988 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.962 & -0.536 & -20.983 & 0.713 \end{bmatrix};$$

	Elevator	Aileron	Rudder	Flap
Maximum (deg)	17	20	25	20
Minimum (deg)	-23	-20	-25	-20

Table 2: Control surface deflection ranges [2]

4.3 Performance Evaluation, Analysis and Explanation:
Simulations are based on the Boeing 747-100 numerical model at 30% damage degree. The control surface deflection points of confinement are set by the table. Two execution estimations are presented, to be specific the maximum response value and settling time. In particular, maximum response value here is the maximum deviation of the final settling value to the reference. Settling time is the time moment at which states finally are contained in the locale of 2% around the equilibrium.

In this simulation, Note that the result of the standard LQR control is also listed for comparison. Simulation results are given for each proposed fault tolerant control approach. The results of flight states (u, w, q, θ, v, p, r,Ø) are given for both the control strategies below

Table 3: LQR vs. MRAC.

	Damage 5%				Damage 30%			Damage 50%		
	Maximum Response Value		Settling Time		Maximum Response Value	Settling Time		Maximum Response Value	Settling Time	
	LQR	MRAC	LQR	MRAC	LQR	LQR	MRAC	LQR	LQR	MRAC
u	1.43×10^{-4}	1.175×10^{-3}	10.5	10.4	1.25×10^{-4}	9.8	10.3	3.4×10^{-4}	9	10.35
w	-13.7×10^{-3}	-3.31×10^{-3}	6	6	-13.9×10^{-3}	7	6	-13.9×10^{-3}	7	6
q	-23×10^{-6}	-5.7×10^{-6}	9	8.5	-24×10^{-6}	8	9	-23×10^{-6}	7	9
θ	-10.8×10^{-6}	-2.8×10^{-6}	6	8.5	-10.5×10^{-6}	9	8.5	-10.8×10^{-6}	8	8.5
v	1.8×10^{-3}	1.23×10^{-3}	7	7	2.12×10^{-3}	6	4.8	2.8×10^{-3}	4	2.7
p	-4.54×10^{-3}	-3.36×10^{-3}	6	6.5	-6.31×10^{-3}	9.5	4.7	-9.07×10^{-3}	7	2.4
r	5.16×10^{-4}	2.65×10^{-4}	8.5	4	4.15×10^{-4}	9	3	2.19×10^{-4}	5	2.1
Ø	8×10^{-3}	3.42×10^{-3}	8	6	6.82×10^{-3}	10	5	3.76×10^{-3}	6	2.4

The maximum response value for the MRAC control scheme is almost same for different damage degrees, as the main objective of the control design is to mimic the performance of the damaged aircraft as that of undamaged. The key observation in analysis of the model reference controller is the settling time and whereas in the nominal controller both the response value and the settling time change based on the damage degree.

A number of perceptions are produced using the data from the table 3 and figures 2 to 5. From that we know, the aircraft longitudinal dynamics are nearly not influenced. It is normal since the harmed vertical tail basically causes the progressions of aerodynamic derivatives in the horizontal course. The expansion of damage degree is directly proportional to the downfall of the performance.

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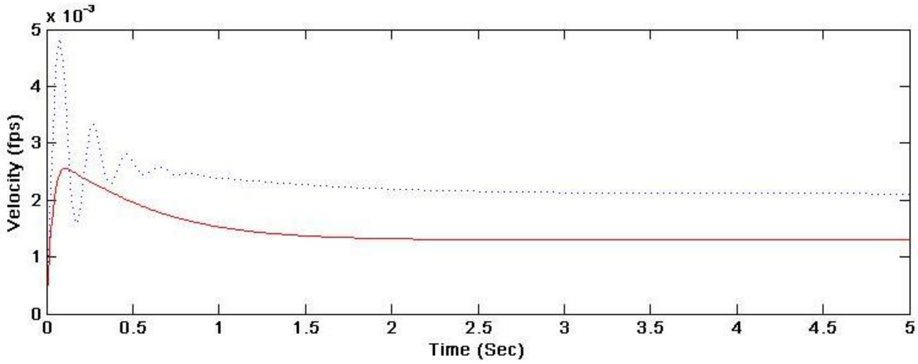

Fig. 2: Vertical Velocity

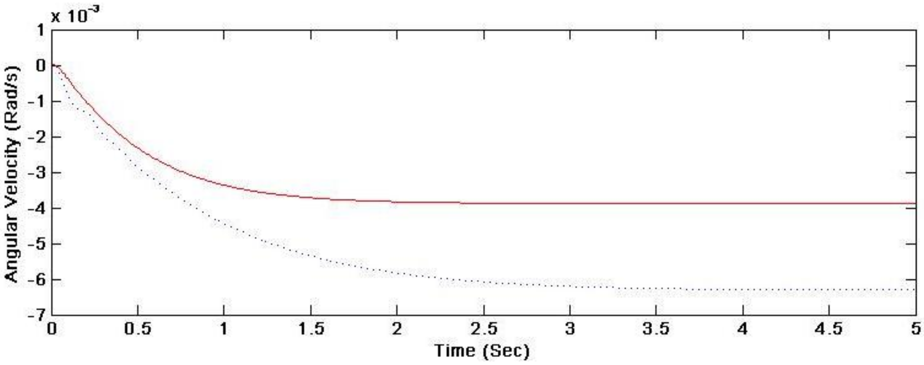

Fig. 3: Roll Rate

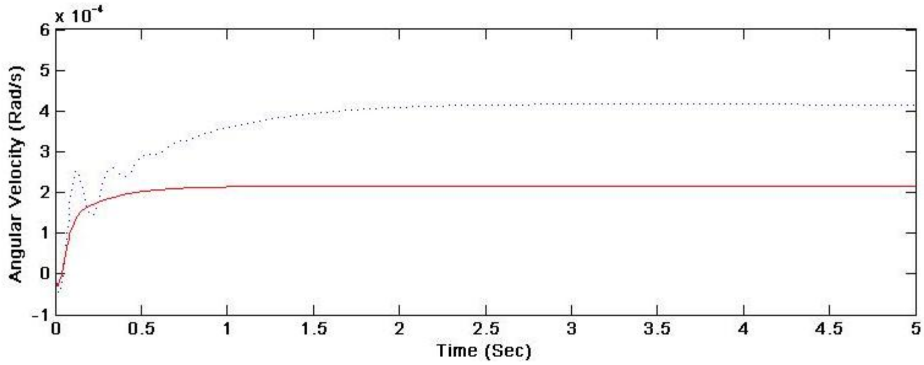

Fig. 4: Yaw Rate

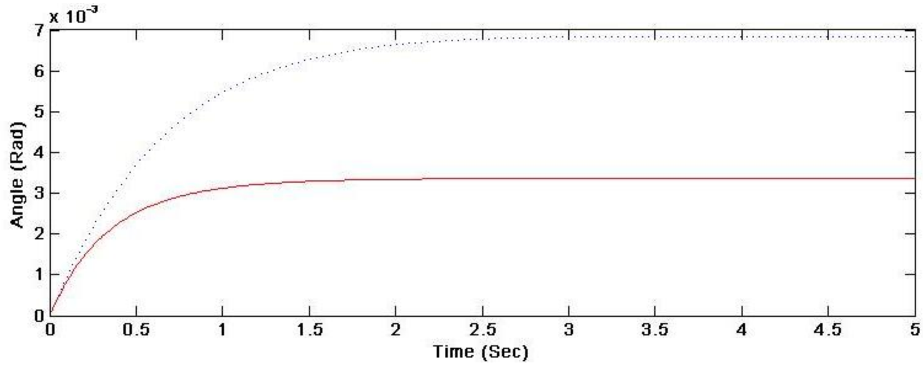

Fig. 5: Roll Rate

The response values for MRAC are represented using the solid red line in the graphs. And the blue dotted line in the above specified graphs represent the LQR controller response for various state vectors for 30% vertical tail loss.

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5. CONCLUSION

This paper explored fault tolerant control to airplane that experiences structural damage, particularly the vertical tail damage or loss. A damage degree was embraced to parameterize the damaged flight dynamics as a linearized dynamics model. The outline calculations were exhibited and connected to a Boeing-747 100 model through numerical recreations. The conventional control design LQR results and the adaptive control MARC results are compared and the maximum response value is much lower in MARC. Various degrees of damage are introduced to evaluate the performance of both the controllers. The reproduction comes about likewise demonstrated that the fault tolerant control and its execution are restricted once the requirements on control surfaces are considered. Right now, the control approaches give design parameters that permit to appropriate tuning to diminish the impact of control immersion to some degree. Its ability is as yet constrained. Outline change that considers the actuator faults is under scrutiny. Advanced controller designs are also under investigation for fine response values and maximum tolerance for structural damage.

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