

# A Review of Reconstruction of Hyperspectral Images from Random Projections

Layana Raj<sup>1</sup>, Nishanth Augustine<sup>2</sup>

Department of Electronics and Communication Engineering, LBS College of engineering, Kasaragod, India<sup>1,2</sup>

**Abstract:** Hyperspectral imagery provide accurate and detailed information extraction than possible with any other type of remotely sensed data. This improvement comes with computational complexity and over dimensionality. There is an increase in interest in dimensionality reduction through random projections due in part to the emerging paradigm of compressed sensing. CS exploits the fact that many signals are sparse in the sense that they have concise representation in certain basis called dictionary. Traditionally some transform based fixed dictionaries such as DFT, DCT, DWT are used which are relatively easy to analyse. But they are over simplistic and for certain real time data such as hyperspectral images they offer less accuracy. An approach that has been recently proven to be very effective is adaptive dictionary learning technique in which the dictionary is constructed adaptively using the input image for better sparsity. While learning the dictionary the most important computational challenge is the solution of corresponding optimization problem. The reconstruction strategies like compressive projection principal component analysis, multi-hypothesis prediction method and several class dependent strategies were proposed for the reconstruction of hyperspectral imagery from random projections. In this paper, a brief review of various dictionary learning methods and reconstruction techniques for HSI is presented along with their performance evaluation.

**Keywords:** Hyper Spectral Images, Compressive Sensing, Principal Component Analysis (PCA), Multi Hypothesis Prediction.

## 1. INTRODUCTION

Remote sensing refers to the technology [1] of recording, observing and recognizing (sensing) objects or events at far away (remote) places. Instruments may use a part or several parts of the electromagnetic spectrum as an information carrier. Now the current remote sensing technique is in a path of emergence, which is able to capture multispectral and even a hyperspectral images. The invention of hyperspectral images is a milestone in remote sensing, it provide image data containing both spectral and spatial information.

Hyperspectral imaging is now a reasonably familiar concept in the world of remote sensing [2]. The ‘hyper’ in hyperspectral image refers to large number of measured wavelength bands. If a picture is worth 100 words, then a hyperspectral image is equivalent to 10,000 words. It collects and processes information from the electromagnetic spectrum. The main advantage of hyperspectral images are obtaining the spectrum for each pixel in the image of a scene, the sensor acquires the light intensity for a large number of contiguous spectral bands. The rich spectral information is expected to improve the performance of image-analysis techniques and provides extraction of more accurate and detailed information than possible with any other type of remotely sensed data, this improvement often comes with the cost of over-dimensionality, computational complexity, and statistical ill-conditioning. Besides this, high-dimensional HSI data set also increases communication costs associated with the data transmission from the remote sensor. As a consequence, some efficient method for spectral dimensionality reduction of HSI data is almost always required before it can be used in image-analysis applications.

To overcome the logistical and computational challenges involved in dealing with high-dimensional data, we often find the most concise representation of a signal. The fundamental fact that builds up Compressive sensing is, we can represent many signals using only a few non-zero coefficients based on suitable basis or dictionary. A signal is said to be sparse signal if that can be compactly expressed as a linear combination of a small number of elementary signals. CS [3,5] relies on two principles: sparsity, which related to the signals of interest and incoherence, which pertains to the sensing modality. The two methods that can be used to reduce the dimension are band selection and feature extraction. Sparse representation seeking for the most compact representation of a signal using a dictionary. The signal representation error in sparse representation varies largely depending on the dictionary [4]; therefore, the dictionary is chosen so as the error is minimized. A framework for finding optimal dictionaries for simultaneous sparse signal representation is important. The dictionary can be either fix as a pre-specific set of functions such as analytic-based dictionary or learned from a training set to fit a given set of signals (learning-based dictionary). Construction of analytical-based dictionary is simple in which the atoms are created using a stationary function such as cosine and sine

or wavelet functions. Time-Frequency Dictionaries, DCT, Gabor Transform, Wavelet Transform, Contourlet Transform, etc., are existing. Amongst these DCT and Wavelet Transform are commonly used.

The purpose of this paper is to survey and provide some of the key mathematical insights underlying dictionary construction for sparse representation of images. The rest of this paper organized as follows. The basis of sparse representation and dictionary learning is presented in section II. Various dictionary construction algorithms are explained in section III. Analysis of different dictionary construction scheme along with their comparison by using different images is included in section IV. Finally the paper is concluded in section V.

## II. SPARSE REPRESENTATION AND DICTONARY LEARNING

Sparse representation is a popular method to acquire, represent and compress signal using a few atoms in an adequate dictionary. Sparse coding is the process of computing the representation matrix  $X$  for the given input data  $Y$  and the dictionary  $D$ , and it is commonly performed by pursuit algorithm.

A vector space  $S$  of finite dimension  $L$  can be represented as the linear combination of  $M$  vectors ( $M > L$ ). These  $M$  vectors of finite dimension  $L$  are not independent but they spans  $S$ . The overcomplete set  $F$  of  $M$  vectors is not a basis of  $S$  but it is a frame also called dictionary or codebook. Any vector,  $s$ , in the set  $S$  can be expressed as the linear combination of the frame vectors, but because of the linear dependence of the frame vectors, the expansion is not unique. The basic idea when using a frame is that we have more vectors and thus a better chance of finding a small number of vectors whose linear combination matches with the signal vector well. Finding the optimal vectors to use in this approximation is an NP-hard problem and requires extensive calculation. So a suboptimal technique is preferable in order to limit the computational complexity.[4]

A dictionary learning problem is a matrix factorization in which the goal is to factorize a training data matrix,  $Y$ , as the product of a dictionary,  $D$ , and a sparse coefficient matrix,  $X$ ; i.e.  $Y = DX$ . Here the aim of dictionary learning is to learn the dictionaries  $D$  and an associated sparse code  $X$  for each band of  $Y$ .

## III. DICTIONARY CONSTRUCTION ALGORITHMS

The major problem associated with sparse representation-based processing is how to choose the dictionary. There are many pre-specified dictionaries, e.g. Fourier, Gabor, Discrete Cosine Transform (DCT), and wavelet. Though being simple and having fast computations, these non-adaptive dictionaries are not able to efficiently (sparsely) represent a given class of signals. The alternate choice is to use adaptive dictionaries. In this approach, a dictionary is learned from some training signals belonging to the signal class of interest. These adaptive dictionaries outperform the non-adaptive ones in many signal processing applications.

### A. DISCRETE COSINE TRANSFORM (DCT)

DCT has excellent properties of energy compaction[6]; This represents an image as a sum of sinusoidal functions of varying magnitudes and frequencies, thus transforming the image from spatial domain to frequency domain. Most of the visually significant information about the image is concentrated only in a few coefficients. This helps to discard the coefficients that are relatively small without introducing any visual distortion in the reconstructed image. Generally there are mainly two types of DCT: one dimensional (1-D) DCT and two dimensional (2-D) DCT. Since 2D matrix is used for representing an image ,so 2-D DCT is considered in this research work. The input image to be transformed is divided into square blocks each and compute DCT coefficient for each block.

The dictionary coefficients corresponding to sparse representation are obtained by taking inverse DCT.

The 2-D IDCT for an input image of dimension of  $M \times N$  can be defined as follows,

$$f(x, y) = \frac{2}{\sqrt{MN}} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} c(u)c(v)F_c(u, v) \cos \left[ \frac{\Pi}{2N} (2x + 1)u \right] \cos \left[ \frac{\Pi}{2M} (2y + 1)v \right]$$

where

$$C(x) = \begin{cases} \frac{1}{\sqrt{2}}, & x = 0 \\ 1, & x = 1, 2, \dots, N - 1 \end{cases}$$

The discrete cosine transform (DCT) helps to separate the image with respect to its visual quality into different spectral sub-bands of differing importance.

In this paper in sparse coding stage for approximating the input data under the selected dictionary orthogonal matching pursuit algorithm (OMP) is used. Advantages of DCT are it independent on the input image data thus we obtain a

general transformation for every image, computational complexity and computation time can be reduced, the coefficients obtained will be real values, and reduce the correlation between the pixels.

### B. DISCRETE WAVELET TRANSFORM (DWT)

Wavelet can represent a signal in time-frequency domain [7], the transformation will preserve spatial information. Analysing a signal gives more information by this kind of representation about when and where of different frequency components. Because of the multi-resolution property of DW it can analyze different frequencies by different resolutions. The discrete wavelet transform (DWT) is an implementation of the wavelet transform using a discrete set of the wavelet scales and translations will obey some defined rules. In other words, by this transform the signal is decomposed into mutually orthogonal set of wavelets.

The basic concept behind wavelet transform [8] is to hierarchically decompose an input signal into a series of successively lower resolution reference signals and their associated detail signals, so it decomposes the image signal into number of sub-band signals which are with different spatial resolution. This is effected by choosing suitable basis functions that allow for this changes in the time extension are expected to conform the corresponding analysis frequency of the basis function. At each level, the reference signal and the detail signal for the information needed to reconstruct the reference signal at the next higher resolution level. Wavelet transform (WT) is first applied on the signals to get their sparse representation of frequency and directional characteristics. Hence, the low frequency characteristics of long and high frequency characteristics of short features could be dealt simultaneously.

Wavelet transform is the sum of over all time of the signal multiplied by the scaled and shifted versions of the wavelet function

Step 1: choose a wavelet and compare it to a section at the start of original signal

Step 2: Calculate a number 'c' that represent how closely correlated with this section of signal. The higher the value of c, more the similarity.

Step 3: Shift the wavelet to the right and repeat the step 1,2 until you have covered the whole signal.

Step 4: Stretch the wavelet and repeat step 1 – 3

Wavelet are obtained by single wavelet prototype called mother wavelet by dilation and shifting

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}} \Psi\left(\frac{t-b}{a}\right)$$

a → scaling parameter      b → shifting parameter

Images can be decomposed into four parts by two-dimensional Wavelet Transform. In fact, the decomposition can continue until the size of the sub-image is as small as you want. By setting some parts of its sub-images, we can reduce the quantity of information; in other word by setting the useless data we can compress the image.

Analysis proves that wavelet based CS techniques is provide better image reconstruction quality than obtained by the practical image compression standards like JPEG. Wavelet transform based CS techniques are more accurate and powerful as the wavelets have time-frequency location and multiresolution characteristics. The block artefacts introduced in the reconstructed image by the DCT can overcome by wavelet transform. They are spatially compact and fast transforms and scalable to large images.

Table1: examples of wavelet transform

Mother wavelet	Wavelet function
Haar transform	$\Psi(t) = \begin{cases} 1 & 0 \leq t < \frac{1}{2} \\ -1 & \frac{1}{2} \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$
Mexican hat	$\Psi(x,y) = -\frac{1}{\pi\sigma^4} \left( 1 - x^2 + y^2 \right) e^{-x^2 + y^2}$
Dyadic transform	$\psi_2^j(x) = \frac{1}{2^j} \psi\left(\frac{x}{2^j}\right)$

### C. K SINGULAR VALUE DECOMPOSITION ALGORITHM (K-SVD)

This section introduces the K-SVD [9] algorithm for training of dictionaries. This algorithm is flexible and works in simultaneously with any pursuit algorithm. It is simple and designed to be a truly direct generalization of the K-Means. The K-SVD is highly efficient, for an effective sparse coding, and a Gauss-Seidel-like accelerated dictionary update method. K-SVD is an iterative method that alternates between sparse coding of the examples based on the current dictionary and updating of the dictionary atoms to better fit the data. The update of the dictionary columns is combined with an update of the sparse representations, and thus accelerating convergence. The algorithm's steps are coherent with each other, both working towards the minimization of a clear objective function.

The KSVD algorithm has 2 steps [10]

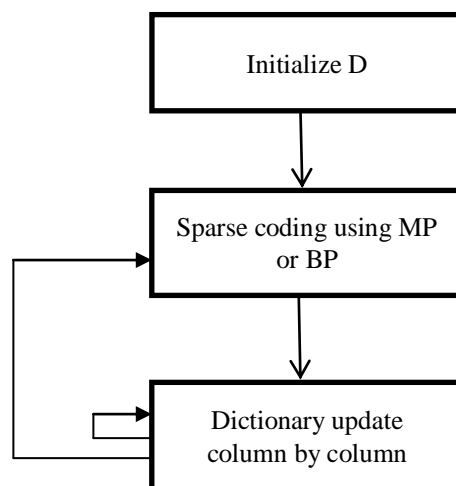
(i) Sparse coding : Producing the sparse representation matrix  $X$ , given the current dictionary  $D$

(ii) Dictionary update (the main innovation) : updating the dictionary atoms, given the current sparse representation

Once the sparse coding task is done, second stage get performed to search for a better dictionary. This process updates one column at a time, fixing all columns in  $D$  except  $d_k$ , and finding a new column  $\tilde{d}_k$  and new values for its coefficients that best reduce the MSE.

The dictionary atoms are formed by KSVD technique, and orthogonal matching pursuit algorithm is used for decomposing the signal to adapt the dictionary.

The main contribution of KSVD is it perform dictionary update atom by atom in a simple and efficient way rather than a matrix inversion method.



### D. METHOD OF OPTIMAL DIRECTION

Many algorithms are available for learning the dictionary. Such as K Singular Value Decomposition (K-SVD), method of optimal direction (MOD) and Sequential generalization of K means (SGK). These methods are computationally less complex than ODL and their performance is superior to that of ODL. The main contribution of the MOD method is its simple and efficient way of updating the dictionary atoms.

The MOD [12] algorithm is summarized as follows.

Choose an initial frame  $F_0$ . Set  $i=1$ ;

Find an approximation for each training vector  $U_i$  using a vector selection algorithm and calculate all the residuals,  $r_i = U_i - \tilde{U}_i$ .

All frame vectors  $f_i$  are adjusted according to equation

$f_i = f_i + \delta_i$ , where  $\delta_i$  is the residual. The frame vectors are then normalized to unit length to get  $F_{i+1}$

Repeat step 2 and 3 until the stopping criterion is satisfied.

MOD updates all the atoms in parallel as the minimum mean square error (MMSE) solution for given  $Y$  and  $X_i$ , while the MOD method assumes known coefficient at each iteration it derive best possible dictionary at each stage.

K-Singular Value Decomposition (K-SVD) is well-known algorithm, which has been very successful. In its dictionary update stage, only one atom is updated at a time. Moreover, while updating each atom, the non-zero entries in the associated row vector  $X$  of are also updated. This leads to a matrix rank-1 approximation problem which is then solved via performing an SVD operation. This algorithm derives its name from its sequential update of atoms using singular value decomposition (SVD).

K-SVD is advantageous over MOD in terms of speed and accuracy. K-SVD in its present form fails to retain any structured sparsity while MOD retains any structured sparsity. In K-SVD, the use of SVD interferes with the sparse coding, and also restricts the signal-atoms to unit norm. MOD gives the optimal adjustment of frame vectors in each iteration for a given frame and approximations. It typically converges through a few iterations and an overall effective method, relatively high complexity of matrix inversion makes it a little complicated. The sparse coefficients are determined using OMP.

The error matrix used to compare various dictionary techniques is Peak Signal to Noise ratio (PSNR). It is the ratio between the maximum possible power of a signal and the power of corrupting noise. The PSNR is most commonly used as a measure of quality of reconstruction in image compression etc.

$$\text{PSNR} = 20 * \log_{10} (255 / \sqrt{\text{MSE}})$$

### III. RESULT AND PERFORMANCE COMPARISON

An analysis and comparison of various dictionary construction such as spatial image representation in frequency map using DCT, a hierarchical decomposition method of input image into a series of successively lower resolution reference signals and their associated detail signals by DWT, a singular value decomposition method KSVD and method of optimal direction (MOD) is exploited here. The efficiency of various methods depends on the accuracy of reconstructed data and complexity of the algorithm.

DCT makes an image sparse in the transform domain, through experimentation it is found that with 80% of the coefficients the images were reconstructed in a visually feasible way. Mild distortions will be produced when fewer coefficients are selected for sparse representation. The main advantage of this technique is its independency of input data that reduces its computation complexity. But the drawback of this scheme is even it resulting a high compression ratio, produces blocking artefacts in the reconstructed image that reduces the quality of retrieved data.

By using DWT it is observed that it provide better image quality and compression ratio than DCT algorithm. The main disadvantage is its independency on input data, thus its not efficient for all type of data.

KSVD and MOD are dictionary learning algorithm instead of a predefined ones. MOD is a little bit faster than KSVD, since while updating the dictionary atoms the whole elements will be considered rather than updating one atom at a time as in KSVD. Thus it has a better convergence property. The matrix inversion dictionary updating method in MOD makes computational difficulty. Both KSVD and MOD suffers from a common weakness, as these results a non-structured training dictionary these method are suitable for signals of small size. But as concentrated as dictionary construction overall compression ratio and quality of the recovered data will be almost similar in both cases and provide an almost equivalent sparse approximations.



Figure 1(a): Original terrain image

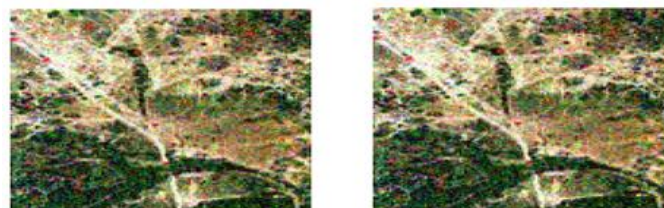


Figure1 (b): Reconstructed image in DCT and DWT basis





Figure1 (c): Reconstructed image in KSVD and MOD basis

Table 2 comparison of the PSNR (in dB) for different dictionary based image reconstruction

Image	DCT	DWT	KSVD	MOD
TERRAIN IMAGE	31.5	33	35	34.3
INDIAN PINES	30.9	32.1	34.5	35.1
WASHINGTON DC MALL	31.8	32.6	33.8	34.8

**IV.CONCLUSION**

This paper deals with an analysis of various techniques for effective sparse representation of an image including analytic based and learning based dictionary based on PSNR value. Although DCT has many advantage as the transform independent on input, on-board distortions will be resulted in the reconstructed data. As the result of the analysis obtained by using PSNR it is clear that the DWT is better than DCT for both imperceptibility and robustness. Both KSVD and MOD is an iterative dictionary learning algorithm that updates the dictionary atoms to better fit the input data. It will result relatively better reconstructed data compared to other two.

**REFERENCES**

[1] José M. Bioucas-Dias et.al “hyperspectral remote sensing data analysis and future challenges” ,IEEE geoscience and remote sensing magazine, June 2013

[2] D. Ramakrishnan and Rishikesh Bhart “Hyperspectral remote sensing and geological applications”,current science,vol.108,no.5,10 march 2015

[3] E.J Candes and M.B Walkin, An introduction to compressive sampling IEEE signal processing magazine volume 25 No.2, 2008

[4] V. K. Goyal, M. Vetterli, and N. T. Thao, “Quantized overcomplete expansions in RN: Analysis, synthesis, and algorithms,” IEEE Trans. Inform. Theory, vol. 44,pp. 16-31, Jan. 1998

[5] Saad Qaisar, Rana Muhammad Bilal et.al “Compressive Sensing: From Theory to Applications, A Survey”

[6] Maneesha Guptha , Dr. Amit Kumar garg and Mr. Abishek kaushik Review : image compression algorithm in international journal of sustainable construction engineering technology (IJSCT) volume 1, issue 10,( 2011)

[7] Ronald A DeVore , BJron Jaweth , Bradley J Luicer Image compression through wavelet transform coding in IEEE transaction on information theory volume 38, No.2 (2009)

[8] Ronald. A. DeVore, B Jorn Jawreth et.al “Image compression through wavelet transform coding”, IEEE Transactions on information theory, Vol.38,No. 2 March 1992

[9] Michal Aharon , Michal Elad and Alfred Bruckstein KSVD : An algorithm for designing overcomplete dictionaries for sparse representation in IEEE transaction on sigal processing volume 54 No.11 (2006)

[10] Ender M. Eksioğlu, and Ozden Bayir, “K-SVD Meets Transform Learning: Transform K-SVD” IEEE signal processing letters, vol. 21, no. 3, march 2014

[11] Zhuolin Jiang,Zhe Lin et.al “Learning A Discriminative Dictionary for Sparse Coding via Label Consistent K-SVD”.

[12] kjersti Engan, Sven Ole Aase, and John Hackon Husoy “method optimal direction for fame design” IEEE transaction.1999