

# Comparative Analysis of Type-1 and Type-2 Fuzzy Controllers with different Membership Functions for CSTR

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**Abstract:** All the real systems exhibits non-linear nature, conventional controllers are not always able to provide good and accurate results. Fuzzy Logic Control is used to control non linear processes and obtain better response. Type 1 Fuzzy control is traditional fuzzy controller which is prone to uncertainties. Type 2 Fuzzy control generalizes type 1 fuzzy control. Type 2 fuzzy set incorporates instances of uncertainties in its membership function. Membership Function plays very crucial role in the Fuzzification process. The values of Membership Function must be defined precisely. Membership functions allow us to graphically represent a fuzzy set. The continuous stirred tank reactor (CSTR) is a non linear process and the concentration maintenance is very difficult. So it is prone to uncertainties more as a result, type 2 fuzzy controller must be used. In this paper we perform comparative analysis on different membership functions used in type 1 and type 2 fuzzy controllers. The application used for the comparative analysis is CSTR and the software used is MATLAB/SIMULINK.

**Keywords:** CSTR, Type- 1 fuzzy Logic, Type-2 fuzzy logic.

## 1. INTRODUCTION

The continuous stirred tank reactor is a very common processing unit in chemical and polymer industries. Its name suggests, it is a reactor in which the contents are well stirred and uniform throughout. The CSTR is normally run at steady state and is usually operated so as to be quite well mixed. The CSTR is generally modelled as having no spatial variations in concentration, temperature or reaction rate throughout the vessel. Since the temperature and concentration are identical everywhere within the reaction vessel, they are the same at the exit points as they are elsewhere in the tank.

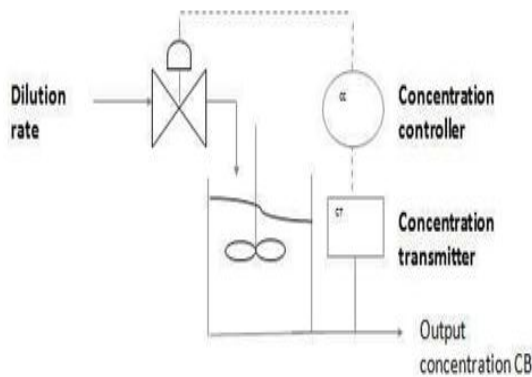


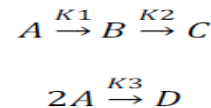
Fig.1: CSTR control system

This system may exhibit highly non-linear dynamics, especially when consecutive and side reactions are present. The one particular control problem for this system

is analysed and solved by using a novel combination of gain scheduling and linear frequency domain design techniques [1]. Sequentially different controllers are implemented in Lab VIEW are as follows PID, Fuzzy, a hybrid of Fuzzy-PID, Fuzzy self tuning PID, A hybrid of Fuzzy and Fuzzy self tuning PID and their time domain specifications are observed which are obtained in better manner from the previous controller[2] [3].

## DESCRIPTION OF THE PROCESS

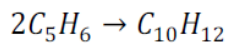
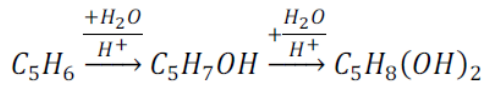
An example for general reaction scheme of the above type can be written as:



Where A is the educt, B is the desired product; C and D are unwanted byproducts. Here the main reaction which yields the desired product is accompanied by the consecutive and parallel reactions which produce the unwanted byproducts. In general, one tries to make the reaction rates  $K_2$  and  $K_3$  small in comparison to  $K_1$  by an appropriate choice of catalyst and reaction conditions. If this is not possible or possible to certain degree, the concentration of B in the product stream which leaves the reactor may be controlled by the inflow to the reactor or reaction temperature.

We consider the process where cyclopentenol is produced in a CSTR from cyclopentadiene by acid catalysed

electrophilic addition of water from dilute solution. Due to strong reactivity of the educt and the product, dicyclopentadiene is produced as a side product and cyclopentanediol as a consecutive product by addition of another water molecule. Thus the complete reaction scheme is

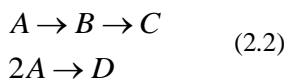


In this CSTR system we treat the production of cyclopentanol from cyclopentadiene. The mathematical model of this process is a fourth order nonlinear dynamical unstable system which has unstable zero dynamics and thus cannot be controlled using the well known techniques of exact linearization by nonlinear coordinate transformations and nonlinear feedback. Here the reactor is an isothermal reactor, as the temperature is maintained constant. For an isothermal CSTR, the product concentration can be controlled by manipulating the feed flow rate, which changes the residence time. We here analyze the situation where both flow through the reactor and the amount of heat removed per unit of time are available as manipulated variables to control the product concentration in a specified range. A fast temperature control loop is designed so that the reactor temperature can be kept approximately constant over the whole range of operation. Thus the relevant dynamics for the concentration control problem can be reduced to a second order nonlinear system with unstable zero dynamics. Then derive a control law for the reduced problem which is based on linear controller design techniques for the linearized plant and gain scheduling.

**2. MATHEMATICAL MODELING**

In our application we are considering isothermal CSTR (that is temperature is constant). The reactant conversion in a chemical reactor is a function of residence time or its inverse, the space velocity. For isothermal CSTR, the product concentration can be controlled by manipulating the feed flow rate which changes the residence time

Here we take the reaction



The desired product is the component B, the intermediate component in the series reaction. In this module A is the Cyclopentadiene, B=cyclopentanol, C=cyclopentanediol, D=dicyclopentadiene

We model the CSTR by taking the state space model. Initially we use instantaneous balance method for CSTR modelling as it is a lumped process [1].

The rate of formation of each component is given by

$$r_A = -k_1C_A - k_3C_A^2 \tag{2.3}$$

$$r_B = k_1C_A - k_2C_B \tag{2.4}$$

$$r_C = k_2C_B \tag{2.5}$$

$$r_D = \frac{1}{2}k_3C_A^2 \tag{2.6}$$

$r_A$  -represents rate of reaction of species A

$r_B$  -represents rate of reaction of species B

$r_C$  -represents rate of reaction of species C

$r_D$  -represents rate of reaction of species D

$k_1, k_2, k_3$  -constants

The component material balance equations are given by

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - k_1C_A - k_3C_A^2 \tag{2.7}$$

$$\frac{dC_B}{dt} = -\frac{F}{V}C_B + k_1C_A - k_2C_B \tag{2.8}$$

$$\frac{dC_C}{dt} = -\frac{F}{V}C_C + k_2C_B \tag{2.9}$$

$C_A$  - molar concentration of A

$C_B$  - molar concentration of B

$C_C$  - molar concentration of C

$C_D$  - molar concentration of D

$C_{Af}$  - represents the final steady state concentration of A

Here we are concerned only with our desired output B, we are not bothered about other equations so we need to solve the first two equations only

They can be represented generally in steady state form

The linear state space model is

$$\dot{x} = Ax + Bu \tag{2.11}$$

$$y = Cx + Du \tag{2.12}$$

$\dot{x}$  -input  
y-output

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{2.13}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{2.14}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} C_A - C_{As} \\ C_B - C_{Bs} \end{bmatrix} \tag{2.15}$$

$C_A$  - molar concentration of A

$C_B$  - molar concentration of B

$C_{As}$  - molar concentration of A at steady state

$C_{Bs}$  - molar concentration of B at steady state

$$u = \begin{bmatrix} F/V - F_s/V \\ C_{Af} - C_{Afs} \end{bmatrix} \quad (2.16)$$

$F/V$  - space velocity

$$y = x_2 = [C_B - C_{Bs}] \quad (2.17)$$

$$a_{11} = \frac{\partial [f(C_A)]}{\partial C_A} \quad (2.18)$$

$$a_{12} = \frac{\partial [f(C_A)]}{\partial C_B} \quad (2.19)$$

$$a_{21} = \frac{\partial [f(C_B)]}{\partial C_A} \quad (2.20)$$

$$a_{22} = \frac{\partial [f(C_B)]}{\partial C_B} \quad (2.21)$$

$$b_{11} = \frac{\partial [f(C_A)]}{\partial (F/V)} \quad (2.22)$$

$$b_{12} = \frac{\partial [f(C_A)]}{\partial C_{Af}} \quad (2.23)$$

$$b_{21} = \frac{\partial [f(C_B)]}{\partial (F/V)} \quad (2.24)$$

$$b_{22} = \frac{\partial [f(C_B)]}{\partial C_{Bf}} \quad (2.25)$$

Finally derivating the above we get terms

$$A = \begin{bmatrix} -\frac{F_s}{V} - k_1 - 2k_3 C_{As} & 0 \\ k_1 & -\frac{F_s}{V} - k_2 \end{bmatrix} \quad (2.26)$$

$$B = \begin{bmatrix} C_{Afs} - C_{As} & \frac{F_s}{V} \\ -C_{Bs} & 0 \end{bmatrix} \quad (2.27)$$

$$C = [0 \quad 1] \quad (2.28)$$

$$D = [0 \quad 0] \quad (2.29)$$

Substituting the rate constants [2] as

$$k_1 = 50hr^{-1}, k_2 = 100hr^{-1}, k_3 = 10 \frac{mol}{litrehr}$$

$$C_{As} = 3gmol / liter \quad C_{Bs} = 1.117 gmol / liter$$

$$F_s / V = 0.5714 min^{-1}$$

On substituting the above values in the equations (2.26) and (2.27) we get

$$A = \begin{bmatrix} 0.78625 & 0 \\ 0.06607 & 0.78947 \end{bmatrix} \quad (2.30)$$

$$B = \begin{bmatrix} 0.62219 \\ -0.07506 \end{bmatrix} \quad (2.31)$$

Thus the plant model i.e. the CSTR model is now converted into state-space model and this model is implemented in Lab VIEW using different Controllers which is discussed in next chapter. The transfer function of CSTR is obtained by MATLAB as given below [1].

$$G(s) = \frac{-1.117s + 3.1472}{s^2 + 4.6429s + 5.381}$$

By applying Routh criteria and Ziegler's Nichols method, the tuning parameters of the PID controller are obtained as follows

$$Kp=2.4936$$

$$Ti=0.73$$

$$Td=0.18$$

These tuning parameters of controller are useful to get optimum response from CSTR.

### 3. SIMULATION & RESULTS

Block Diagram of Type-1 Fuzzy with Triangular Membership Function shown below.

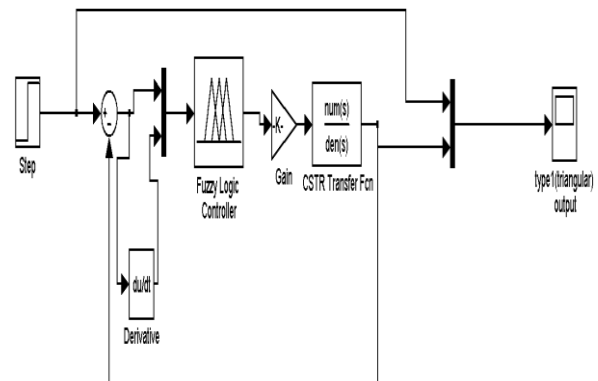


Fig.2: Block diagram of Type -1 fuzzy controller with triangular membership function.

The Fuzzy controller implemented in MATLAB/SIMULINK is shown in the above figure. The Fuzzy loader is used to load the membership functions and rules related to the given CSTR [5] [6]. The set point and change in error are compared by using a comparator and the output is given to the Fuzzy controller. The response of the given controller is observed and shown in the below figure.



Fig. 4: Response of CSTR with Type-1 fuzzy

The response of the CSTR with type-1 fuzzy controller has better time domain specifications than PID controller. For obtaining better time domain specifications we have implemented CSTR using hybrid of fuzzy-PID controller.

Block Diagram of Type-1 Fuzzy with Trapezoidal Membership Function shown below.

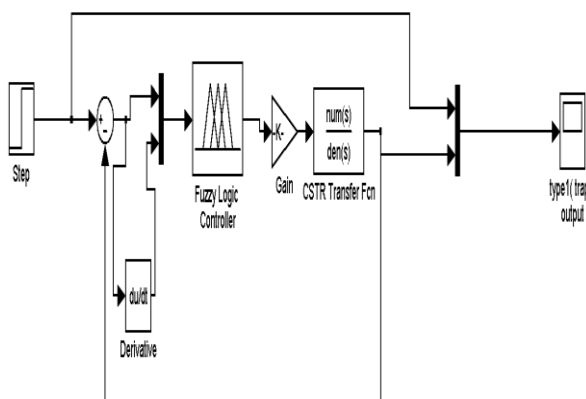


Fig.3: Block diagram of Type -1 fuzzy controller with trapezoidal membership function

The Fuzzy controller implemented in Lab VIEW is shown in the above figure. The Fuzzy loader is used to load the membership functions and rules related to the given CSTR [5] [6]. The set point and change in error are compared by using a comparator and the output is given to the Fuzzy controller. The response of the given controller is observed in the front panel as shown in the below figure.



Fig. 5: Response of CSTR with Type-1 fuzzy

The response of the fuzzy controller which is observed in the front panel has better time domain specifications than PID controller.

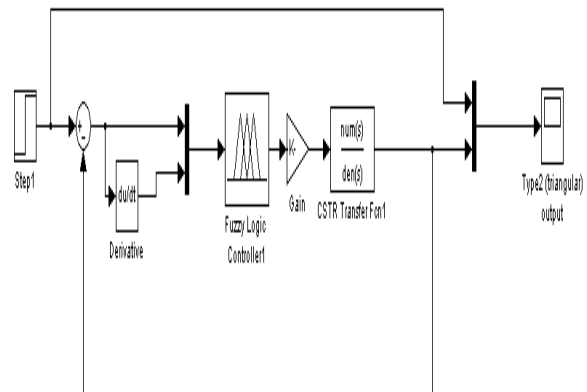


Fig. 6: Block diagram of Type -1 fuzzy controller with triangular membership function

For obtaining better time domain specifications we have implemented CSTR using hybrid of fuzzy-PID controller. Block Diagram of Type-2 Fuzzy with Triangular Membership Function the Fuzzy controller implemented in Lab VIEW is shown in the above figure.



Fig. 8: Response of CSTR with Type-2 fuzzy

The Fuzzy loader is used to load the membership functions and rules related to the given CSTR [5] [6]. The set point and change in error are compared by using a comparator and the output is given to the Fuzzy controller. The response of the given controller is observed in the front panel as shown in the below figure.

The response of the fuzzy controller which is observed in the front panel has better time domain specifications than PID controller. For obtaining better time domain specifications we have implemented CSTR using hybrid of fuzzy-PID controller. Block Diagram of Type-2 Fuzzy With Trapezoidal Membership Function

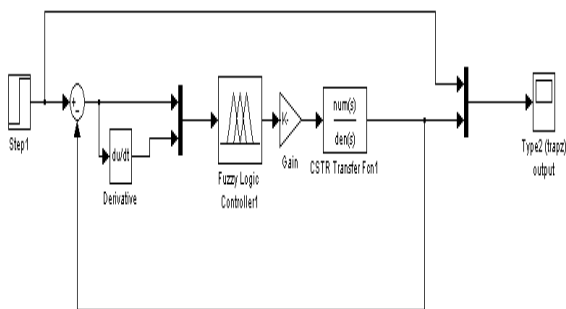


Fig. 7: Block diagram of Type -1 fuzzy controller with trapezoidal membership function

The Fuzzy controller implemented in Lab VIEW is shown in the above figure. The Fuzzy loader is used to load the membership functions and rules related to the given CSTR [5] [6]. The set point and change in error are compared by using a comparator and the output is given to the Fuzzy controller. The response of the given controller is observed in the front panel as shown in the below figure.



Fig. 9: Response of CSTR with Type-2 fuzzy

The response of the fuzzy controller which is observed in the front panel has better time domain specifications than PID controller. For obtaining better time domain specifications we have implemented CSTR using hybrid of fuzzy-PID controller.

The comparison of PID, fuzzy, Hybrid of fuzzy-PID, fuzzy self-tuning PID, Hybrid of fuzzy and fuzzy self-tuning PID controllers with respect to time domain specifications are shown in the below table. It is observed that the rise time ( $T_r$ ) as well as settling time( $T_s$ ) of the hybrid of fuzzy and fuzzy self-tuning PID are better when compared to the all the above controllers and the delay time( $T_d$ ) does not undergo much variations in all the above controllers.

Table 1: Comparison of Type-1 & Type-2 fuzzy with different membership functions.

Membership Function	Type 1 fuzzy		Type 2 fuzzy	
	Rise time	Settling time	Rise time	Settling time
Triangular	2.05 sec	3.85 sec	1.75 sec	3.80 sec
Trapezoidal	1.85 sec	3.90 sec	1.65 sec	3.85 sec

### 5. CONCLUSION

In this paper, we developed the mathematical model of three tank water level control system and simulated with conventional PID controller and Fuzzy controllers (Type-1 and Type-2) using Matlab/Simulink. From the analysis we conclude that three tank water level control system with conventional PID controller has a problem of overshoot & undershoot for unit step input. In order to achieve an optimum response, we simulated the three tank water level control system with Type-1 and Type-2 fuzzy logic controllers. Type-2 fuzzy controller gives better response in terms of time domain specifications.

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