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DOA Estimation Using MOD MUSIC Method and Calculation of Signal and Noise Power

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Abstract: The DOA estimation algorithms are designed based on the ideal assumption where the antenna arrays are free from imperfect conditions. But in practical situation, ideal conditions are extremely difficult to obtain and thus due to imperfect conditions, the performance of DOA estimation will be severely degraded. Results of classical methods like MUSIC or Beamforming for direction of arrival estimation degraded when sources are close or coherent or having low SNR. Based on the investigation to classical DOA algorithm (MUSIC and Improved MUSIC), a mod-MUSIC estimation algorithm is proposed which makes full use of signal subspace and noise subspace characteristics. It has good performance in imperfect condition and compare this algorithm with classical algorithms in different conditions.

Keywords: DOA estimation; Array signal processing; Eigen space; Eigen vector.

1. INTRODUCTION

DOA estimation is one of the important contents of array signal processing, and extensively applied in the field of radar, sonar, communication, astronautics, aeronautics, etc. In lots of DOA estimation algorithms with excellent performance, the most classic one is the multiple signal classification (MUSIC) based on eigenvalue decomposition. It can search for the unidimentional peak value of spectrum to get the direction of information source. The operational measurement of MUSIC is much less than that of maximum likelihood method (ML), weighted subspace fitting method (WSF) and other tridimensional search methods. Based on MUSIC, weighted MUSIC and ameliorative MUSIC are developed in the condition of coherent sources. We will propose a MOD MUSIC DOA estimation algorithm based on the eigen space in the text which makes full use of the characteristic of signal subspace and noise subspace. Its performance is better than that of MUSIC in the condition of incoherent sources and ameliorative MUSIC in the condition of coherent sources, and the algorithm still show a good performance in the condition of small snapshot and low SNR so it is predominant and robust.

2. MUSIC AND IMPROVED MUSIC

2.1 MUSIC

Suppose that there are K sources, the receiving signal of M elements uniform linear antenna array is given by

$$x = \sum_{i=1}^{K} a(\theta_i)s_i + n = AS + n \qquad (1)$$

where s_i is the transmitting signal of source i; (θ_i is the DOA of the i^{th} source. $a(\theta_i)$ is the normalized direction vector; n is white noise whose mean is 0 and variance is $\sigma^2 I$. The direction matrix A =[$a(\theta_1)$, $a(\theta_2)$,….., $a(\theta_K)$] and the source matrix S = [s_1; s_2; ….; s_k].

The covariance matrix of the array signal is $R = E[xx^H]$. The eigen-decomposition is

$$R = U\Lambda U^H = \sum_{i=1}^{M} \lambda_i u_i u_i^H \qquad (2)$$

where $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_M$ and it's eigen -values are complied with this sequence:
$\lambda_1 > \lambda_2 \ldots > \lambda_{K+1} = \ldots\ldots \lambda_M = \sigma^2$
That is the first K eigenvalues are in connection with the signal and their numeric value are all more than σ^2. The eigen vectors corresponding with the k larger eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_K$ are $u_1, u_2, \ldots\ldots, u_K$. Λ_s is the diagonal matrix composed of the k larger eigenvalues.
The later M-P eigenvalues are totally depended on the noise and their numeric value areσ_n^2. The eigen vectors corresponding with $\lambda_{K+1}, \lambda_{K+2}, \ldots\ldots, \lambda_M$ are $u_{K+1}, u_{K+2}, \ldots\ldots, u_M$ and they construct noise subspace U_N =[$u_{K+1}, u_{K+2}, \ldots\ldots, u_M$] . Λ_N is the diagonal matrix composed of the M-P smaller eigenvalues. So R could be divided into

$$R = U_s \Lambda_s U_s^H + U_N \Lambda_N U_N^H \qquad (3)$$

According to each column vector is orthogonal to noise subspace: $U_N^H \, a(\theta_i) = 0$ i= 1,2,…...K, the spatial frequency spectrum of MUSIC is concluded:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)\, U_N\, U_N^H\, a(\theta)} \qquad (4)$$

Through this Eq(4) we estimate the DOA by searching the peak value. But several coherent signals may merge into one signal in the condition of coherent sources so that the independent sources reaching antenna will reduce, then the rank of output covariance matrix will be less than k. When decomposing the signal covariance matrix, the number of

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larger eigenvalues is less than k and then the number of eigenvalues equal to σ^2 is more than M-K. The number of vectors of the signal subspace will be less than k too which means the number of the signal subspace's dimension will be less than the number of columns of direction matrix. Some direction vectors of coherent sources will not be orthogonal to noise subspace so DOA estimation will miss report in spatial frequency spectrum.

2.2 Improved MUSIC

The Improved MUSIC is to reconstruct the receiving signal's covariance matrix

$$R_x = R + IR^*I \qquad (5)$$

R^* is the conjugate of R.

$$I = \begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & \dots & 1 & 0 \\ \dots & \dots & \dots & \dots \\ 1 & 0 & \dots & 0 \end{bmatrix}_{M\times M} \qquad (6)$$

The estimation of receiving signal covariance matrix is R = $(1/N)xx^H$ .N is the number of snapshot. Generally R is Hermite matrix rather than Toeplitz matrix. We revise R and get the estimation of covariance Toeplitz matrix Rx = $R_x = R + IR^*I$. Obviously R_x is Toeplitz matrix so let R_x replace R, then decomposeR_x. Other steps of Improved MUSIC are the same as MUSIC. Improved MUSIC had better DOA estimation performance in the condition of coherent sources because it's reconstruction of receiving signal covariance matrix.

3. NOVEL DOA ESTIMATION

This algorithm takes full use of signal subspace and noise subspace.

Define matrix as

$R_A = ASS^HA^H = APA^H = U_s\Lambda_sU_s^H$

where $P = SS^H$, U_s is signal subspace. Then,

$$a^H(\theta_t)R_A^+a(\theta_t) = a^H(\theta_t)(APA^H)^+a(\theta_t) = ((A^+a(\theta_i))^HP^+A^+a(\theta_i) \qquad (8)$$

$= \delta_i^T P^+ \delta_i$

$= [P]_{i,j} = 1/P_i$

Where $[]^+$ is generalized inverse. $\delta_i = [0,..,1,0,..0]^T$ is M×l vector which element i is 1 and the other are all 0. P_i is the power of source i.

Define a novel function of spatial frequency spectrum

$$P_{ES}(\theta) = \frac{a^H(\theta)R_A^+a(\theta)}{a^H(\theta)U_N U_N^H a(\theta)} \qquad (9)$$

When $\theta=\theta_i$ (i=1,2,..,K), $a^H(\theta)U_N U_N^H a(\theta) = 0$ $a^H(\theta)R_A^+a(\theta)$ = l/ P_i, the peak value of P_{ES}'s spatial frequency spectrum is appeared at 0. $\theta=\theta_i$ (i=1,2,..,K) . So the function of P_{ES} 's spatial frequency spectrum can be used for DOA estimation. In the meantime we reconstruct the receiving signal covariance matrix according to Eq (5) to make sure estimating DOA right in the condition of coherent sources. The function of spatial frequency spectrum defined in the text can not only estimate DOA right but also estimate the source power according to Eq(8).

3.1 mod-MUSIC Method

MUSIC and Improved MUSIC both use noise subspace only. In the text we will study an estimation which uses signal subspace and noise subspace both.

The defined matrix R_A can be further normalised to improve performance in case of small snapshots,coherent source and small SNR.

$R_A=R+IR^*I$

$R_A=R_A+(R_A^{-1})$

$$R_A=R_A+\frac{R_A}{\max R_A} \qquad (10)$$

$$P_{mod-MUSIC}(\theta) = \frac{U_S U_S^H}{a^H(\theta) U_N U_N^H a(\theta)} \qquad (11)$$

$$Pn = 10log[a^H(\theta)U_n \Lambda_n U_n^H a(\theta)] \qquad (12)$$

$$Ps = 10\log[\frac{1}{a^H(\theta)U_s\Lambda_sU_s^H a(\theta)}] \qquad (13)$$

Where Ps and Pn are source and noise power respectively.

So a mod-MUSIC method based on eigen space is proposed, the steps are:

(1) Calculate the covariance matrix according to the receiving signal, then reconstructing it :Rx=R+IR*I.

(2) Eigen-decompose the covariance matrix: Rx = $U\Lambda U^H$ the sequence from big arrive small, into signal subspace and noise subspace, that is Rx= R = $U_s\Lambda_sU_s^H + U_N\Lambda_NU_N^H$

(3) (3) According to R_A = $U_s\Lambda_sU_s^H$, calculating R_A^+=$U_s\Lambda_s^{-1}U_s^H$

(4) (4) Make θ changed and calculating the spectrum function according to (9), get the estimation of DOA by searching the peak value.

(5) Estimate the power of the DOA estimated:

$Pn = 10log[a^H(\theta)U_n \Lambda_n U_n^H a(\theta)]$

$Ps = 10\log[\frac{1}{a^H(\theta)U_s \Lambda_s U_s^H a(\theta)}]$

4. SIMULATION AND ANALYSIS

Simulation conditions: uniform linear array with 16 antenna, element spacing is half-wavelength; the number of sources is 5; DOA is given as 10°, 20°, 30°, 40°, 50° separately, and the corresponding power are 1, 0.9, 0.8, 0.7, 0.6.

Simulation l: The performance of mod-MUSIC under different SNR is investigated in this simulation. We compare mod-MUSIC with MUSIC and ES-DOA. The

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number of snapshot is 200. Fig.l-Fig.3 are the spatial spectrums in the condition of SNR (-5, 5, 10dB). It can be seen that when SNR=10 in Fig 3, there isn't obvious peak value at 10° and 30° using MUSIC and ES-DOA but obvious peak values are formed in each DOA direction using mod-MUSIC. In Fig.1 and Fig.2, The peak values of MUSIC and mod-MUSIC are in the DOA direction, but the peak values of mod-MUSIC are more actuate and side beam are lower in the condition of the same SNR so its performance is better among MUSIC and ES-DOA.

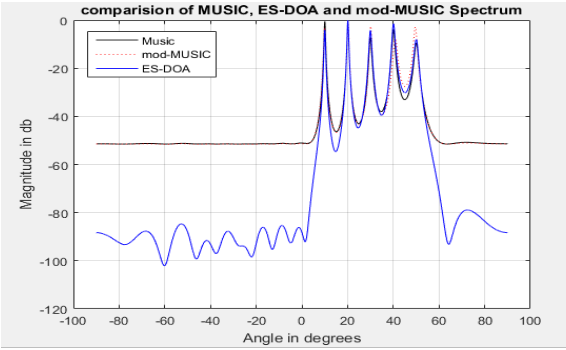

Fig.1. Performance of MUSIC, ES-DOA and mod-MUSIC when SNR = 5dB

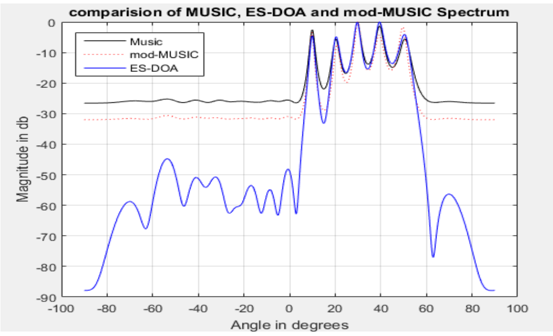

Fig.2. Performance of MUSIC, ES-DOA and mod-MUSIC when SNR = -5dB

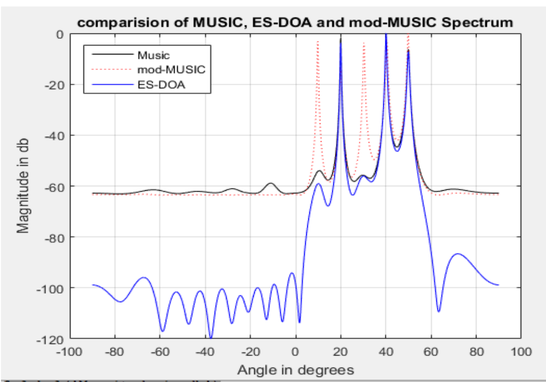

Fig.3. Performance of MUSIC, ES-DOA and mod-MUSIC when SNR = 10dB

Simulation 2: The performance of mod-MUSIC in the condition of small snapshots is investigated in this simulation. We comapre MUSIC, ES-DOA and mod-MUSIC in Fig. 4, SNR= 10dB, the number of snapshot is 20 in this simulation. There isn't obvious peak value at 10°, 30°, 40° and 50° using MUSIC and ES-DOA but obvious peak values are formed in each DOA direction using mod-MUSIC. So mod-MUSIC shows better performance in condition of small snapshot.

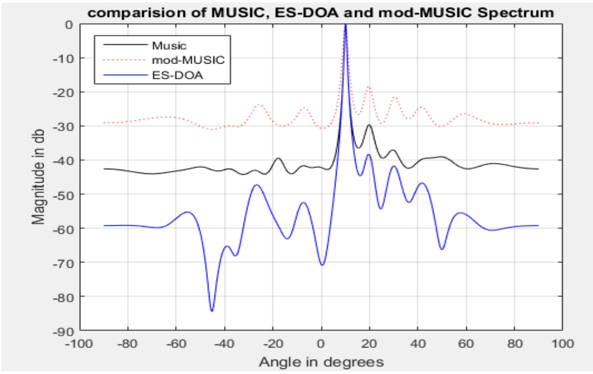

Fig.4. Performance of MUSIC, ES-DOA and mod-MUSIC when SNR = 10dB and number of snapshot is 20.

Simulation 3: The performance of mod-MUSIC in coherent sources is investigated in this simulation. The sources of DOA=10° and DOA=30° are the same. The number of snapshot is 200, and SNR=15. The performance of MUSIC and ES-DOA is showed in Fig 5 in coherent sources. It is concluded that there aren't obvious peak values at 30° and 10° for MUSIC algorithm, so MUSIC algorithm doesn't work well in coherent sources. The obvious peak values are formed in each DOA direction using mod-MUSIC. Fig 5 shows the performance of MUSIC, ES-DOA and mod-MUSIC. It can be seen that obvious peak values are formed in each DOA direction using mod-MUSIC. But there is no peak at 10° and 30° using MUSIC, ES-DOA method. So the performance of mod-MUSIC is better than that of MUSIC and ES-DOA in coherent sources.

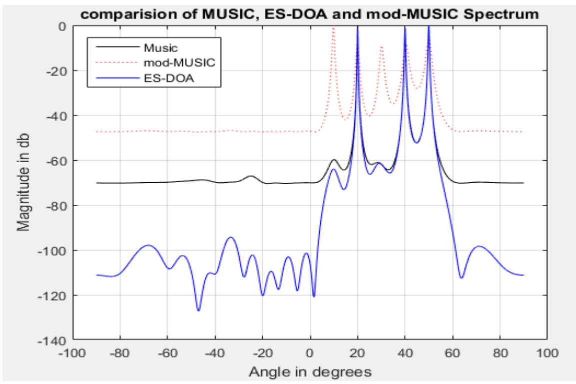

Fig 5. Performance of MUSIC,ES-DOA and mod-MUSIC in coherent sources

Simulation 4: The ES-DOA can not only estimate DOA exactly but also estimate the source power. Defining PMSE $=(p_i - p_{ie})^2$ where p_i is the pefect power value of i[th] source and p_{ie} is the estimated power value of i[th] source. Both of them are the PMSE average value of many sources. We can conclude that the performance of source power estimation is getting better along with increasing the number of snapshot.

5. CONCLUSION

From the above analysis it can be proved that MUSIC, ES-DOA method gives better result for DOA estimation in ideal conditions. But mod-MUSIC method gives best performance when conditions are practical like when sources are coherent or SNR is low or small snapshots are used.

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