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The Root Locus and Polynomial Approach based Controller Comparison for Magnetic Ball Suspension System

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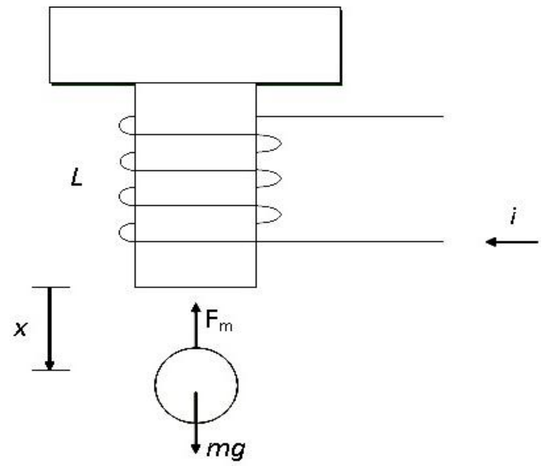
Abstract: Magnetic Levitation System (MLS) is an example of a non-linear and inherently unstable system. To overcome this instability an electromagnet will have to be constructed. The current in the electromagnetic coil is to be varied suitably using a compensator to generate varying magnetic field and hence a varying force to be exerted on any object in its vicinity. Two approaches were followed to design compensator. Firstly Root locus approach where in the pole in unstable region is moved to stable region. Another approach is Inward approach where in the poles are placed at desired location & co-efficient of compensator are obtained.

Keywords: Compensator, Root Locus, Magnetic Lavitation, Controller Design.

I. INTRODUCTION

The physical system, as shown in Figure 1.1, consists of a steel ball that is to be levitated under an electromagnet. For the purpose of theoretical analysis and system behavioural study, the system parameters assumed suitably. And hysteresis effects of the electromagnet are assumed to be negligible.



Where,
m= Mass of the ball
g = Gravitational force
F_m=Force due to electromagnet
x = Displacement of the ball
L= Inductance of the coil
i = Current through the coil

The paper concentrates on the design of a controller for keeping a steel ball suspended in the air. In the ideal situation, the magnetic force produced by current from an electromagnet will counteract the weight of the steel ball. The main function of this controller is to maintain the balance between the magnetic force and the ball's weight. System linearization and compensation are employed to design the controller for this unstable nonlinear system. The controller designed in this project provides a robust closed-loop stabilization which can abide considerable range of disturbances on suspended mass.

$$G(s)=\frac{V_x(s)}{V(s)}=\frac{\textit{Ball}\text{ Position}}{\textit{Voltage}\text{ to Coil}}=\frac{-K_1\beta}{(R+sL_1)(ms^2-K_x)}$$

Or,

$$G(s)=\frac{K_{11}}{(s+p_3)(s^2-K_2)} \quad (1.1)$$

Where

$$K_{11}=\frac{-\beta K_1}{mL_1} \quad K_2=\frac{K_x}{m} \quad p_3=\frac{R}{L_1}$$

The following parameters are used to obtain the complete transfer function of the system.

Resistance of coil R	3Ω
Inductance of coil L	0.0425 H
Constant C	9.07×10^{-5}
I_0	0.5 A
Mass of ball M	0.02312 kg
x_0	0.01m

Substituting the above values in expression (1.1) we obtain the open loop transfer function of the system as,

$$G(s)=\frac{V_x(s)}{V(s)}=\frac{-472053.5}{(s-44.29)(s+44.29)(s+70.588)} \quad (1.2)$$

The open loop poles are at s_1=-44.29, s_2=44.29 and s_3=-70.588.The locus starts at poles and ends at zero.

Since one of the pole lies in right half of's' plane, the system in unstable. The uncompensated root locus is shown in Fig (1.1).

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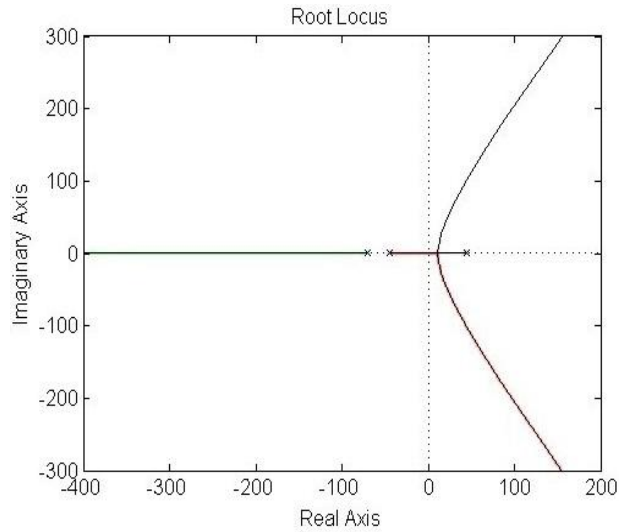

Fig 1.1. Root locus of the uncompensated system

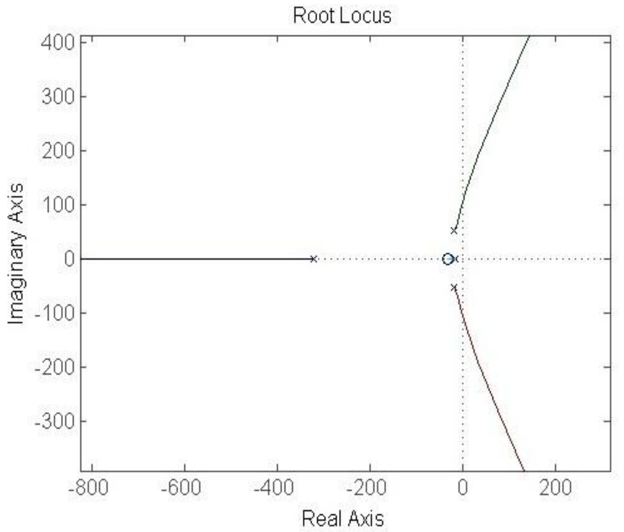

Fig 2.1. Root locus of the compensated system (RL Method)

II. COMPENSATOR USING ROOT LOCUS APPROACH

The controller for magnetic levitation system is designed by Root locus method (RL Method) to provide maximum gain margin G_m, minimum damping oscillation and minimum settling time. Now Let us consider a magnetic levitation system with unity feedback and design a compensator based on root locus approach. The open-loop magnetic levitation system transfer function without pole cancellation is given by

$$G(s) = \frac{-472053.5}{s^3 + 70.588s^2 - 1961.51s - 138459.458}$$

A compensator, $G_c(s)$, is used with a plant $G(s)$ so that the overall loop gain can be set to satisfy desired time and frequency domain specifications and G_C is used to adjust the system dynamics favorably without affecting the steady-state error. Consider the first-order compensator with the transfer function as

$$G_c(s) = K\frac{(s+Z)}{(s+P)} \quad (2.1)$$

The design problem is to select zero Z, pole P and gain K in order to provide a suitable performance. As mentioned earlier, by selecting the location of Z arbitrarily and then as a rule of thumb, the pole location is selected 10 times that of zero. And then by trial and error method the value of K can found out.

To provide some robustness in the system model, the controller is designed at zero equals -30 and pole is chosen to be an order 10 away from the controller zero and hence is set to -300. The transfer function of the compensator becomes

$$G_c(s) = K\frac{(s+30)}{(s+300)} \quad (2.2)$$

The locations of the zeros and pole are selected so as to result in a satisfactory root locus for the compensated system. The closed loop transfer function of the given system along with compensator is given by expression (2.3). The root locus of the compensated system and step response of the same is show in Fig 2.1 and Fig 2.2 respectively.

$$G(s) = \frac{1.884\times10^6 s + 5.625\times10^7}{s^4 + 370.6s^3 + 1.921\times10^4 s^2 + 1.157\times10^6 + 1.498\times10^7} \quad (2.3)$$

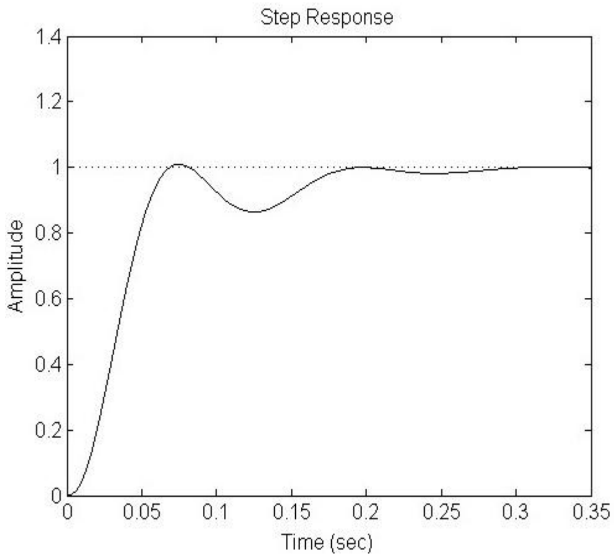

Fig 2.2. Step Response of the compensated system (RL Method)

From the Fig 2.1, root locus of the system is stabilized i.e. all the roots of the magnetic levitation system have moved to the left half of s-plain and the system achieves steady state quickly with least oscillations after using the compensator

With the compensator mentioned in (2.2).The system time domain specifications are,

i. System gain = 3.991
ii. settling time =0.175 seconds
iii. Phase margin= 6.3145
iv. Peak overshoot = 0.786%

III.INWARD APPROACH

Another control system design approach is called Inward approach (Polynomial Approach). In this approach the reverse of the outward approach i.e. first a desired closed loop transfer function is designed, and then solve for required controller. This method is called a linear algebraic methodology for controller design

In this approach we shall discuss the Diophantine equation. Consider the LTI system defined by transfer function

$$\frac{Y(s)}{U(s)} = \frac{N(s)}{D(s)} \quad (3.1)$$

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$$D(s) = s^n + D_1 s^{n-1} + ... + D_{n-1}s + D_n$$

$$N(s) = N_0 s^n + N_1 s^{n-1} + ... + N_{n-1}s + N_n$$

Assume that this transfer function system is completely state controllable and completely observable. That is there is no pole zero cancellation in transfer function, or D(s) and N(s) have no common factors. When polynomials D(s) and N(s) have no cancellation, these polynomials are called co-prime polynomials Then there exists an unique *(n-1)*th degree polynomial $\alpha(s)$ and $\beta(s)$ such that

$D_c(s)\ D(s) + N_c(s)\ N(s)$ =D_0(s)

$$D(s) = D_0 + D_1 s + ... + D_n s^n$$

$$N(s) = N_0 + N_1 s + ... + N_n s^n$$

The Diophantine equation can be solved for $D_c(s)$ and $N_c(s)$ by use of following *2n×2n* Sylvester Matrix E, which is defined in terms of the coefficient of co-prime polynomials D(s) and *N(s)* as follows [3]. The controller co-efficient matrix is given by

$$X \times Y = F$$

$$\begin{bmatrix} D_n & 0 & ... & 0 & N_n & 0 & ... & 0 \\ D_{n-1} & D_n & ... & 0 & N_{n-1} & N_n & ... & 0 \\ . & & & & . & & & \\ . & D_{n-1} & ... & 0 & . & N_{n-1} & ... & 0 \\ . & & & & . & & & \\ & . & & . & & . & & . \\ D_1 & . & ... & . & N_1 & . & ... & . \\ & . & & . & & . & & . \\ 1 & D_1 & ... & D_{n-1} & N_0 & N_1 & ... & N_{n-1} \\ 0 & 1 & ... & D_{n-2} & 0 & N_0 & ... & N_{n-2} \\ . & . & & . & . & . & & . \\ . & . & ... & . & . & . & ... & . \\ . & . & & . & . & . & & . \\ 0 & 0 & ... & D_1 & 0 & 0 & ... & N_1 \\ 0 & 0 & ... & 1 & 0 & 0 & ... & N_0 \end{bmatrix} \times \begin{bmatrix} D_{cn-1} \\ D_{cn-2} \\ . \\ . \\ . \\ . \\ D_{c0} \\ N_{cn-1} \\ N_{cn-2} \\ . \\ . \\ . \\ N_{c0} \end{bmatrix} = \begin{bmatrix} F_{n+m} \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ F_1 \\ F_0 \end{bmatrix} \qquad (3.2)$$

The co-efficient D_{c0}, N_{c0}, D_{c1}, N_{c1}.Can be determined by the following equation

$$Y = X^{-1} \times F$$

Where,

X = system Sylvester matrix
F = desired root location
Y = coefficients of Diophantine equation

To achieve arbitrary pole-placement, the degree of controller ($G_C(s)$) configuration must be *m = n - 1* or higher [3]. If it is less than n-1, it may be possible to assign some of the poles but not all. The degree of the Inward Approach $D_0(s)$ of overall function $G_0(s)$ is *n + m.*

$$G_0(s) = \frac{N_c(s)N(s)}{D_c(s)D(s) + N_c(s)N(s)}$$

Referring to unstable plant transfer function given by (1.2), the order of the plant transfer function is n=3, and it needs controller $G_c(s)$ of the order m = n-1 = 2. This shows the degree of the characteristic equation $D_0(s)$ of the overall transfer function $G_0(s)$ is n + m = 5

Then polynomial of the desired characteristic equation $D_0(s)$ is given by equation

$$D_0(s) = (s+100)^5$$

The controller transfer function obtained by this approach is,

$$G_c(s) = \frac{-12.55s^2 - 1483s - 4.22\times10^4}{s^2 + 429.4s + 7.165\times10^4} \qquad (3.3)$$

The closed loop transfer function of the system is given by

$$G_0(s) = \frac{5.923\times10^6 s^2 + 7\times10^8 s + 1.992\times10^{10}}{s^5 + 500s^4 + 10^5 s^3 + 10^7 s^2 + 10^8 s + 10^{10}} \qquad (3.4)$$

The root locus of the compensated system and step response of the same is show in Fig 3.1 and Fig 3.2 respectively.

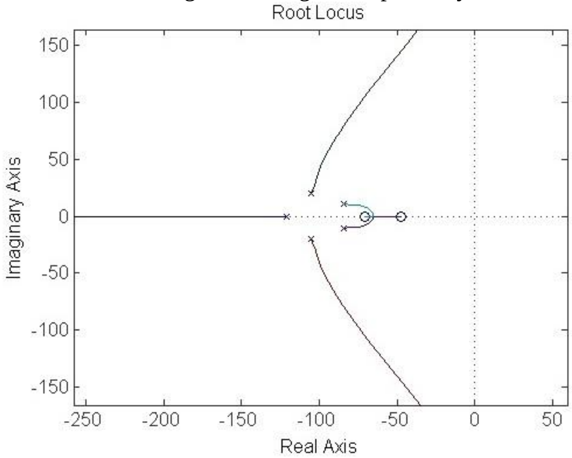

Fig 3.1. Root locus of the compensated system (Inward Method)

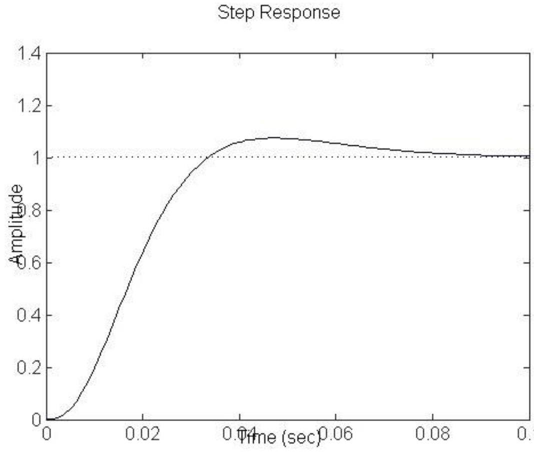

Fig 3.2. Step Response of the compensated system (Inward Method)

From the Fig3.1., root locus we can see that all the poles of the system are on left hand side of the 's' plane, showing the system is stable. With the compensator mentioned in equation (3.3) .The system parameters are

i. system gain = 2.8584
ii. settling time = 0.0771seconds
iii. Phase margin= 47.8662
iv. Peak overshoot = 70.32%

Using algebraic technique discussed above, the stability of the magnetic levitation system can be satisfactory achieved by selection of arbitrary stable poles of overall transfer function of the system. However, the performance specifications of the overall system are met. But this controller is unrealizable for practical implementation because the coefficients of the controller transfer functions are very large that may cause saturation of the components during practical implementation.

IV. RESULTS AND COMPARISION

For different locations of compensator's pole and zero the system performance parameters like gain margin (G_m), phase margin (P_m), settling time (t_s) and peak overshoot (%M_p) will also get affected. The various system parameters are determined and the following results are observed

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A. Root Locus method

As mentioned in section II the controller is designed for different Pole-zero location based on the root locus approach. And the comparative results are tabulated in Table 4.1.

POLE	ZERO	G_m	P_m (degree)	t_s(sec)	%Mp
-200	-20	0.532	-12.763	0.459	0
-300	-30	1.367	6.3145	0.175	0.78
-400	-40	1.841	9.0474	4.95	31.1
-500	-50	1.644	4.3426	6.14	62.8

Table 4.1 Results for different Pole-zero location (RL method)

B. Inward Approach method

As mentioned in section III the controller is designed for different desired pole location based on the inward approach. And the comparative results are tabulated in Table 4.2.

DESIRED POLE POLYNOMIAL	G_m	P_m (degree)	t_s(sec)	%Mp
$(s+200)^5$	3.2132	56.6904	0.0345	35.6%
$(s+100)^5$	2.8584	47.8662	0.0771	70.32%

Table 4.2 Results for different Pole-zero location (RL method)

SYSTEM GAIN	G_m	P_m (degree)	t_s(sec)	%Mp
2.93	2.2213	18.1823	Infinite	NaN
2.95	2.1995	17.9217	10.8	0
3.991	1.3674	6.3145	0.175	0.786
9	0.0485	-22.9846	4.1	77.2

Table 4.3 Results for different Pole-zero location (Inward method)

By observing the table 4. 1 for different location of poles and zeros and variable system gain, we find that for pole at-300 and zero at -30 the gain margin Gm=1.3674 and settling time 0.175 seconds which is found very optimum from the table.

A comparative study is also done for the different system gains when desired pole polynomial is $(s+100)^5$ and are tabulated in Table 4.3.

From Table 4.1,4.2,4.3the controller design using root locus is more effective as compared improved gain margin, better time response.

V. CONCLUSION & FUTURE SCOPE

Magnetic Levitation System (MLS) is inherently unstable because of system non linearity. So the system has been linearised and compensated with suitable compensators and then results are compared to find the best possible choice. The further scope would be to design the controller using advanced control technique such as adaptive control technique, fuzzy logic, sliding mode control etc. The system can be made more reliable, stable and precise using such advanced controllers

REFERENCES

[1] SintayehuChalla, *"Thesis on Magnetic Levitation on A Steel Ball"* Addis Ababa University School Of Graduate Studies April 2007.

[2] Norman Nise,*Control Systems Engineering*, Wiley India (P.) Ltd., Delhi, Third Reprint, 2007.

[3] K Ogata, *Discrete Time Control System*, Second Edition,PHI arning Private Limited,NewDelhi,2009.

[4] Milaca B NaumovicBoban R Veselic, "Magnetic Levitation System In Control Engineering Education Udc681.537".

[5] Marwan K. Abbadi, "Modelling And Control Of A Magnetic Levitation System" Department Of Electrical And Computer Engineering Bradley University May 13,2004.

[6] ValerDolgaLiaDolga, "Modelling And Simulation Of A Magnetic Levitation System" Annals Of The Oradea University.Fascile of Management And Technological Engineering, Volume VI (XVI) 2007.

[7] Benjamin C. K "Automatic Control System" fifth edition. Prentice hall of Indian Private ltd, New Delhi,1989.

[8] RICHARD C.DORF, ROBERT H.BISHOP ,*Modern Control System*, eleventh edition, Pearson International Edition.

[9] Wong, T. H. "Design of a magnetic levitation system-an undergraduate project," *IEEE Tran. on Education*, vol E-29, no. 4, November 1986.

[10] El Hajjaji, A. and M. Ouladsine, *Modeling and nonlinear control of magneticlevitation systems.* Industrial Electronics,IEEE Transactions on, 2001

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