Long Period Fiber Grating (LPFG) and to Study the Effect of Coupling Length on Its Different Modes

Ms. Himani Dua\textsuperscript{1}, Mr. Puneet Sehgal\textsuperscript{2}

Department of Instrumentation, Assistant Professor, Shaheed Rajguru College of Applied Sciences for Women, University of Delhi, India\textsuperscript{1}
Department of Electronics, Assistant Professor, A.R.S.D. College, University of Delhi, India\textsuperscript{2}

Abstract: As we already know that optical fiber communication is now a days used in the world’s communication network. Optical fiber can be used as a medium for telecommunication over long distances and networking of respective signals as it is light weight and can be bundled in the form of cables. Fiber optics is the only technique known today to have the power to meet the strong demands for flexibility and high bandwidth posed by the rapidly growing communication networks. The optical systems were primarily used in point-to-point long distance links [10]. As long period grating (LPG) is an important component of optical fiber communication system, thus, long period grating can be designed in an effective manner to use it as a important tool to sense the refractive index of any medium. In this paper we have studied about in depth concepts of Long period grating (LPG) and coupled mode equations have been solved for long period grating to analyze the grating structures that exhibit attractive optical properties that make them suitable for optical communication system as a wavelength filter. At the end, we have studied the effect of coupling length on exchange of power and its variation between the core mode and different cladding modes at $\lambda$ =1.55μm for a specific set of parameters defined under the observation for a fiber.

Keywords: Optical Fiber, Communication network, Long Period Grating, Coupled Mode analysis, cladding modes, coupling length.

I. INTRODUCTION

Fiber gratings are often classified as fiber Bragg gratings (FBGs) or long-period gratings (LPGs), according to grating period. LPGs typically have a grating period in the range of from 100 μm to 1 mm, whereas FBGs have a sub-micron period. A long-period fiber grating (LPG), which couples light from a fundamental guided core mode into co-propagating cladding modes at various wavelengths, was first reported by Vengsarkar and co-workers in 1996 [1]. LPGs have also been used as gain-flattening filters for erbium-doped fiber amplifiers [2], and as optical fiber polarizer’s [3]. LPGs have a number of unique advantages such as the fact that simple techniques are required to fabricate them, their compact construction (they are intrinsic fibre devices) and non-conducting (dielectric) structure that is immune to electromagnetic interference (EMI). LPGs have a number of unique advantages such as the fact that simple techniques are required to fabricate them, their compact construction (they are intrinsic fibre devices) and non-conducting (dielectric) structure that is immune to electromagnetic interference (EMI) [4]. Long period gratings are periodic photo-induced devices which couple light from core mode to various cladding modes of a single mode fiber. The cladding modes are quickly attenuated and this result in series of loss bands in the transmission spectra of the grating as shown in Fig 1 below. Each of these loss bands corresponds to coupling to distinct cladding modes [5].

The phase matching condition between the fundamental mode and the forward propagating cladding mode for the long-period grating (LPG), is given by [6]

$$\lambda_0 = \left(n_{eff}^{co} - n_{eff}^{cl,m}\right) \Lambda$$

Where $\lambda_{res}$ is the resonance wavelength, $n_{co}$ is the effective refractive index of the core mode and $n_{eff}^{cl,m}$ is the effective index of the m\textsuperscript{th} cladding mode[9]. $\Lambda$ is the grating period which is much longer for co propagating coupling at a given wavelength than for the counter propagating coupling [7]. Thus, the rejection wavelengths of LPG are sensitive to such environmental changes [8]. As mentioned earlier a long period fiber grating is formed typically by introducing a periodic refractive index modulation in the core of the optical fiber. LPG couples an incident fundamental core mode ($LP_{01}$) to forward propagating cladding modes ($LP_{m0}$) when the phase
The core cladding mode index profile and first order perturbation compared with the fiber at $\lambda = 1.55\mu m$ is tabulated in below Table 2.

To begin with the coupled mode analysis for a long period grating, we consider an optical fiber with a general refractive index profile $n(r)$ in which there is a sinusoidal $z$-dependent periodic index variation $\Delta n^2(z) = \Delta n^2 \sin Kz$ in the core region. The total field at any value of $z$ can thus be described as

$$\psi = A(z) \psi_{co}(r)e^{-i\beta_{co} z} + B(z) \psi_{cl}(r)e^{-i\beta_{cl} z}$$

(1)

where $\psi_{co}(r)$ and $\beta_{co}$ represent normalized modal field and propagation constant of the core mode, $\psi_{cl}(r)$ and $\beta_{cl}$ represent normalized modal field and propagation constant of the phase matched cladding mode of the fiber. $A(z)$ and $B(z)$ are the amplitudes corresponding to core and cladding mode (where $z$ is the direction of propagation). Since the modes are orthonormal and normalized for unit power, $|A(z)|^2$ and $|B(z)|^2$ directly give the power in the core mode and cladding mode respectively. In the absence of perturbation, $A$ and $B$ are constants and equal to their value at $z=0$; the perturbation couples power among the modes as they propagate and hence, $A$ and $B$ are $z$-dependent. Since $\psi_{co}(r)$ and $\psi_{cl}(r)$ are the modal fields of the fiber in the absence of any perturbation, they satisfy the following equations

$$\nabla_t^2 \psi_{co}(r) + \left(k_0^2 n^2 - \beta_{co}^2\right)\psi_{co}(r) = 0$$

(2)

$$\nabla_t^2 \psi_{cl}(r) + \left(k_0^2 n^2 - \beta_{cl}^2\right)\psi_{cl}(r) = 0$$

(3)
The total field satisfies the following wave equation

$$\nabla^2 \psi + k_0^2 \left( n^2 + \Delta n^2(z) \right) \psi = 0 \quad (4)$$

We now substitute the total field $$\psi$$ from equation (1) into equation (4) and use the slowly varying approximation (i.e. $$\lambda \frac{d^2 A}{dz^2} \ll \frac{dA}{dz}$$ and $$\lambda \frac{d^2 B}{dz^2} \ll \frac{dB}{dz}$$) to obtain the following equation

$$-2\beta e^{i\beta t} \sum_{n} \frac{d}{dz} \left[ \psi_{cl} m(z) e^{i\beta l} + \frac{d}{dz} \left[ \psi_{co} m(z) e^{i\beta l} \right] \right] = 0 \quad (5)$$

where equations (2) and (3) have been used to simplify the expression. Multiplying equation (5) by $$\psi_{co}(r)$$, integrating over the whole cross-section of the fiber and using the orthonormality condition $$\langle \psi_{co} \psi_{cl}^* \rangle = 0$$ we get the coupled mode equation depicting evolution of modal amplitude of core mode with propagation distance, $$z$$, as

$$\frac{da}{dz} = \frac{k_0^2}{2} \left( \frac{\psi_{co} \Delta n^2}{\psi_{cl} m(z)} \right)$$

$$\frac{db}{dz} = \frac{k_0^2}{2} \left( \frac{\psi_{co} \Delta n^2}{\psi_{cl} m(z)} \right)$$

Where, $$\Delta \beta = \beta_{co} - \beta_{cl}$$, Similarly, by multiplying $$\psi_{cl} m(r)$$ and integrating over the whole cross-section of the fiber we get the coupled mode equation depicting evolution of modal amplitude of cladding mode with propagation distance, $$z$$, as

$$\frac{da}{dz} = \frac{k_0^2}{2} \left( \frac{\psi_{cl} \Delta n^2}{\psi_{co} m(z)} \right)$$

$$\frac{db}{dz} = \frac{k_0^2}{2} \left( \frac{\psi_{cl} \Delta n^2}{\psi_{co} m(z)} \right)$$

Equations (6) and (7) can be further simplified by defining of coupling coefficients $$\kappa_{11}, \kappa_{12}, \kappa_{21}$$ and $$\kappa_{22}$$ as

$$\kappa_{11} = \frac{k_0^2}{2} \left( \frac{\psi_{co} \Delta n^2}{\psi_{cl} m(z)} \right)$$

$$\kappa_{12} = \frac{k_0^2}{2} \left( \frac{\psi_{co} \Delta n^2}{\psi_{cl} m(z)} \right)$$

$$\kappa_{21} = \frac{k_0^2}{2} \left( \frac{\psi_{cl} \Delta n^2}{\psi_{co} m(z)} \right)$$

$$\kappa_{22} = \frac{k_0^2}{2} \left( \frac{\psi_{cl} \Delta n^2}{\psi_{co} m(z)} \right)$$

With $$\Delta n^2(z) = \Delta n^2 \sin K z$$. The equations (6) and (7) can now be expressed as

$$\frac{da}{dz} = -2i\kappa_{11} A(z) \sin K z - 2i\kappa_{12} B(z) e^{i\Delta \beta} \sin K z$$

$$\frac{db}{dz} = -2i\kappa_{22} B(z) \sin K z - 2i\kappa_{21} A(z) e^{-i\Delta \beta} \sin K z$$

In order to understand the physical significance of the self coupling coefficients $$\kappa_{11}, \kappa_{22}$$, we now define a new set of variables, $$\psi_{co} = A(z) e^{-i\Delta \beta}$$ and $$\psi_{cl} = B(z) e^{-i\Delta \beta}$$ and rewrite the coupled mode equations in terms of $$a(z)$$ and $$b(z)$$ as

$$\frac{da}{dz} = -i(\beta_{co} + 2\kappa_{11} \sin K z) a - 2i\kappa_{12} \sin K z b$$

$$\frac{db}{dz} = -i(\beta_{cl} + 2\kappa_{22} \sin K z) b - 2i\kappa_{21} \sin K z a$$

As can be seen from equations (11) and (12) and $$2\kappa_{11} \sin K z$$ and $$2\kappa_{22} \sin K z$$ are the small first order perturbation corrections ($$\Delta n \sim 10^{-4}$$) to the propagation constants $$\beta_{co}$$ and $$\beta_{cl}$$ of the core mode and cladding mode of $$n$$th order respectively due to presence of perturbation. Hence, in order to simplify the algebra to obtain analytical solution, $$\kappa_{11}$$ and $$\kappa_{22}$$ are neglected in comparison to the propagation constants $$\beta_{co}$$ and $$\beta_{cl}$$. Incorporating this and substituting

$$\sin(Kz) = \frac{1}{2i} [\exp(iKz) - \exp(-iKz)]$$

one can rewrite equations (11) and (12) as

$$\frac{da}{dz} = -\kappa_{12} B(z) \exp(i\Delta \beta) + \kappa_{12} B(z) \exp(-i\Delta \beta)$$

$$\frac{db}{dz} = -\kappa_{21} A(z) \exp(i\Delta \beta) + \kappa_{21} A(z) \exp(-i\Delta \beta)$$

For weak perturbations, $$\kappa_{12}$$ and $$\kappa_{21}$$ are small and hence, the typical length scale over which the mode amplitudes change appreciably $$1/\kappa_{12} \approx 1/\kappa_{21}$$, which is large. If we integrate equations (13) and (14) over a short length $$L$$, we obtain

$$A(L) = A(0) e^{-i\Delta \beta L}$$

$$B(L) = B(0) e^{-i\Delta \beta L}$$

In order to estimate the magnitude of each term in equations (15) and (16), we use the typical values of propagation constants for LP01 and LP00 modes as given in Table 1.2,

$$\Delta \beta = \beta_{co} - \beta_{cl} = 5.8550555 - 5.82592743 = 0.02916807 \mu \text{m}^{-1}$$

$$\lambda_0 = 1.55 \mu \text{m}$$.

We choose $$\Delta \beta$$ to satisfy the phase matching condition, i.e., $$\Delta \beta = K$$ and $$L = 53 \times 10^{-6} \text{ m}$$ leading to

$$\frac{\sin((\Delta \beta - K)L/2)}{(\Delta \beta - K)} \approx \frac{L}{2} = 26.91 \times 10^{-6} \text{ m}$$

$$\frac{\sin((\Delta \beta + K)L/2)}{(\Delta \beta + K)} \leq \frac{1}{2\Delta \beta} \approx \frac{1}{2 \times 17 \times 10^{-6} \text{ m}}$$

Thus, for $$\Delta \beta \approx K$$, the contribution can be
Ignored. The evolution of amplitude of core and cladding modes with $z$ can now be expressed as

$$\frac{dA}{dz} = \kappa_{12}B(z)e^{-i(\Delta \beta - K)z} = \kappa_{12}B(z)e^{-i\Gamma z}$$  \hspace{1cm} (16)

$$\frac{dB}{dz} = -\kappa_{21}A(z)e^{-i(\Delta \beta - K)z} = -\kappa_{21}A(z)e^{-i\Gamma z}$$  \hspace{1cm} (17)

where $\Gamma$ is known as detuning or phase mismatch factor defined as

$$\Gamma = \Delta \beta - \frac{2\pi}{A}$$  \hspace{1cm} (18)

Differentiating equation (23) with respect to $z$ and substituting equation (24), the following second order differential equation is obtained

$$\frac{d^2 A}{dz^2} - i\Gamma \frac{dA}{dz} + \kappa^2 A = 0$$  \hspace{1cm} (19)

Where, $\kappa = \sqrt{\kappa_{12} \kappa_{21}}$.

Using the boundary conditions that $A(z=0)=A_0$ and $B(z=0)=B_0$, we obtain the following analytical solutions for $A(z)$ and $B(z)$

$$A(z) = e^{\frac{i\Gamma z}{2}} \left[ A_0 \left\{ \cos(\gamma z) - i \frac{\Gamma}{2\gamma} \sin(\gamma z) \right\} + B_0 \left\{ \frac{\kappa}{\gamma} \sin(\gamma z) \right\} \right]$$  \hspace{1cm} (20)

$$B(z) = e^{-\frac{i\Gamma z}{2}} \left\{ i \frac{\kappa}{\gamma} \sin(\gamma z) \right\}$$  \hspace{1cm} (21)

Where $\gamma = \sqrt{\frac{\Gamma^2}{4} + \kappa^2}$. We assume that unit power is initially launched only in the core mode, i.e., $A(z=0)=1$ and $B(z=0)=0$ and the above equations reduce to a following simplified form

$$A(z) = e^{\frac{i\Gamma z}{2}} \left\{ \cos(\gamma z) - i \frac{\Gamma}{2\gamma} \sin(\gamma z) \right\}$$  \hspace{1cm} (22)

$$B(z) = e^{-\frac{i\Gamma z}{2}} \left\{ i \frac{\kappa}{\gamma} \sin(\gamma z) \right\}$$  \hspace{1cm} (23)

The power in the core mode and the cladding mode at any $z$ can now be expressed as

$$P_{co} = A(z)A^*(z) = 1 - \frac{\kappa^2}{\gamma^2} \sin^2(\gamma z)$$  \hspace{1cm} (24)

$$P_{cl} = B(z)B^*(z) = \frac{\kappa^2}{\gamma^2} \sin^2(\gamma z)$$  \hspace{1cm} (25)

For phase matched condition, i.e., the detuning factor $\Gamma = 0$, $\gamma = \kappa$ and the power in the core mode and cladding mode can be expressed as

$$\kappa^2 \left[ \frac{\sin^2(\kappa z)}{\gamma^2} \right]$$  \hspace{1cm} (26)

$$\kappa^2 \sin^2(\kappa z)$$  \hspace{1cm} (27)

As can be predicted from the above equations power is continuously exchanged between the core mode and the phase matched cladding mode. Complete power transfer from the core mode to the cladding mode takes place after a propagation distance of $z = \frac{\pi}{2\kappa}$ also referred to as coupling length. Hence coupling length $l_c$ is defined as

$$l_c = \frac{\pi}{2\kappa}$$  \hspace{1cm} (28)

### III. COUPLING COEFFICIENTS

It may be noted that ($\kappa = \kappa_{12} = \kappa_{21}$) as defined in equation using the normalization condition equation can be expressed as

$$\kappa_{12} = \kappa_{21} = \frac{k_0}{4} \frac{2n_1\Delta n}{\psi_{cl}^{\psi_{co}}} dr$$

$$\kappa_{11} = \frac{k_0}{4} \frac{2n_1\Delta n}{\psi_{cl}^{\psi_{co}}} dr$$

$$\kappa_{22} = \frac{k_0}{4} \frac{2n_1\Delta n}{\psi_{cl}^{\psi_{co}}} dr$$

hence, the coupling coefficient $\kappa$ is a function of the index change and the modal overlap between the core guided mode and phase matched cladding mode over the region of perturbation. Since the coupling coefficient is directly proportional to the overlap integral, the modal distribution of the cladding mode will have a strong influence on its magnitude. It may be reiterated that the index modulation has no azimuthal variation, the coupling of the symmetric core guided mode can occur only to the symmetric cladding modes ($l=0$), since, for all other cladding modes, the overlap integral is zero. The coupling coefficient can be expressed analytically as in equation (30) below

![Fig2: Variation of coupling coefficient with cladding mode order for 1.55µm.](image-url)
is not an integral multiple of coupling length, an incomplete exchange of power between the core mode and the phase matched cladding mode takes place even at resonant wavelength.

![Graph showing exchange of power and variation in coupling length between the core mode and different cladding modes at a wavelength of 1.55 μm.](image)

Fig 3: Exchange of power and variation in coupling length between the core mode and different cladding modes at a wavelength of 1.55 μm.

<table>
<thead>
<tr>
<th>Lp02 mode</th>
<th>Grating period (μm)</th>
<th>Coupling length (μm)</th>
<th>Kappa (μm⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>57.92520</td>
<td>269500.00</td>
<td>5.83e-6</td>
</tr>
<tr>
<td>0.3</td>
<td>54.8970</td>
<td>147500.00</td>
<td>1.06e-5</td>
</tr>
<tr>
<td>0.4</td>
<td>48.3850</td>
<td>106950.00</td>
<td>1.47e-5</td>
</tr>
<tr>
<td>0.5</td>
<td>43.1866</td>
<td>8785.00</td>
<td>1.79e-5</td>
</tr>
<tr>
<td>0.6</td>
<td>37.3309</td>
<td>7755.00</td>
<td>2.02e-5</td>
</tr>
<tr>
<td>0.7</td>
<td>32.1924</td>
<td>7169.00</td>
<td>2.19e-5</td>
</tr>
<tr>
<td>0.8</td>
<td>27.7171</td>
<td>6582.00</td>
<td>2.29e-5</td>
</tr>
<tr>
<td>0.9</td>
<td>23.6106</td>
<td>6753.00</td>
<td>2.32e-5</td>
</tr>
<tr>
<td>1.0</td>
<td>20.7252</td>
<td>6830.00</td>
<td>2.3e-5</td>
</tr>
<tr>
<td>1.1</td>
<td>18.5814</td>
<td>7118.00</td>
<td>2.21e-5</td>
</tr>
<tr>
<td>1.2</td>
<td>16.8514</td>
<td>7640.00</td>
<td>2.05e-5</td>
</tr>
<tr>
<td>0.13</td>
<td>15.3413</td>
<td>8565.00</td>
<td>1.83e-5</td>
</tr>
<tr>
<td>0.14</td>
<td>13.7595</td>
<td>10062.00</td>
<td>1.56e-5</td>
</tr>
<tr>
<td>0.15</td>
<td>11.8318</td>
<td>12120.00</td>
<td>1.3e-5</td>
</tr>
<tr>
<td>0.16</td>
<td>9.83032</td>
<td>19706.00</td>
<td>7.97e-6</td>
</tr>
<tr>
<td>0.17</td>
<td>8.93348</td>
<td>35000.00</td>
<td>4.41e-6</td>
</tr>
</tbody>
</table>

Table 3: Coupling length and kappa of LPG for coupling to different cladding modes at 1.55 μm

Thus, we can plot the propagation curve for coupling length between the LP (01) mode with other cladding modes at wavelength 1.55 μm (as shown in Fig 3) and at wavelength 1.54 μm (as shown in Fig 4 above) using the set of points tabulated in Table 3. Hence, we observed that we cannot get complete power exchange from core mode to cladding mode for any length of fiber.

V. RESULT AND CONCLUSION

Thus we have studied the detailed theory of Long fiber grating (LPG) by deriving the coupled mode equations. We have analysed the Variation of coupling coefficient with cladding modes order at the wavelength 1.55μm and also plotted the propagation curve for coupling length between LP(00) mode with other cladding modes at wavelength 1.55μm and 1.54μm respectively. Thus, it can be concluded that we cannot get complete power exchange from core mode to cladding mode for any length of fiber.

This analysis of LPG shows that they can be used as gain-flattening filters for erbium-doped fiber amplifiers and optical fiber polarizer’s.

ACKNOWLEDGEMENT

The authors are thankful to University of Delhi and their respective colleges. Thanks are also due to Department of Electronic Science, South Campus, University of Delhi and our colleagues for their constant guidance and unconditional support.

REFERENCES