ISSN 2321 – 2004
ISSN 2321 – 5526

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 1, Issue 3, June 2013

Delay Dependent Stability Criteria for Linear Systems with Time Varying Delay in a Range

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Abstract: Time delays are a common problem in many physical systems. Presence of delay often leads to instability and poor control performance. So study of stability of systems with time delay has gained much popularity. This paper is concerned with the stability analysis of systems with time delay. Time delay is assumed to be continuous time varying function and it belongs to a specified range. This states that the lower and upper bounds of the delay are assumed to be known. Here the upper delay bound is considered to be known and lower delay bound is considered to be zero. Using an appropriate Lyapunov functional, delay dependent stability criteria is proposed. Stability conditions are defined using Linear Matrix Inequalities (LMI). Effectiveness of the proposed method is demonstrated using a suitable numerical example.

Keywords: Time delays, Asymptotic stability, Lyapunov functional, Linear Matrix Inequalities.

I. INTRODUCTION

Time delays are often encountered in many practical systems such as biological systems, chemical processes, nuclear reactors and so on. In these systems, the rate of variation of system states depends on past states also. This often leads to instability and poor control performance. Stability criteria for these systems can be classified into two classes; 1) delay independent stability criteria 2) delay dependent stability criteria. In delay independent stability criteria, stability and performance of the system is ensured for delays of any size. In delay dependent stability criteria, stability and performance of the system is ensured for delays lesser than a given bound. When delay is comparatively small, delay independent case proves to be more conservative.

There are two approaches as far as delay dependent criteria is concerned, frequency domain and time domain approach. Because of powerful tools such as direct Lyapunov method, time domain approach is the widely used one. Lyapunov-Krasovskii approach and Lyapunov-Razumikhin approach are widely used and these can be used to handle systems with time varying delay. Considerable attention has been given to problems with delay dependent stability analysis and control of time delay systems [1~8]. A system with time varying delay was considered in [9] and delay dependent stability criteria was proposed. An improved delay dependent stability criterion for systems with time varying delay was proposed in [10] by using Jensens inequality. In both cases upper bound for the delay was defined. Problem of delay dependent stability of systems with time varying delays and structured uncertainties were considered in [11]. Delay dependent robust control of systems with time varying delay and norm bounded uncertainties were considered in [12]. Information about the derivative of the delay was included in all the works. But, in most cases derivative of the delay is unknown. So a separate stability condition was given for this case.

Delay dependent H_∞ control for time delay systems was introduced in [13]. Introducing equality with some free weighting matrices, delay dependent stability criteria with H_∞ performance was presented. Delay dependent robust H_∞ stability for uncertain time delay systems with stochastic perturbations was introduced in [14]. Criteria were developed by taking into account the relationship between terms in the Leibniz-Newton formula. Free weighting matrices were used to define the criteria. By using, new Lyapunov-Krasovskii functional and using slack matrix variables, a less conservative delay dependent stability criteria was derived in terms of Linear Matrix Inequalities in [15]. Based on this, a condition for the existence of static state feedback controller which ensures asymptotic stability was proposed. Delay range dependent stability and stabilization for systems with time varying delay in a range was proposed in [16]. By using a new Lyapunov functional and by making use of some free weighting matrices, a less conservative criterion was proposed. Based on these criteria, existence of a delay range dependent state feedback $H\infty$ control was proposed. All these conditions were obtained in terms of Linear Matrix Inequalities (LMI's).

ISSN 2321 – 2004
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INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 1, Issue 3, June 2013

The composition of the paper is as follows. Section 2 describes the problem statement, in which the representation of the system and conditions of delay are defined. Section 3 gives the main results in which the asymptotic stability of time delay system is specified using a particular Lyapunov functional. The simulation results are discussed in Section 4.

II. PROBLEM FORMULATION

A Consider a class of linear systems with time varying delay in a range:

$$\dot{x}(t) = Ax(t) + A_1 x(t - d(t))$$
$$x(t) = \phi(t)$$
$$t \in [-\tau, 0] \qquad (1)$$

where x(t) ϵ R^n is the state vector; u(t) ϵ R^p is the input vector; A and A_1 are known parameter matrices with appropriate dimensions and $d(t)$ is a time varying continuous function satisfying

$$0 \le d(t) \le \tau$$
$$\dot{d}(t) \le \mu \le 1 \qquad (2)$$

where τ & μ are constants.

$\phi(t)$ is a continuous vector valued initial function defined in

t ϵ [- τ ,0].

The main aim of this paper is to analyse the asymptotic stability of a given system with time varying delay using a less conservative stability criteria derived in terms of Linear Matrix Inequalities.

III. MAIN RESULTS

A. *Stability Analysis*

Theorem 1: Given scalars $\tau > 0$ & μ , the linear system (1) with time varying delay satisfying (2) is asymptotically stable if there exist matrices $P = P^T > 0, Q = Q^T > 0, Z = Z^T > 0$ such that the following LMI holds:

$$\begin{bmatrix} \Pi & * & * \\ A_1^T P + \tau A_1^T ZA & -(1-\mu)Q + \tau A_1^T ZA_1 & * \\ \tau^{-1} Z & 0 & -\tau^{-1} \end{bmatrix}$$

< 0

Proof:
Considering a modified Lyapunov functional,

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

Where $V_1(t) = x^T P x$

$$V_2(t) = \int_{t-d(t)}^{t} x^T(s) Q x(s) ds$$

$$V_3(t) = \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z \dot{x}(s) ds d\theta$$

Now considering the derivative of $V(t)$

$$\dot{V} \le \dot{x}^T Px + x^T P \dot{x} + x^T Qx - (1-\mu) x^T (t-d) Q x(t-d)$$
$$+ \int_{-\tau}^{0} \dot{x}^T(t) Z \dot{x}(t) d\theta - \int_{-\tau}^{0} \dot{x}^T(t+\theta) Z \dot{x}(t+\theta) d\theta$$

Applying Jensens inequality, which can be described as:

$$-\int_{t-\tau}^{t} \dot{x}^T(\theta) Z \dot{x}(\theta) d\theta \le$$
$$-\tau^{-1} [x(t) - x(t-\tau)]^T Z [x(t) - x(t-\tau)]$$

Applying it to above equation and expanding it along the solutions of (1), we get:

$$\le [x^T A^T + x^T(t-d) A_1^T] Px + x^T P [Ax + A_1 x(t-d)]$$
$$+ x^T Qx - (1-\mu) x^T(t-d) Q x(t-d)$$
$$+ \tau [x^T A^T + x^T(t-d) A_1^T] Z [Ax + A_1 x(t-d)]$$
$$- \tau^{-1} x^T(t) Z x(t) + \tau^{-1} x^T(t) Z x(t-\tau)$$
$$+ \tau^{-1} x^T(t-\tau) Z x(t) - \tau^{-1} x^T(t-\tau) Z x(t-\tau)$$

Rearranging the terms, we get:

$$\dot{V} \le x^T [A^T P + PA + Q + \tau A^T ZA - \tau^{-1} Z] x$$
$$+ x^T(t-d) [A_1^T P + \tau A_1^T ZA] x$$
$$+ x^T(t-\tau) [\tau^{-1} Z] x$$
$$+ x^T(t-d) [-(1-\mu) Q + \tau A_1^T ZA_1] x(t-d)$$

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$$+x^T(t)[\tau^{-1}Z]x(t-\tau)$$
$$+x^T(t-\tau)[-\tau^{-1}Z]x(t-\tau)$$
$$+x^T[PA_1+\tau A^T ZA_1]x(t-d)$$
$$=\zeta^T(t)\phi\zeta(t)$$

Where

$$\zeta(t)=[x^T(t)\;\; x^T(t-d)\;\; x^T(t-\tau)]$$

ϕ =

$$\begin{bmatrix} \Pi & * & * \\ A_1^T P+\tau A_1^T ZA & -(1-\mu)Q+\tau A_1^T ZA_1 & * \\ \tau^{-1}Z & 0 & -\tau^{-1}Z \end{bmatrix}$$

$$\Pi = A^T P + PA + Q + \tau A^T ZA - \tau^{-1}Z$$

With this, it concluded that system (1) is asymptotically stable and hence this ends the proof. □

When the information about the time derivative of delay is not known, following result can be derived from Theorem 1.

Corollary 1: Given scalar $\tau > 0$, the linear system (1) subject to (2) is asymptotically stable if there exists matrices $P=P^T>0, Q=Q^T>0, Z=Z^T>0$ such that the following LMI holds:

$$\begin{bmatrix} \Pi & * & * \\ A_1^T P+\tau A_1^T ZA & -Q+\tau A_1^T ZA_1 & * \\ \tau^{-1}Z & 0 & -\tau^{-1} \end{bmatrix} < 0$$

Proof:

By putting $\mu = 0$ in the above given proof, above stability condition can be obtained.

$\prod$ is the same as defined in Theorem: 1.

IV. NUMERICAL EXAMPLE

To show the effectiveness of the proposed method, consider system (1) with

$$A=\begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad A_1=\begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

All simulations were carried out using YALMIP toll box. Table I shows the admissible values of upper delay bound for various values of μ. For unknown μ values, admissible delay bound is given in table II.

TABLE I

μ	0.3	0.5	0.66
τ	1.2	2	2.25

TABLE II

μ	0
τ	1.199

The delay range bound provided by the stability criterion of Theorem 1 is given in Table 1. For various values of the derivative of time delay, the maximum admissible delay bound is obtained. Table 2 shows the maximum admissible delay bound for unknown or constant time delay. This is defined in Corollary 1, which turns out to be a special case of Theorem 1.

V. CONCLUSION

In this paper, delay dependent stability criteria for a class of linear systems with time varying delay has been considered. A new Lyapunov functional was proposed to construct the stability criteria. With the use of Leibniz-Newton formula and Jensens inequality, a simplified delay dependent stability criteria is proposed. Even though the proposed criteria has less matrix variables, the results obtained are less conservative.

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BIOGRAPHY

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