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High Resolution Spectral Analysis useful for the development of Radar Altimeter

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Abstract: This paper presents a comparative study of high resolution spectral estimation methods applied to Radar Altimeter. Spectral estimation methods such as Yule-Walker, Burg, Covariance, modified Covariance, MUSIC, minimum norm, ESPRIT methods are briefly reviewed. Computer simulations have been made using a test signal with six frequencies in order to evaluate the probability of detection of each frequency. High resolution spectral estimates showed much more spectral details than the Fourier spectrum and also it exhibits consistent peak position when compared to the lower resolution technique.

Keywords: Radar altimeter, High resolution, spectral estimation, Yule-Walker, Burg, Covariance, Modified Covariance, MUSIC, minimum norm, ESPRIT

I. INTRODUCTION

Radar Altimeter is used to measure the altitude of an aircraft during low altitude flights and landing phase. It is an all-weather altitude measurement system. The transmitter antenna transmits RF signal at 4.25 to 4.35 GHz and receiving antenna receives it after some delay. The delay is measured internally and is proportional to the altitude of the vehicle above ground level. It measures altitude more directly, using the time taken for a radio signal to reflect from the surface back to aircraft. In Radar altimeter, transmitter output is frequency modulated. The signal received from the ground is of a different frequency than the transmitter at the time of arrival. The frequency of the return signal is the frequency of the transmitter at the time signal left the transmitter. If the rate of frequency change is noted, the frequency difference between the oscillator and the received signal is measured; the time can be determined. A block diagram of a FM-CW Radar Altimeter is shown in fig. 1

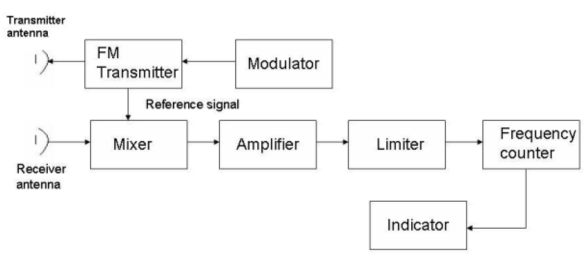

Figure1.Principle of FM-CW Radar Altimeter.

In the frequency modulated CW Radar Altimeter, the transmitter frequency is changed as a function of time, in a known manner . Assume that the transmitter frequency increases linearly with time as shown in fig 2 a. If there is a reflecting surface at a distance R, an echo will return after a time T=2R/C. If the echo signal is heterodyned with a portion of the transmitter signal in a nonlinear element such as diode, a beat note f_b will be produced. The beat note is a measure of the altitude of the vehicle. In any practical CW radar altimeter, the frequency cannot be continuously changed in one direction only. Periodicity in the modulation is necessary, as in the triangular frequency modulation waveform shown in fig 2 (b). The resulting beat frequency as a function of time is shown in fig 3. The beat frequency is of constant frequency except at the turnaround region. If the frequency is modulated at a rate f_m over a range Δf, the beat frequency is given by,

$$f_b = \frac{4Rf_m\Delta f}{c} \quad (1)$$

Thus the measurement of beat frequency determines the range R [1].

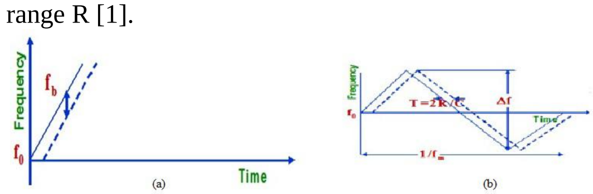

Figure 2. (a)Linear modulation (b) Triangle modulation

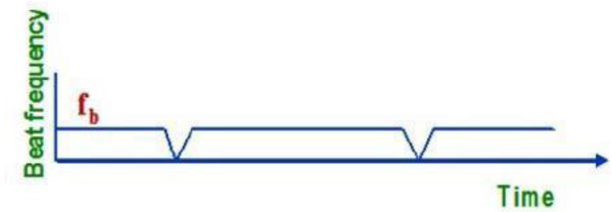

Figure 3. Beat frequency as a function of time.

A portion of the transmitter signal acts as a reference signal required to produce the beat frequency. The beat frequency is amplified and limited to remove any amplitude fluctuations. The frequency of the amplitude limited beat note is measured using DSP processor with digital techniques. The frequency is obtained by using the Discrete Fourier Transform. This paper describes an investigation of high resolution spectral analysis applied to the FMCW Radar Altimeter for estimating the range.

II. HIGH RESOLUTION SPECTRAL ANALYSIS

Spectral analysis is any signal processing method that characterizes the frequency content of a measured signal[2]. Spectral analysis with DFT computes a uniformly spaced set of N spectral amplitudes from an N-point data window. High-resolution determination of amplitude, frequency, and phases requires that N be large enough for the chosen resolution[3]. Spectral estimation is useful when the spectrum is rapidly changing. It tries to find the best fit of the data to a finite set of trial functions in the data domain, also allows the frequency values and other parameters be finely resolved. Spectrum estimation is useful,even for static spectra, instead of using sinusoids to fit the signal, we can use any arbitrary set of trial functions. The observation period or number samples is finite for real world applications. Thus, using the limited available information spectral content is estimated. In this paper, we have considered seven different methods: Yule-Walker, Burg, Covariance, modified Covariance, MUSIC (MUltiple Signal Classification), Minimum norm, ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique). When a block of signal samples is available, it may be too short to provide enough frequency resolution in the periodogram spectrum [4]. In autoregressive method of spectral estimation, instead of trying to estimate power spectrum directly from the data, we model the data as the output of a linear system driven by white noise, and then attempt to estimate the parameters of that linear system. Estimates are found by solving a system of linear equations, and the stability of the estimated AR polynomial can be guaranteed [5]. High resolution methods aim to separate the observation space in a signal subspace, containing only useful information, and its orthogonal complement, called noise subspace. This decomposition makes the spectral analysis more robust and highly improves the spectral resolution [6].

A. Yule-Walker method

The Yule-Walker AR method of spectral estimation computes the AR parameters by forming a biased estimate of the signal's autocorrelation function, and solving the least squares minimization of the forward prediction error. This results in the Yule-Walker equations. The use of a biased estimate of the autocorrelation function ensures that the autocorrelation matrix above is positive definite. Hence, the matrix is invertible and a solution is guaranteed to exist. Moreover, the AR parameters thus computed always result in a stable all-pole model. The AR coefficients are obtained by solving the normal equations [7].

$$\begin{bmatrix} r_x(0) & r_x^*(1) & r_x^*(2)\cdots & r_x^*(p) \\ r_x(1) & r_x(0) & r_x^*(1)\cdots & r_x^*(p-1) \\ r_x(2) & r_x(1) & r_x(0)\cdots & r_x^*(p-2) \\ \vdots & \vdots & \vdots & \vdots \\ r_x(p) & r_x(p-1) & r_x(p-2)\cdots & r_x(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_p(1) \\ a_p(2) \\ \vdots \\ a_p(p) \end{bmatrix}$$

$$= -b(0) \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2)$$

Once the AR parameters have been estimated, then the AR spectral estimate is computed as

$$P_{AR}(e^{j\omega}) = \frac{b(0)}{1+\sum_{k=1}^{p} a_p(k)e^{-jk\omega}} \quad (3)$$

where,

$$r_x(k) = \frac{1}{N}\sum_{n=0}^{N-1-k} x(n+k)x^*(n)\,; k = 0,1,\dots,p \quad (4)$$

$$b(0) = r_x(0) + \sum_{k=1}^{p} a_p(k) r_x(k) \quad (5)$$

An artifact that may be observed in this method is spectral line splitting.

B. Covariance method

The covariance method for AR spectral estimation is based on minimizing the forward prediction error. This method fits an autoregressive (AR) model to the signal by minimizing the forward prediction error in the least squares sense. For short data records the covariance method produces higher resolution spectrum estimates than the Yule-Walker method. The covariance method requires finding the solution to the set of linear equations [7],

$$\begin{bmatrix} r_x(1,1) & r_x(2,1) & \cdots & r_x(p,1) \\ r_x(1,2) & r_x(2,2) & \cdots & r_x(p,2) \\ \vdots & \vdots & & \vdots \\ r_x(1,p) & r_x(2,p) & \cdots & r_x(p,p) \end{bmatrix} \begin{bmatrix} a_p(1) \\ a_p(2) \\ \vdots \\ a_p(p) \end{bmatrix} = -\begin{bmatrix} r_x(0,1) \\ r_x(0,2) \\ \vdots \\ r_x(0,p) \end{bmatrix} \quad (6)$$

where,

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$$r_x(k,l) = \frac{1}{N}\sum_{n=p}^{N-1} x(n-l)x^*(n-k)\,; \quad k = 0,1,\dots,p \qquad (7)$$

In this method, no windowing is performed in the data for the formation of autocorrelation estimates.

C. Modified Covariance method

The modified covariance method is based on minimizing the forward and backward prediction errors. This method fits an autoregressive (AR) model to the signal by minimizing the forward and backward prediction errors in the least squares sense [7]. Here autoregressive parameters are found by solving a set of linear equations given in equation 6 with

$$r_x(k,l) = \sum_{n=p}^{N-1}[x(n-l)x^*(n-k) + x(n-p+l)x^*(n-p+k] \qquad (8)$$

This method is not subjected to spectral line splitting, gives a statistically stable spectrum estimates with high resolution. Also peak locations are tend to be less sensitive to phase.

D. Burg method

The Burg method for AR spectral estimation is based on minimizing the forward and backward prediction errors while satisfying the Levinson-Durbin recursion [7],[8]. The primary advantages of the Burg method are resolving closely spaced sinusoids in signals with low noise levels, and estimating short data records, in which case the AR power spectral density estimates are very close to the true values. The accuracy of the Burg method is lower for high-order models, long data records, and high signal-to-noise ratios. In contrast to other AR estimation methods, the Burg method avoids calculating the autocorrelation function, and instead estimates the reflection coefficients directly. In the analysis of sinusoids in noise, this algorithm is subject to spectral line splitting and peak locations are highly dependent upon the phases of the sinusoids. As the model order for the Burg method is reduced, a frequency shift due to the initial phase of the sinusoids will become apparent.

E. MUSIC (MUltiple SIgnal Classification)

This method is based on the Eigen decomposition of autocorrelation matrix into subspace, a signal subspace and a noise subspace . Once these subspaces have been determined, a frequency estimation function is then used to extract estimates of frequencies. Let R_x be the M*M autocorrelation matrix of x(n) .If the eigen values are arranged in the decreasing order $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_M$, and if v_1, v_2, …v_M are the corresponding Eigen vectors, then we divide Eigen vectors into two groups: p signal Eigen vectors, M-p noise Eigen vectors. The frequency estimation function is given by

$$P_{MU}\ (e^{jw}) = \frac{1}{\sum_{i=p+1}^{M}|e^H v_i|^2} \qquad (9)$$

The frequencies of the complex exponentials are taken as the locations of the p large peaks in $P_{MU}\ (e^{jw})$ [7]. Theoretically, according to equation 9 the estimate value tends towards infinity whenever it is evaluated at a frequency corresponding to a signal spectral component. Practically, because of the limited calculation accuracy and estimation errors, the estimate values are always finite. Since very sharp peaks are detected, the spectral resolution is highly improved. Also their amplitudes lose any physical significance. Due to this, MUSIC algorithm is often considered a frequency estimate rather than a power spectral density estimate

F. Minimum norm method

Another Eigen decomposition based method is the minimum norm algorithm. This algorithm uses a single vector a that is constrained to lie in the noise subspace, and the complex exponential frequencies are estimated from the peaks of the frequency estimation function [7].

$$P_{MN}\ (e^{jw}) = \frac{1}{|e^H\ a|^2} \qquad (10)$$

The problem is to determine which vector in the noise subspace minimizes the effects of the spurious zeros on the peaks of $P_{MN}\left(e^{jw}\right)$. The minimum-norm method, as its name implies, seeks to minimize the norm of a in order to avoid spurious peaks in its pseudospectrum. We formulate constrained minimization problem as

$$min\ v^H P_n v \qquad (11)$$

$subject\ to$

$$v^H(P_n^H u_1)=1 \qquad (12)$$

By solving we get, $v = \lambda P_n^{-1}(P_n^H u_1) = \lambda u_1$ (13)

Minimum norm solution is

$$a = P_n \text{v} = \lambda P_n\, u_1 = \frac{P_n\, u_1}{u_1^H P_n\, u_1} \qquad (14)$$

which is simply the projection of the unit vector onto the noise subspace, normalized so that the first coefficient is equal to one. In terms of the eigenvectors of the autocorrelation matrix, the minimum norm solution may be written as

$$a = \frac{(V_n V_n^H)u_1}{u_1^H(V_n V_n^H)u_1} \qquad (15)$$

G. ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique)

It exploits a deterministic relationship between subspaces. In this method, signal subspace is estimated from the data matrix rather than the estimated correlation matrix. The essence of ESPRIT lies in the rotational property between staggered subspaces that is invoked to produce the frequency estimates [9]. In the case of a discrete-time signal this property relies on observations of the signal over two

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identical intervals staggered in time. Consider a single complex exponential
$s_0(n)=e^{j2\Pi fn}$ with complex amplitude α frequency f .

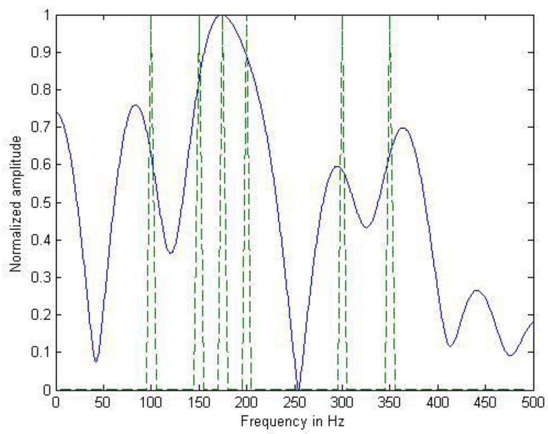

Figure 4. Test signal used for simulation

This signal has the following property

$$s_0(n+1)=\alpha e^{j2\Pi f(n+1)} = s_0(n)e^{j2\Pi f} \quad (16)$$

that is, the next sample value is a phase-shifted version of the current value. This phase shift can be represented as a rotation on the unit circle $e^{j2\Pi f}$. Consider a signal consisting of a signal made up of complex exponentials, and the noise component w(n).

$$x(n)=\sum_{p=1}^{P} \alpha_p \ v(f_p)e^{j2\pi n p} + w(n) = V\varphi^n \alpha + w(n)$$
$$= s(n) + w(n) \quad (17)$$

The matrix ∅ is the diagonal matrix of phase shifts between neighboring time samples of the individual, complex exponential components of s(n)

$$\emptyset = diag\{\emptyset_1, \emptyset_2, \dots, \emptyset_p\}$$

$$= \begin{bmatrix} e^{j2\pi f_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{j2\pi f_p} \end{bmatrix} \quad (18)$$

where $\emptyset_p=e^{j2\pi f_p}$ for p=1,2,…,P. Since the frequencies of the complex exponentials f_p completely describe this rotation matrix, frequency estimates can be obtained by finding ∅ .

III. EXPERIMENTAL RESULTS

Numerical experiments have been made with a sinusoidal test signal containing six frequencies having same amplitude and same phase. Fig 4 shows the Fourier spectrum of the test signal. Note that no windowing has been performed, to avoid a decrease in range resolution, resulting in lower side lobes. Dashed lines indicate the position of each scattering points.
Performance criteria have been selected using the following guiding rules: different methods should be compared on a common basis with respect to their capabilities to detect groups of closely spaced and isolated spectral lines; they should provide an estimation of the variability of the frequency estimate as well as some other parameters like amplitude and phase, and also a measure of the bias.
Fig 5, 6, 7, 8, shows an overlay plot of 10 different realization of spectrum estimation using four different methods of autoregressive spectral estimation. Among the four autoregressive PSD estimate, such as Yule Walker, Covariance, Modified Covariance, and Burg method, Yule-Walker method produces AR spectra for short data records with least resolution. The Burg and covariance methods produce comparable AR spectral estimates. The modified covariance method is best for sinusoidal components in the data. Some problems with the Burg method, including spectral line splitting and bias of the frequency estimate, appear to be eliminated when modified covariance method is used. The modified covariance is fairly insensitive to the initial phase and is an accurate estimate of the sinusoid frequency. The modified covariance method, unlike the Burg algorithm, does not guarantee a stable linear prediction filter. If a large model order is selected relative to the number of data samples, AR spectral estimates tend to exhibit spurious peaks. Lowering the selected order to prevent spurious peaks also reduces the resolution. The table I shows output of the frequency estimation using ESPRIT technique. Fig 9, 10 shows an overlay plot of 10 different realization of frequency estimation using MUSIC and minimum norm method. With the high resolution techniques such as MUSIC and minimum norm method, we can see that the spectrum is consistent with the Fourier transform and shows much more spectral details. Closely spaced spectral pairs can easily be seen in the MUSIC compared with Fourier spectrum. MUSIC, minimum norm, ESPRIT requires the knowledge of number of complex exponentials in the signal. Note that
MUSIC and minimum norm method are nonlinear frequency estimator and consequently, the power of each spectral line cannot be related with the signal power. These are not true PSD estimators because they do not preserve the measured process power, also the autocorrelation sequence can't be recovered by Fourier transforming the frequency estimators.

Table I
OUTPUT OF ESPRIT TECHNIQUE

Input Frequency	100	150	175	200	300	350
Estimated frequency	99.9	150.20	175.06	200.1	300.00	349.99

We perform a statistical study on the estimation of one spectral component with the frequency 200 Hz by means of the four methods of parametric estimation. Consider that the SNR sweeps the range from 0 dB to 20 dB and figure 11 shows a plot of the variance variation of each obtained estimate over 1,000 outcomes. The variance of modified covariance method is slightly lower than other parametric methods.

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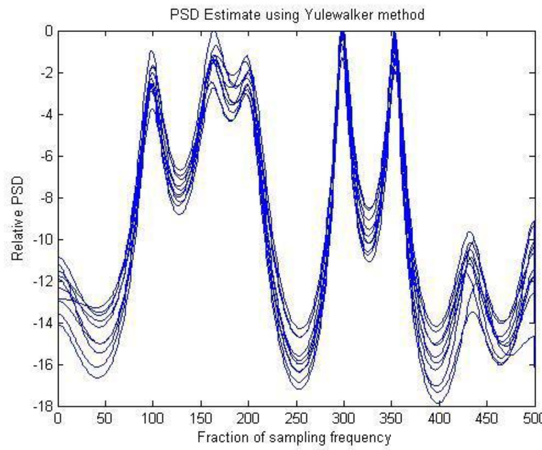

Figure 5. Spectrum estimation using Yule-Walker method

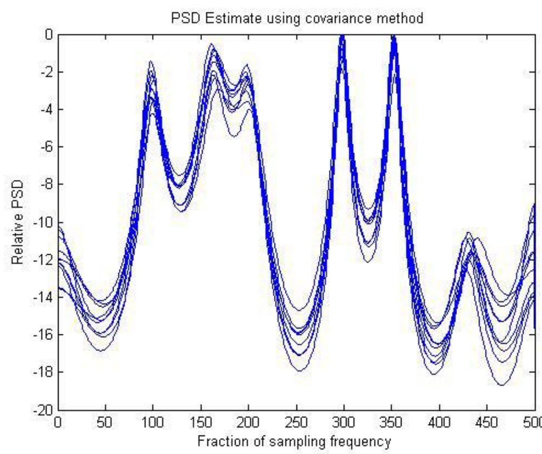

Figure 6. Spectrum estimation using Covariance method

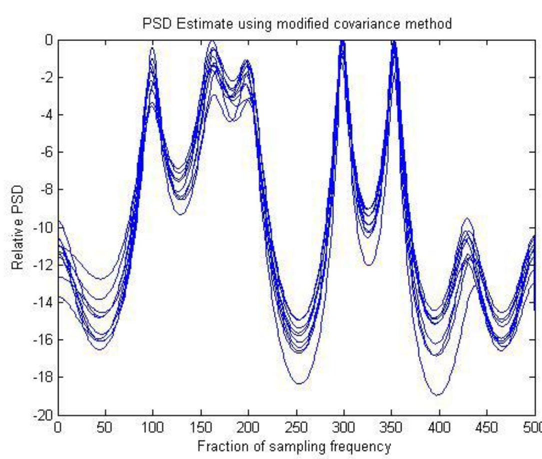

Figure 7. Spectrum estimation using Modified Covariance method

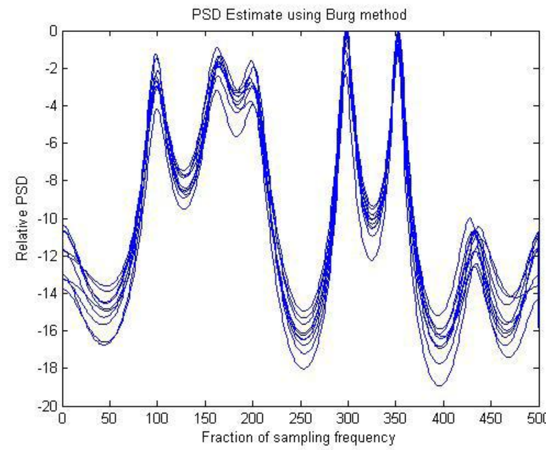

Figure 8. Spectrum estimation using Burg method

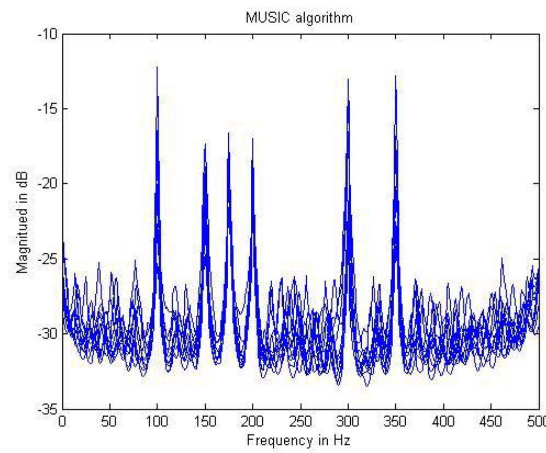

Figure 9. Frequency estimation using MUSIC algorithm

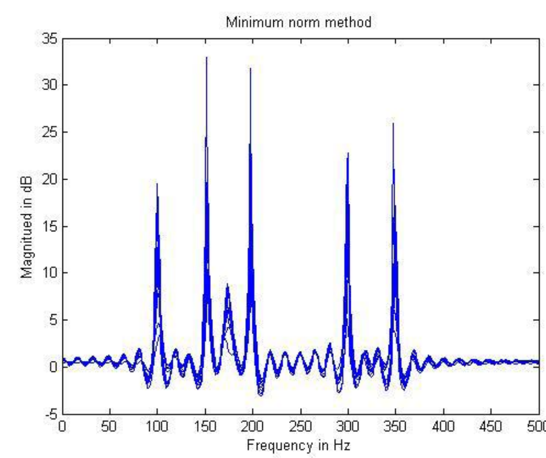

Figure 10. Frequency estimation using Minimum norm algorithm

Figure 12 shows a plot of variance variation using MUSIC, ESPRIT, and Fourier Transform method. The variances of ESPRIT estimate is slightly lower than two other estimates due to the exact calculation of the signal frequency, while

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the accuracy of the Fourier and MUSIC methods is limited by the number of considered calculation points.

IV. CONCLUSION

High resolution frequency estimation methods such as MUltiple SIgnal Classification (MUSIC), minimum norm method, ESPRIT have been promoted in the research literature as having better resolution and better frequency estimation characteristics than the autoregressive spectral estimators. This super resolution technique can be used in Radar Altimeter for estimating the range. Improved performance is due to division of information in the autocorrelation matrix into two vector subspaces i.e., signal subspace and noise subspace.

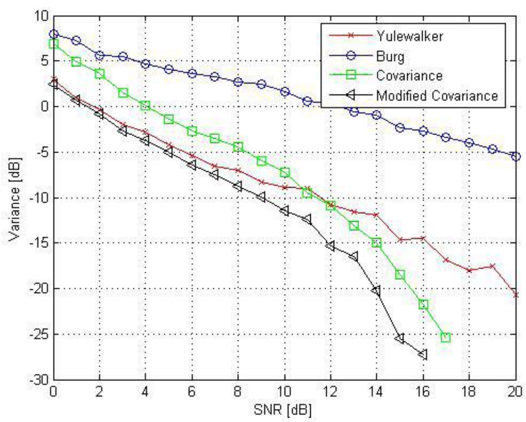

Figure 11. Statistical behavior of parametric methods.

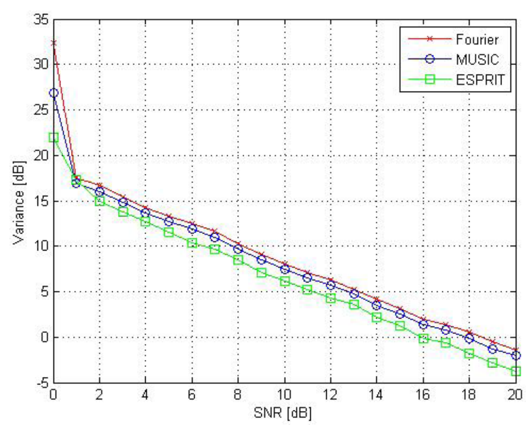

Figure 12. Statistical behavior using MUSIC, ESPRIT, Fourier Transform.

Functions of the vectors in either the signal or noise subspace can be used to create frequency estimators that, when plotted, show sharp peaks at the frequency locations of sinusoids or other narrow band spectral components.

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