IJIREEICE
ISSN (Online) 2321 – 2004
ISSN (Print) 2321 – 5526
INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 4, Issue 6, June 2016

Design of MRAC for Multi Input Multi Output Coupled Tank System

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Abstract: This paper presents a controller design methodology using Model Reference Adaptive System (MRAS) technique for a multi input multi output (MIMO) coupled tank system using Lyapunov design approach. Adaptation law designed using Lyapunov approach, ensures the stability of the system and the system has better transient response. Mathematical modeling of a coupled tank system has been presented in this paper. The change in the response of the system with the change in the value of adaptation gain parameters has also been studied. Simulation using MATLAB has been carried out to show the behavior of the system after applying the control algorithm.

Keywords: Mathematical Modeling, MRAC, Lyapunov Theory, adaptation gains, and MIMO.

I. INTRODUCTION

Among various approaches used in adaptive control, Model Reference Adaptive Control (MRAC) system is one that has been used widely [1]. Direct and Indirect approaches can be used to design model reference adaptive system [2]. In direct approach, controller parameters are directly estimated [2]. In the present paper, direct approach has been used to design the adaption laws.

For the estimation of controller parameters in adaptive control system adaptation laws can be designed using various methods [3]. In this paper, adaptation laws are designed based on Lyapunov method. Lyapunov approach guarantees the stability of the system and gives better transient response characteristic compared to MIT rule [3].

Regulating the level of liquid in tanks and rate of flow of liquid from one tank to another tank in coupled tank system is one of the basic control problems in process industries [4]. Under fixed operating conditions, conventional PID controllers can be used optimally where PID parameters can be tuned using various algorithms such as Ziegler-Nichols method. But under dynamic conditions where operating condition changes PID controller operates under sub-optimal condition if PID parameters remains unchanged [4]. This requires the use of adaptive system such as model reference adaptive system [5]. In [4] MRAC and Modified MRAC system using MIT rule has been presented for SISO (Single Input Single Output) coupled tank system. In [5] direct model reference adaptive control based on Command Generator Tracker (CGT) approach on a nonlinear model of SISO coupled tank system has been presented. In [6] control algorithm based on Hankel norm for MIMO coupled tank system has been given.

This paper is organized as follows: In section II, an overview on MRAC and Lyapunov theory for controller parameter adaptation purpose is given. In section III, a MIMO coupled tank system and its mathematical modeling is given. In section IV, control algorithm has been designed using Lyapunov theory. In section V, simulation results are given. And Section VI concludes the effect of adaptation gains based on the simulation results obtained.

II. MODEL REFERENCE ADAPTIVE CONTROL

A reference model has been chosen based on the required specifications in model reference adaptive system [2]. The output of both plants, the reference model and the actual plant, has been compared and an error called as tracking error can be obtained. Fig.1 shows the general schematic structure of MRAC system.

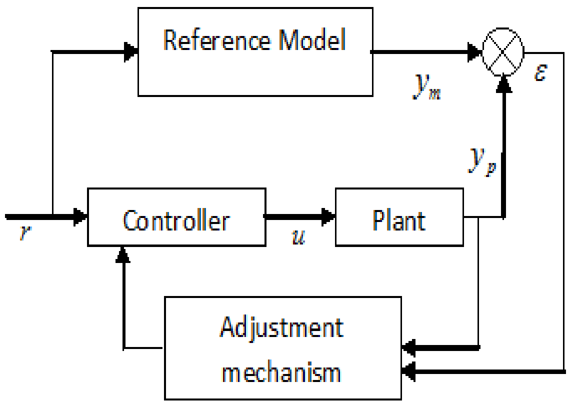

Fig. 1.Block diagram of MRAC

This error signal obtained is applied at the input of the plant as feedback, after passing it through the block that changes the parameter of the controller. The parameters of the controller are then adjusted itself until error becomes zero [7].

A. Lyapunov Theory

The nonlinear differential equation as given below is investigated by Lyapunov [7].

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Vol. 4, Issue 6, June 2016

$$\frac{dx}{dt} = f(x) \qquad \text{where, } f(0) = 0 \tag{1}$$

As $f(0) = 0$, the equation: $x(t) = 0$ has the solution.

If any positive definite (pd) Lyapunov function $V : R^n \to R$ exists such that for the solution of (1) the derivative of Lyapunov function $'V'$ is negative semi definite (nsd), then the solution is stable. If derivative of Lyapunov function $'V'$ is negative definite (nd), then the solution is asymptotically stable [6].

Assume the linear system as given below

$$\frac{dx}{dt} = Ax \tag{2}$$

If this system is asymptotically stable, then according to the Lyapunov equation given as

$$A^T P + PA = -Q \tag{3}$$

A symmetric positive definite matrix $'P'$ is obtained for each symmetric positive definite matrix $'Q'$ [7].

III. MATHEMATICAL MODELING OF A MIMO COUPLED TANK SYSTEM

Fig.2 shows the structure of the two coupled tanks [6].

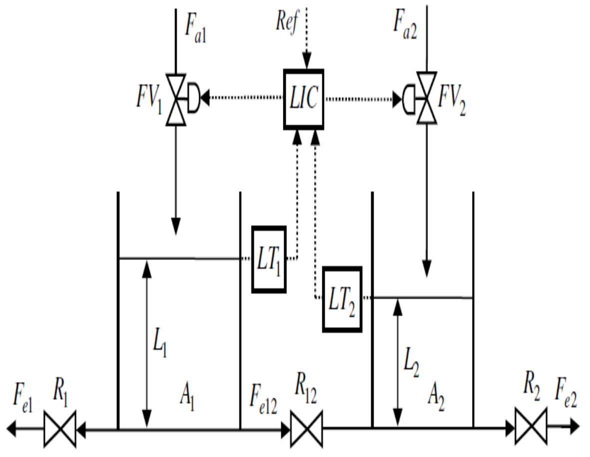

Fig. 2.Structure of MIMO coupled tanks

where,

F_{a1} & F_{a2} = Fluid input flows,

F_{e1}, F_{e12} & F_{e2} = Fluid output flows,

A_1 & A_2 = Area of tank's base,

L_1 & L_2 = Level of fluid in tank,

LT_1 & LT_2 = Level transducer,

LIC = Controller and level indicator,

FV_1 & FV_2 = Input flow control valves,

R_1, R_{12} & R_2 = Hydraulic resistances

The variation of liquid level in tanks can be given as [6]:

$$\frac{dL_1}{dt} = \frac{1}{A_1}\left(F_{a1} - F_{e1} - F_{e12}\right) \tag{4}$$

$$\frac{dL_2}{dt} = \frac{1}{A_2}\left(F_{a2} - F_{e2} + F_{e12}\right) \tag{5}$$

Considering:

$$F_{e1} = \frac{L_1}{R_1}; F_{e2} = \frac{L_2}{R_2}; F_{e12} = \frac{L_1 - L_2}{R_{12}} \tag{6}$$

Equation (4) and (5) can be given as:

$$\frac{dL_1}{dt} = -\frac{R_1 + R_{12}}{A_1 R_1 R_{12}} L_1 + \frac{1}{A_1 R_{12}} L_2 + \frac{1}{A_1} F_{a1} \tag{7}$$

$$\frac{dL_2}{dt} = -\frac{1}{A_2 R_{12}} L_1 - \frac{R_2 + R_{12}}{A_2 R_2 R_{12}} L_2 + \frac{1}{A_2} F_{a2} \tag{8}$$

Then state space equation can be given as [6]:

$$\begin{bmatrix} \frac{d}{dt} L_1 \\ \frac{d}{dt} L_2 \end{bmatrix} = \begin{bmatrix} -\frac{R_1 + R_{12}}{A_1 R_1 R_{12}} & \frac{1}{A_1 R_{12}} \\ \frac{1}{A_2 R_{12}} & -\frac{R_2 + R_{12}}{A_2 R_2 R_{12}} \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} & 0 \\ 0 & \frac{1}{A_2} \end{bmatrix} \begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} \tag{10}$$

The values of the plant parameters are given below in Table. I [6]

TABLE I. Values of Plant Parameters

Plant Parameter	Values
A_1	0.8
A_2	0.4
R_1	0.5
R_{12}	0.3
R_2	1

Putting the above plant parameters in (9) the state space equation can be obtained as [6]:

$$\begin{bmatrix} \dot{L}_1 \\ \dot{L}_2 \end{bmatrix} = \begin{bmatrix} -6.667 & 4.1667 \\ 8.3333 & -10.8333 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} 1.25 & 0 \\ 0 & 2.5 \end{bmatrix} \begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} \tag{11}$$

IV. DESIGNING OF CONTROL ALGORITHM

A. Choosing Reference Model

In model reference adaptive system, before designing the control algorithm for updating controller parameters, select the reference model depending on our desired requirements.

Suppose our requirement is to make coupled tanks system to operate as a decoupled tanks system having rise time less than 2 seconds and overshoot less than .01%. To achieve this reference model can be given as:

$$\begin{bmatrix} \dot{L}_1 \\ \dot{L}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} \quad (13)$$

B. MRAC using Lyapunov Theory

The state space equation of the reference model given in (12) can be given by [2]:

$$\dot{x}_m = A_m x_m + B_m r \quad (14)$$

where $A_m \in R^{n\times n}$ is a stable matrix, $B_m \in R^{n\times q}$, $r \in R^q$ is a bounded reference input vector, and 'n' is the order of the plant.

Let us consider that the state space equation of the plant model is given as:

$$\dot{x}_p = A_p x_p + B_p u \quad (15)$$

where $A_p \in R^{n\times n}, B_p \in R^{n\times q}$ are unknown constant matrices and (A, B) is controllable.

Suppose plant parameters are known. Then control law can be given as [2]:

$$u = -M^* x_p + N^* r \quad (16)$$

Putting control law from (16) in (15), close loop plant obtained can be given as [2]:

$$\dot{x}_p = \left(A_p - B_p M^*\right)x_p + B_p N^* r \quad (17)$$

where $M^* \in R^{q\times n}$ & $N^* \in R^{q\times q}$ are controller parameters

Comparing (14) and (17)

$$\left(A_p - B_p M^*\right) = A_m \quad (18)$$

$$B_p N^* = B_m \quad (19)$$

But (18) and (19) cannot be used directly for finding out controller parameters if plant parameters are unknown. For unknown plant parameters, adaptation laws have to be designed which is as follows [2]:

Let, $u = -Mx_p + Nr$ (20)

By adding and subtracting the desired input term $-B_p\left(M^* x_p - N^* r\right)$ in (15) and utilizing (18) and (19), the state space equation of the plant can be obtained as given below:

$$\dot{x}_p = A_m x_p + B_m r + B_p\left(M^* x_p - N^* r + u\right) \quad (21)$$

Substitute (20) in (21)

$$\dot{x}_p = A_m x_p + B_m r + B_p\left(-\tilde{M} x_p + \tilde{N} r\right) \quad (22)$$

where $\tilde{M} = M - M^*$ & $\tilde{N} = N - N^*$

Subtracting (14) from (22)

$$\dot{\varepsilon} = A_m \varepsilon + B_p\left(-\tilde{M} x_p + \tilde{N} r\right) \quad (23)$$

Substituting (19) in (23)

$$\dot{\varepsilon} = A_m \varepsilon + B_m N^{*-1}\left(-\tilde{M} x_p + \tilde{N} r\right) \quad (24)$$

Suppose the Lyapunov function given below [2]:

$$V\left(\varepsilon, \tilde{M}, \tilde{N}\right) = \varepsilon^T P \varepsilon + tr\left[\tilde{M}^T \Gamma \tilde{M} + \tilde{N}^T \Gamma \tilde{N}\right] \quad (25)$$

where $P = P^T > 0$ satisfies the lyapunov equation

$$A_m^T P + P A_m = -Q \quad (26)$$

for some $Q = Q^T > 0$. Then,

$$\dot{V} = -\varepsilon^T Q \varepsilon + 2\varepsilon^T P B_m N^{*-1}\left(-\tilde{M} x_p + \tilde{N} r\right) + 2tr\left[\tilde{M}^T \Gamma \dot{\tilde{M}} + \tilde{N}^T \Gamma \dot{\tilde{N}}\right] \quad (27)$$

Now,

$$\varepsilon^T P B_m N^{*-1} \tilde{M} x_p = tr[\tilde{M}^T \Gamma B_m^T P \varepsilon x_p^T]\,\mathrm{sgn}(l)$$

And

$$\varepsilon^T P B_m N^{*-1} \tilde{N} r = tr[\tilde{N}^T \Gamma B_m^T P \varepsilon r^T]\,\mathrm{sgn}(l)$$

Therefore, for adaptation laws given below [2]:

$$\dot{\tilde{M}} = \dot{M} = B_m^T P \varepsilon x_p^T\,\mathrm{sgn}(l) \quad (28)$$

$$\dot{\tilde{N}} = \dot{N} = -B_m^T P \varepsilon r^T\,\mathrm{sgn}(l) \quad (29)$$

Equation (27) can be simplified as:

$$\dot{V} = -\varepsilon^T Q \varepsilon$$

Hence (28) and (29) can be successfully used to update controller parameters.

Equation (28) and (29) is slightly modified by multiplying gain constant in order to improve the response of the system [3] and can be given as:

$$\dot{\tilde{M}} = \dot{M} = \gamma_1 B_m^T P \varepsilon x_p^T\,\mathrm{sgn}(l) \quad (30)$$

$$\dot{\tilde{N}} = \dot{N} = -\gamma_2 B_m^T P \varepsilon r^T\,\mathrm{sgn}(l) \quad (31)$$

where γ_1 & γ_2 are called as adaptation gains.

If N^* is 'pd' then $l = 1$, and if N^* is 'nd' then $l = -1$ [2]

Solving (26) for $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ we get symmetric matrix.

$$P = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

V. SIMULATION RESULTS

This section comprises of the simulation results obtained through MRAC system. Fig.3 and Fig.6 shows the liquid level in coupled system when only input F_{a1} is applied for two different sets of adaptation gains. Fig.4 and Fig.7 shows the liquid level in the coupled tank system when only input F_{a2} is applied. Fig.5 and Fig.8 shows the liquid

 DOI 10.17148/IJIREEICE.2016.4631

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INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 4, Issue 6, June 2016

level in coupled tank system when both F_{a1} and F_{a2} are applied for different adaptation gains.

A. When $\gamma_1 = 10, \gamma_2 = 10$

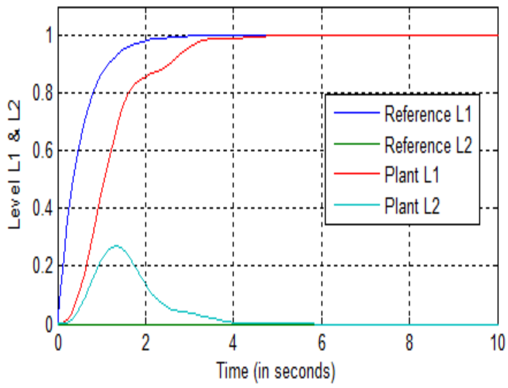

Fig. 3.Plant output for Fa1=1, and Fa2=0

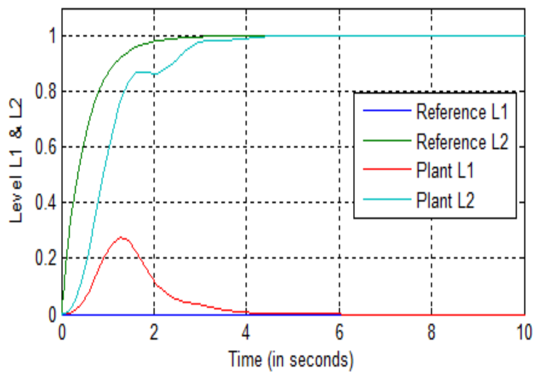

Fig. 4.Plant output for Fa1=0, and Fa2=1

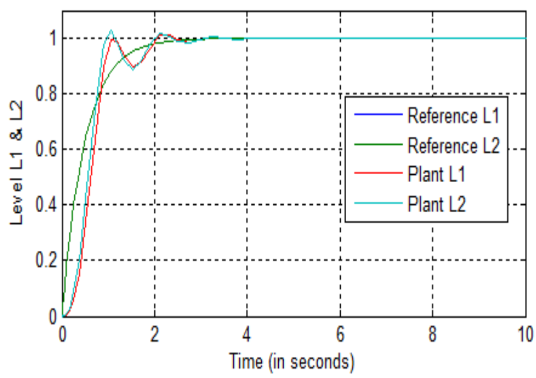

Fig. 5.Plant output for Fa1=1, and Fa2=1

B. When $\gamma_1 = 100, \gamma_2 = 1000$

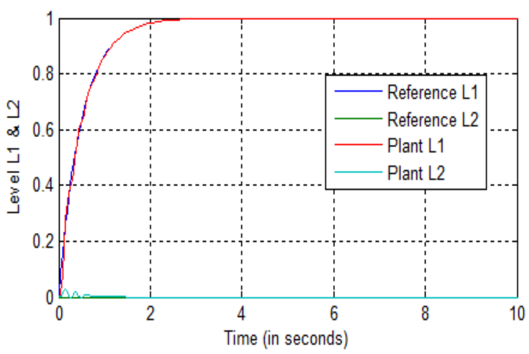

Fig. 6.Plant output for Fa1=1, and Fa2=0

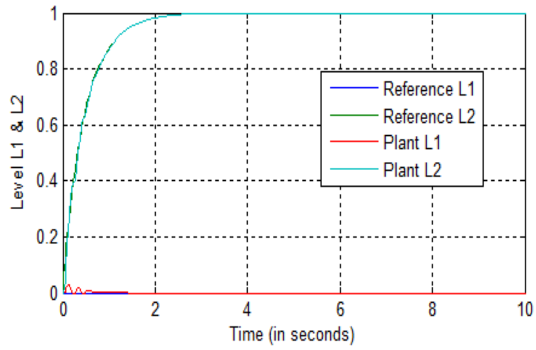

Fig. 7.Plant output for Fa1=0, and Fa2=1

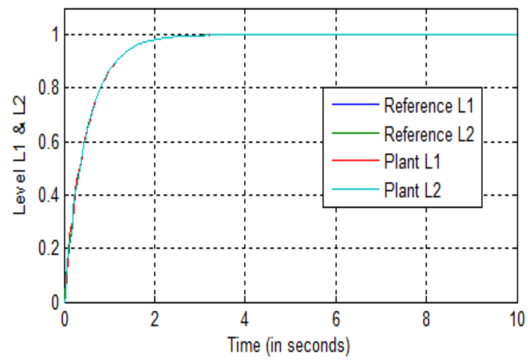

Fig. 8.Plant output for Fa1=1, and Fa2=1

VI. CONCLUSION

From Fig.3 and Fig.4 it can be seen that when only one input is applied then the time domain specifications such as rise time, settling time are higher for plant model compared to reference model for low adaptation gains. If we increase the value of adaptation gains the time domain specifications of the plant reduces and the plant follows reference model more appropriately as can be seen in Fig.6 and Fig.7. Thus it can be concluded that the response of the system improves by increasing the value of adaptation gains.

REFERENCES

[1] Jing Sun, "A Modified Model Reference Adaptive Control Scheme for Improved Transient Performance," IEEE Transactions on Automatic Control, Vol. 38, No. 8, pp. 1255-1259, August 1993.
[2] Petros A. Ioannou, and Jing Sun, Robust adaptive Control. New York: Dover, 1996.
[3] Swarnkar Pankaj, and Jain Shailendra Kumar, "Comparative Analysis of MIT rule and Lyapunov Rule in Model Reference Adaptive Control Scheme," Innovative System Design and Engineering, Vol. 2, No. 4, ISSN 2222-1727, 2011.
[4] K. Asan Mohideen, G. Saravana Kumar, K. Valarmathi, D. Devaraj, and T. K. Radhakrishnan, "Real Coded Genetic Algorithm for system Identification and Tuning of a Modified Model Reference Adaptive Controller for a hybrid tank System," Applied Mathematical Modelling, Vol. 37, pp. 3829-3847, 2013.
[5] Muhammad Nasiruddin Mahyuddin, Mohd. Rizal Arshad, and Zaharuddin Mohamed, "Simulation of Direct Model Reference Adaptive Control on a Coupled-Tank System using Nonlinear Plant Model," International Conference on Control, Instrumentation and Mechatronics Engineering, May 28-29, 2007, Johor, Malaysia.
[6] Mircea Dulau, and Tudor-Mircea Dulau, "Multivariable System with Level Control," 9th International Conference Interdisciplinary in Engineering,, October 8-9, 2015, Tirgu-Mures, Romania.
[7] R. J. Pawar, and B. J. Parvat, "Design and Implementation of MRAC and Modified MRAC technique for Inverted Pendulum," International Conference on Pervasive Computing, 2015.

DOI 10.17148/IJIREEICE.2016.4631