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Fractional Calculus based Dynamic Analysis of Stepper Motor with Leadscrew

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Abstract: Mechatronics system design involves the modelling of physical system before the prototype developments which has been beneficial in any system design in recent years. In an application such as controlling of robot manipulator through the actuator like stepper motor, servo motor has always been challenging with respect to the performance of controller. In this paper, the transfer function of stepper motor as well as Leadscrew is derived. The Leadscrew converts the rotational motion to linear motion. The overall system for actuation is designed with both stepper motor & Leadscrew with taking account of fractional order controller (FOPID) along with performance metrics such as ITSE, ITAE & ISE. The result distinguishes the dynamic performance of the system with the designed fractional order controller & conventional fractional PID controller in terms of transient response characteristics such as peak time, rise time, settling time & in further shows that not only fractional controller (FOPID) is better than conventional PID controller but also the result characterized with performance indexes like ITSE, ITAE & ISE holds good.

Keywords: Mechatronics, Fractional order controller, Stepper Motor, Leadscrew transfer function.

I. INTRODUCTION

Integrated design of physical system has given the importance of mechatronics either in the field of robotics or industrial automation gives rise to modular mathematical modelling, control system design & design optimization. Notwithstanding Stepper motor are different from others in such a way that they perform on discrete control pulses & rotate in discrete steps for which they are employed in industrial applications. Leadscrew are used either for transmission of power or transmission of force [1][6]. This paper reports the mathematical modelling of two phase hybrid stepper motor as well as Leadscrew. The overall system taking both stepper motor & Leadscrew is designed which acts as a robot manipulator that converts rotational motion to linear motion. The main objective is to design the controllerwith different performance indexes having objective functions and also the transient behaviour of the system is analysed. Firstly we have designed the stepper motor through its different aspects. In this design taking parameters of stepper such as inertia, torque, self-inductance, mutual inductance, the transfer function derived & its time domain response analysed. Then we have modelled Leadscrew with taking parameters such as pitch, moment of inertia, gear ratio, the transfer function derived. The controller performance analysed without PID, with PID & fractional order controller (FOPID). It is found that by the application of fractional order controller to the system, the transient characteristics showed better response than conventional PID controller & the observations with ITAE, ITSE & IAE are also discussed.

II. FRACTIONAL CALCULUS & CONTROLLER

Fractional calculus gained more importance due to non-integer calculus over classical integer ones, robust control performance. The fractional calculus is the principle of diffentiation & integration to non-integer order operator ${}_aD_t^{\alpha}$,where a & t denotes the limits of operation & α denotes the fractional order, then the fractional order continuous time dynamic system can be generalized with the differential equations as follows[7]:

$$a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \ldots + a_0 D^{\alpha_0} y(t) = b_m D^{\beta_n} u(t) + b_{m-1} D^{\beta_{n-1}} u(t) + \ldots + b_0 D^{\beta_0} u(t). \qquad (1)$$

Applying Laplace transform to equation-1 with zero initial conditions the input-output representation of the fractional order system can be in the representation of transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots + a_0 s^{\alpha_0}} \qquad (2)$$

The fractional controller ($PI^{\lambda}D^{\mu}$) contains five parameters to be varied: proportional constant K_P, derivative constant K_D, integral constant K_I, fractional orders integration λ and fractional order derivative μ whereas the conventional PID or three term controller contains only three parameters. The equation contains error signal that shows working of controller can be as follows [7]:

$$U(t) = K_P e(t) + K_i D^{-\lambda} e(t) + K_D D^{\mu} e(t) \qquad (3)$$

Where U (t) =Step Stimulus, e (t) = error signal

By applying Laplace Transform to equation-3, with assuming zero initial conditions, the transfer function for the controller will be as follows:

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$$G_C(s) = K_P + \frac{K_i}{S^{\lambda}} + K_d S^{\mu} \qquad (4)$$

Due to more variance parameters we can have large no of controller for tuning. When both fractional order integration & fractional order derivative are equals to '1', the fractional controller can acts as a conventional PID controller [8]. The fig.1 shows the block diagram of the system where u(t) & e(t) are the input step signal & error signal respectively.

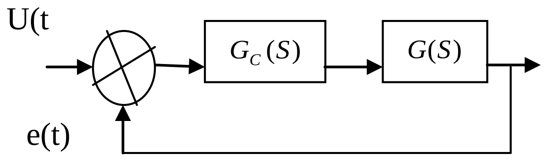

Fig. no.1 Block diagram for overall system

The optimization for controller tuning contains five fractional parameters K_P, K_I, K_D, λ, μ uses the performance criterion that contains following objective functions:

$$Integral\ square\,error(ISE) = \int_0^t e^2(t)\,dt$$

$$Integral\ time - square\,error\ (ISTE) = \int_0^t te^2(t)\,dt$$

$$Integral\ time - absolute\,error\ \ (ITAE) = \int_0^t t|e(t)|\,dt$$

Where e (t) = 1- y (t); y (t) is the closed loop step response.

III. TRANSFER FUNCTION OF OVERALL SYSTEM

The transfer function defines the relationship between the excitation to the system & response from the system. Initially, the transfer function of stepper motor is calculated, then the same is done for Leadscrew. Finally, both the system are cascaded together to form plant model G(S) as shown in fig.1. There is an enormous diversity in the description of stepper motor & its transfer function. The open loop transfer function of stepper by taking parameters such self-inductance, mutual inductance, inertia, step anglefor simulation can be as follows[5]:

$$G_1(S) = \frac{2700000S^2 + 283500000S + 1350000000}{S^4 + 19799S^3 + 650000S + 7500S} \qquad (5)$$

The leadscrew here taken can acts as a robot manipulator that converts rotational motion to linear motion. The rotational motion to the Leadscrew is given through the stepper actuator. The Leadscrew is connected to a mass. It is assumed that there is zero viscous frictional losses & it can be used as positioner mechanism to movement a work part in CNC machine or any other such applications. A gearbox is used in between stepper motor & leadscrew to provide sufficient torque to the system. The various parameters of the leadscrew such as pitch, gear ratio, inertia, and load are shown in table.1 [2].

Table-1: Parameters for Leadscrew arrangement

Sl.No	Parameter	Symbol	Value
1.	Pitch	P	0.005m
2.	Gear ratio	n	10:1
3.	Inertia	I_m	10^{-5} kg.m^2
4.	Load	L	50Kg

The transfer function for leadscrew that is connected to stepper motor can be derived with taking parameters form table.1 is as follows [2]:

$$G_2(S) = \frac{n}{(PL + \frac{n^2 I_m}{P})S^2} \qquad (6)$$

$$G_2(S) = \frac{10}{0.45S^2} \qquad (7)$$

The closed loop block diagram for the stepper motor & stepper motor with leadscrew is shown in fig.2 & fig.3 respectively.

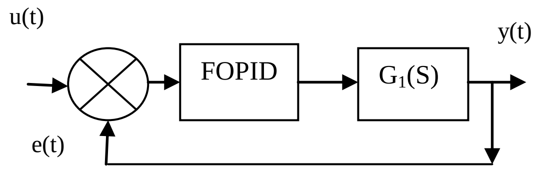

Fig.no.2. Closed loop block diagram of stepper actuator

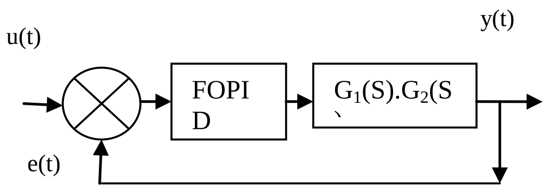

Fig.no.3.Closed loop block diagram of stepper with Leadscrew

IV. SIMULATION & RESULT

For performing on fractional order control, the FOMCON tool box is very useful working under MATLAB environment [7]. The transfer function for stepper & overall system are created in FOTF_GUI & then using optimisation techniques, the performance of the individual systems analysed. The closed loop model of stepper motor shown in fig.2 is simulated. The closed loop performance of the plant (stepper actuator) with fractional PID & integer order PID is given in table.2.

From Table.2, it is found that the closed loop response of stepper motor with FOPID controller having objective function ITAE in comparison with others have better performance. The design parameters of controllers for stepper motor are shown in table.3. The step response according to the controller with respect to different objective functions are shown in fig.4, fig.5, fig.6, fig.7, fig.8, fig.9.

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Table-2: Closed loop performance of stepper motor with fractional controller for distinct indices

Type of Controller	Performance Index	Rise Time (Sec)	Peak time (Sec)	Settling Time (Sec)
FOPID	ITSE	0.055	0.2	0.475
PID	ITSE	0.8575	0.2	0.55
Without PID	ITSE	0.248	0.58	2.187
FOPID	ISE	0.0735	0.18	0.28
PID	ISE	0.775	0.25	0.55
Without PID	ISE	0.2	0.5	1.75
FOPID	ITAE	0.0675	0.19	0.27
PID	ITAE	0.068	0.25	0.55
Without PID	ITAE	0.125	0.311	0.94

Further, the closed loop performance and the design parameters of the plant (stepper with leadscrew) with fractional PID & integer order PID for the system shown in fig.3 is given in table.4 & table.5 respectively.The step response of stepper with leadscrew is shown in fig.10-14.

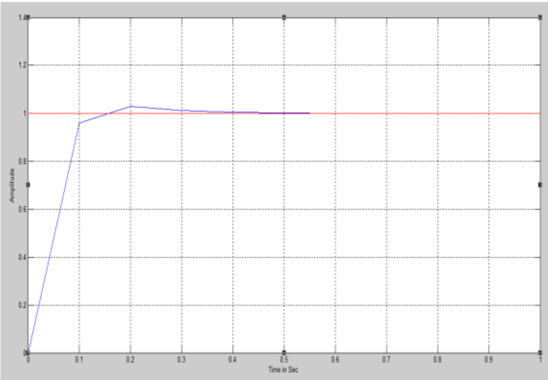

Fig. no.4.Step response of stepper with FOPID for ITSE

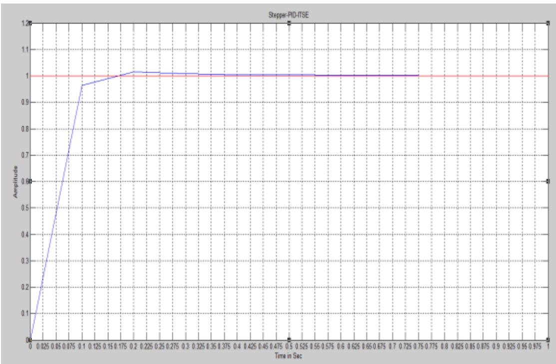

Fig. no. 5 Step response of stepper with PID for ITSE

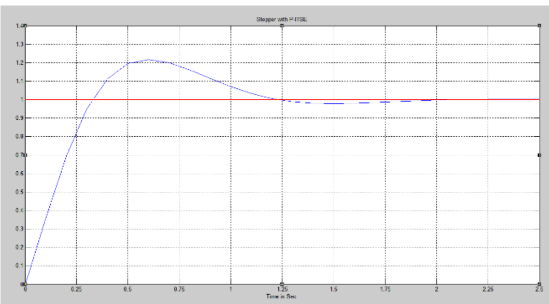

Fig.no.6. Step response of stepper without PID for ITSE

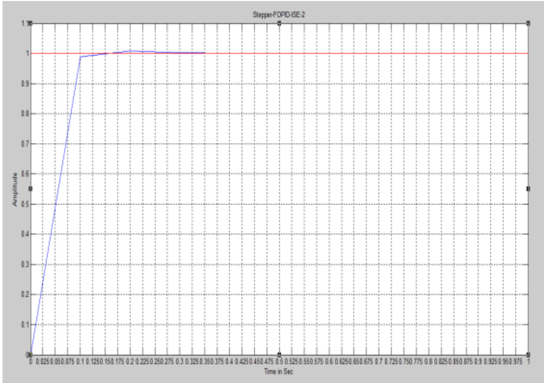

Fig. no.7.Step response of stepper with FOPID for ISE

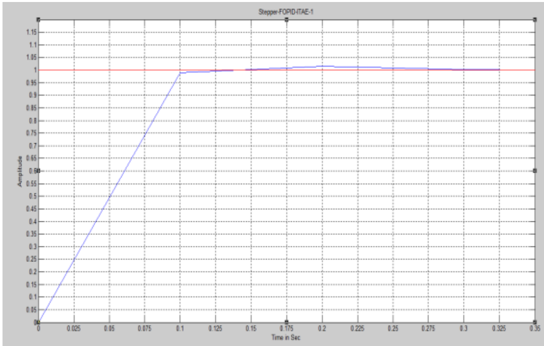

Fig. no.8.Step response of stepper with FOPID for ITAE

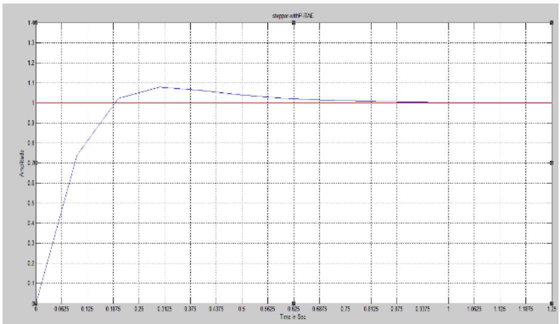

Fig.no.9. Step response of stepper without PID for ITAE

Table-3: Design of fractional PID for stepper motor with distinct performance indices.

Type of Controller	Performance Index	Controller parameters
FOPID	ITSE	K_P =0.0263; K_I =1.0845 K_D =0.0266; λ =0.5005 μ =0.49611
PID	ITSE	K_P =0.46589; K_I =0.12010 K_D =0.00217; λ =1; μ =1
FOPID	ISE	K_P =0.029395;K_I =3.583 K_D =0.0449; λ =0.50021 μ =0.50049
PID	ISE	K_P =0.4589;K_I =0.98545 K_D =0.023; λ =1;μ =1
FOPID	ITAE	K_P =0.025; K_I 15.09 K_D =0.021; λ =0.9999 μ =0.89934
PID	ITAE	K_P =0.569; K_I =0.584 K_D =0.02546; λ =1μ =1

Table-4: Closed loop performance of stepper with Leadscrew by fractional controller for distinct indices

Type of Controller	Performance Index	Rise Time (Sec)	Peak time (Sec)	Settling Time (Sec)
FOPID	ITSE	0.10	0.2	5.8
PID	ITSE	0.18	0.2	19
FOPID	ITAE	0.0813	0.19	2.3
PID	ITAE	0.1085	0.195	4.7
FOPID	ISE	0.03875	0.185	1.5
PID	ISE	0.63	0.195	2.875

Table-5: Design of fractional PID for leadscrew driven by stepper with distinct performance indices.

Type of Controller	Perform. Index	Controller parameters
FOPID	ITSE	K_P =0.059349; K_I =0.0622 K_I =0.4196; λ =0.50082 μ =0.50393
PID	ITSE	K_P =0.069844; K_I =0.0546 K_I =0.04556; λ =1; μ =1
FOPID	ISE	K_P =1; K_I =1; K_I =1; λ =0.5 μ =0.5
PID	ISE	K_P =0.987; K_I =0.2589 K_I =0.0589; λ =1; μ =1
FOPID	ITAE	K_P =1; K_I =1; K_I =1; λ =0.5 μ =0.5
PID	ITAE	K_P =0.9845; K_I =0.345 K_I =0.0674; λ =1; μ =1

From table.4, it is found that the closed loop step for stepper with leadscrew designed with FOPID having objective function ISE has good performance in comparison to others.

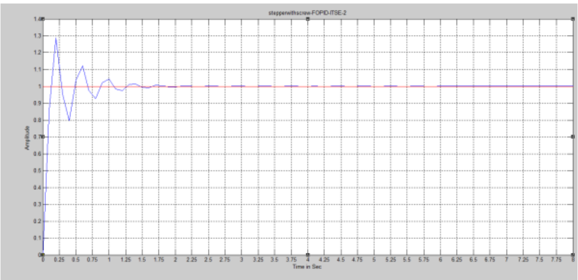

Fig.no.10.Response of stepper with screw using FOPID - ITSE

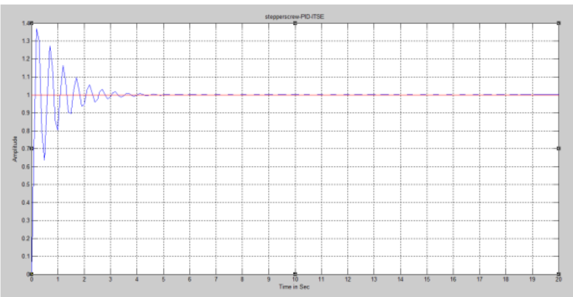

Fig.no.11. Response of stepper with screw using PID - ITSE

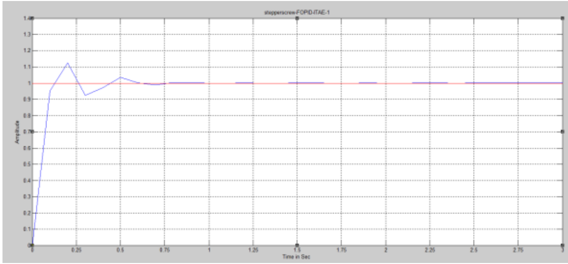

Fig.no.12. Response of stepper with screw using PID - ITAE

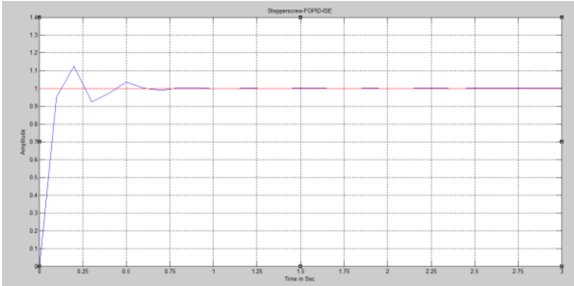

Fig.no.13. Response of stepper with screw using FOPID - ISE

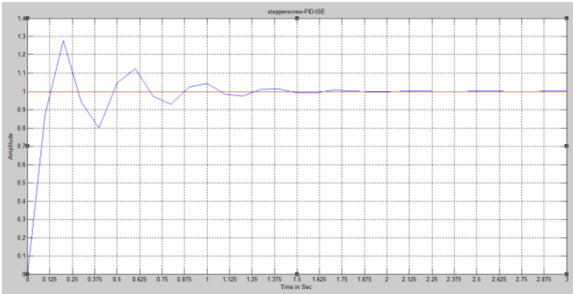

Fig.no.14. Response of stepper with screw using PID - ISE

V. CONCLUSION & FUTURE WORK

This paper takes a case study of stepper motor and defined the fractional order controller performance with different objective functions like ITSE, ITAE etc. Also, it analyses the system that contains leadscrew driven by stepper motor with fractional PID controller. It is found that with fractional PID controller the performance is better than conventional PID & further among different fractional PID with objective functions ITAE method works better for the case of stepper actuator & ISE method for leadscrew driven by stepper actuator. The transient characteristics such as rise time, peak time, settling time also discussed.

In future we can improve the performance of the stepper motor by varying the parameters as well as introducing intelligent control system based on fuzzy or neural network to present system.

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BIOGRAPHIES

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