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H_∞ Based Robust Fixed Structure Controller Design for DC Motor Speed Control using GA

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Abstract: A new technique of designing a robust PID and Polynomial controller for DC motor speed control is proposed. The proposed approach poses the design problem as fixed structure robust controller and mixed sensitivity H_∞ method. Performance weights are designed based on the closed-loop objective and performance requirements which are normally applied in H_∞ optimal control. Further Genetic Algorithm (GA) is adopted to solve the optimization problem for finding the optimal controller by selection of PID and polynomial parameters. Usually in literature, comparison has been carried out based on time domain performance indices; whereas, in this work, mixed sensitivity H_∞ method is considered as the fitness function for the GA techniques in order to assess the robustness of the designed system. Also, comparison of fixed structure PID and Polynomial controller is done for DC motor speed control. The proposed technique can solve the problem of complicated and high order controller of conventional H_∞ optimal control for practical use.

Keyword: DC Motor, genetic algorithm, H-infinity synthesis, robustness, MATLAB

I. INTRODUCTION

These days, a continuous increase in performance of technological processes relaying on improvements in the design of the electrical machines, power electronics, system theory and automatic control, is witnessed. In spite of the development of power electronics resources, the direct current machine is becoming more and more useful. Their uses are not limited in the car applications (electric vehicles), rolling mills, in applications of weak power using battery system or for the electric traction in the multi-machine systems too.

The speed of DC motor can be adjusted to a great extent as to provide controllability and high performance [1,2]. Recent control technique uses conventional intelligence such as genetic algorithms (GA) or Particle Swarm Optimization (PSO) in adaptive or learning control. Duncan McFarlane [3] in 1992 introduced a design procedure which incorporate loop shaping methods to obtain performance and robust stability trade off and a particular H_∞ optimization problem to guarantee closed loop stability. M. D. Minkova [4] in 1998 applied adaptive neural method for speed control and A. A. El- Samahy [5] in 2000 described robust adaptive discrete variable structure control scheme for speed control of DC motor.

In DC motor speed control, many engineers attempt to design a robust controller to ensure both the stability and the performance characteristics of the system under the perturbed conditions. A multi objective formulation for control [6] is introduced by Tapabrata Ray in 2002.

The controllers of the speed that are designed for goal to control the speed of DC motor are numerous: PID Controller, Fuzzy Logic Controller; or the combination between them [7], Particle Swarm Optimization [8, 9], the Augmented Lagrangian Particle Swarm Optimization [11], Fuzzy-Swarm [13], Fuzzy-Neural Networks, Fuzzy-Genetic Algorithm [14], Fuzzy- Ants Colony, Fuzzy-Sliding mode control [15], Neural Network [16].

Unfortunately, the order of the resulting controller from the conventional technique of controller design is usually higher than that of the plant, making it difficult to implement the controller in actual practice. In this paper, the design of the robust design techniques to take care of this is illustrated and robustness against model uncertainties is provided. The robustness can be either stability robustness or performance robustness [17].

One of the most popular techniques is H_∞ optimal control [17-19] in which the uncertainty and performance can be incorporated into the controller design. To obtain parameters in the proposed controller, genetic algorithm is proposed to solve a specified-structure H_∞ loop shaping optimization problem.

Infinity norm of transfer function from disturbances to states is subjected to be minimized via searching and evolutionary computation. The resulting optimal parameters make the system stable and also guarantee robust performance.

The approach in the problem is based on modeling of the real system as a set of linear time-invariant models built around a nominal one, i.e. the model is built as uncertain within known boundaries. The benefit of such a representation of the model is the possibility of getting a robust controller stabilizing a closed loop system even with the uncertainties present in the system [20].

In this paper, design techniques of speed controller of DC motor are illustrated which provides a simple structure for practical use. Section II represents the modelling of DC motor along with the transfer function. Section III illustrates the proposed technique along with the different controller designs. The Genetic Algorithm is described in this section. Section IV describes the design example. Section V shows the results and comparison of characteristics of various controllers that are designed. Section VI concludes the paper.

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II. DC MOTOR MODELING

DC machines are characterized by their versatility. They can be designed to display a wide variety of volt-ampere or speed-torque characteristics for both dynamic and steady state operation by means of various combinations of shunt, series and separately excited field windings and are used in many applications requiring a wide range of motor speeds and a precise output motor control.

The speed of a DC motor is proportional to the voltage applied to it while, its torque is proportional to the motor current. Speed control can be achieved by variable battery tappings, variable supply voltage, resistors or electronic controls.

A schematic diagram of DC motor model is shown in Fig. 1. The armature circuit consist of a resistance (Ra) connected in series with an inductance (La), and a voltage source (e_a) representing the back emf (e_b) induced in the armature when during rotation.

The block diagram of a typical DC motor is shown in Fig. 2.

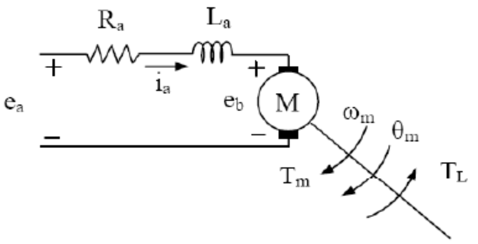

Fig.1. Schematic Diagram of a DC Motor

The motor torque T_m is related to the armature current, i_a, by a torque constant K_i:

$$T_m = K_i i_a \qquad (1)$$

The back emf, e_b, is relative to angular velocity by:

$$e_b = k_b w_m = k_b \frac{d\theta}{dt} \qquad (2)$$

From Fig. 1, we can write the following equations based on the Newton's law combined with the Kirchhoff's law:

$$L_a \frac{di_a}{dt} + R_a i_a = e_a - K_b \frac{d\theta}{dt}$$
$$J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt} = K_i i_a \qquad (3)$$

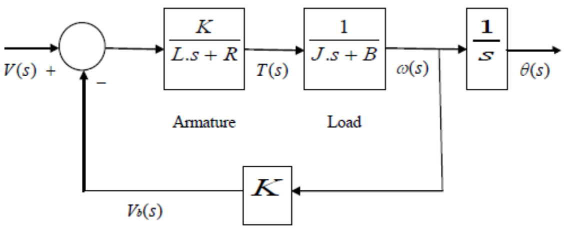

Fig.2. Block Diagram of DC Motor

From Fig. 2, the transfer function from the input voltage, $V(s)$, to the output velocity, $\omega(s)$ and to the output angle, $\theta(s)$ can be written as:

$$\frac{\omega(s)}{V(s)} = \frac{K}{(Ls+R)(Js+B)+K^2}$$
$$\frac{\theta(s)}{V(s)} = \frac{K}{s[(Ls+R)(Js+B)+K^2]}$$

Where K is the electromotive force constant i.e. emf (Nm/A), L is electrical inductance (Henry), R is electrical resistance (ohm), J is the moment of inertia of the rotor (kg.m2/s2), and B is the damping ratio of the mechanical system.

There are several different ways to describe a system of linear differential equations. The plant model will be introduced in the form of state-space representation and given by the equations:

$$X^0 = Ax + Bu$$
$$Y = Cx + Du \qquad (4)$$

In the state space model of a separately excited DC motor, the equations can be expressed by choosing the angular speed (w) and armature current (i) as state variables and the armature voltage (V) as an input. The output is chosen to be the angular speed. According to above equations, the state space model will be:

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dw}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & \frac{Kb}{L} \\ \frac{Kt}{J} & \frac{-B}{J} \end{bmatrix} \begin{bmatrix} i \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V$$
$$y = [0\ 1] \begin{bmatrix} i \\ w \end{bmatrix}$$

TABLE I
VALUES OF MOTOR PARAMETERS

Motor Parameter	Value
K	0.1 (Nm/A)
L	0.5 (H)
R	2 (ohm)
J	0.02 ($kg.m^2/s^2$)
B	0.2 (N-m.s/rad)

The values of different parameters that are taken in MATLAB programs and simulations are as shown in table 1.

III. CONTROLLER DESIGN FOR DC MOTOR SPEED CONTROL

Assume that K (s) is a structure-specified controller. The structure of the controller is specified before starting the optimization process. A set of controller parameters is evaluated to minimize the objective function.

Controller order obtained solving general H_∞controller problem using 'hinfsyn' command is very high i.e. 5. In the approach presented in this section, structured specified controllers are designed solving H_∞ optimization problem using GA.

A. *Controller's Structure Selection*

PID Controller

The Proportional-Integral-Derivative (PID) controller is the most common and simplest form of feedback. Presently, more than 95% of the controllers are of PID type. Most of the industries employ PID controllers in the system because of their simple structure. PID control with its three term functionality covering both transient as well as steady-states response, offers the most efficient and the simplest solution to many real world control problems. Therefore, PID controller with three tuning parameters is selected due to its many advantages.

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The design vector to be obtained solving optimization problem becomes $X = [k_1, k_2, k_3]$ i.e. the tuning parameters of the controller are k_1, k_2, k_3 and the objective function to be minimized is $\|F_l(P,K)\|_\infty$. The PID structure is chosen as:

$$K(s) = k_1 + \frac{k_2}{s} + k_3 s \tag{5}$$

Polynomial Controller Structure

The objective of this controller is to obtain a robust polynomial controller, while satisfying the control objectives, the tracking of a reference trajectory, as well as the rejection of disturbances and noises of measure. Transfer function for Polynomial controller structure having order=3 is defined as:

$$K(s) = \frac{(s+f)*(s+a)}{(s+b)*(s+d)*(s+e)} \tag{6}$$

The GA is run on the design vector $X = [f, a, b, d, e]$.

The Polynomial controller is being optimized using GA technique and the controller unknown parameters are find out.

The performance measure contains the objectives of the third order polynomial controller that are studied in terms of minimization of objective function.

B. *Mixed-Sensitivity Control*

The cost function in the design is the infinity norm based on the concept of robust mixed-sensitivity control, which can be briefly described as follows.

In the mixed-sensitivity control method, initially, the weighting function of the plant's perturbation or performance must be specified. Generally, W1 is specified for the disturbance attenuation of the system and W2 is specified for the uncertainty weight of the plant. The cost function can be written as:

$$J = \left\| \begin{matrix} W_1 S \\ W_2 T \end{matrix} \right\| < 1 \tag{7}$$

Where S is the plant's sensitivity function and T is the plant's complementary sensitivity function.

Assuming that the plant is denoted as P, the controller is denoted as K and the system is the unity negative feedback control, the sensitivity and complementary sensitivity function can be expressed as:

$$S = I + PK \tag{8}$$

$$T = I - PK = PK(I + PK) \tag{9}$$

This cost function is based on frequency domain specifications.

C. *Genetic Algorithm*

The genetic algorithm (GA) is an optimization technique that performs a parallel, stochastic and directed search to evolve the fittest (best) solution. GA is different from conventional optimization methods as it employs the principles of evolution, natural selection and mutation and maximizes the mean fitness of its population through the iterative application of the genetic operators.

Three main operators that comprise GA are reproduction, crossover, and mutation.

The genetic algorithm follows the following steps:

Step1: Generate an initial population of binary string.

Step2: Calculate fitness value of each member of population based on the problem type (minimization or maximization).

Step3: Generate offspring string through reproduction, crossover and mutation and evaluate.

Step4: Calculate fitness value for each string.

Step5: Terminate the process if required solution is obtained or number of generation is attained.

IV. DESIGN EXAMPLE

A speed control system is used to illustrate the effectiveness of the proposed technique. In this example, the system of the speed control of the DC motor has the parameters at the nominal plant as given in table 1. The parameters of PID and Polynomial controllers have been designed using various methodologies with objective function given by equation (7) and simulations have been done using MATLAB.

Thus, the transfer function for speed control of the DC motor can be written as:

$$G = \frac{0.1}{(0.001s^2 + 0.14s + 0.41)} \tag{10}$$

The frequency dependent weighting functions taken are:

$$W_1 = \frac{0.5s + 10}{s + 0.001}$$

$$W_2 = \frac{(0.2619s^2 + 5.649s + 19.06)}{(s^2 + 26.28s + 106.7)}$$

An iterative work with assumed initial values is usually conducted to find out the weighting functions W_1 and W_2. Inverse of W_1 should exhibit the desired shape of the sensitivity function (S); whereas, inverse of W_2 should reflect the shape of the complementary sensitivity function (T). The parameters of W_1 are adjusted such that the singular value curve of S remains below the singular value curve of inverse of W_1. Similarly, the parameters of W_2 are adjusted in such a way that the singular value curve of T remains below the singular value curve of inverse of W_2.

V. RESULTS

A. *Response without Controller*

Fig. 3 shows the step response of DC motor system obtained without any controller.

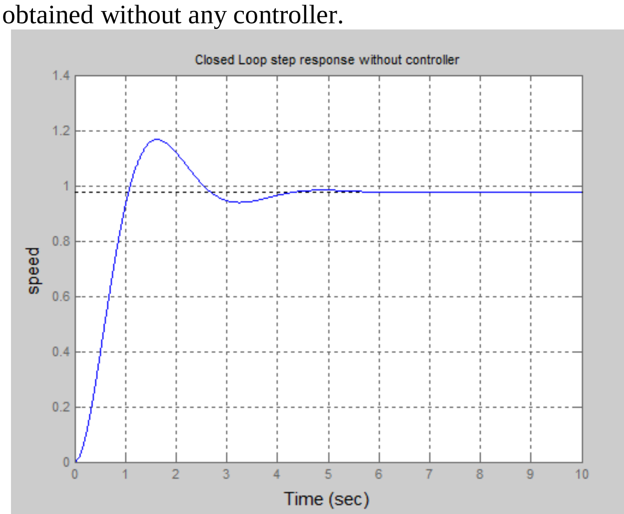

Fig. 3 Closed Loop Step Response without controller

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The specifications of the system response obtained are as follows:
Peak response: 1.17
Overshoot: 19.5%
Rise time: 0.721 sec
Settling time: 3.83 sec
Steady state error: 0.024

B. *PID Controller Using GA*

The size of population of GA is often chosen between [20,100]. For the proposed simulation, the size of population is taken as 50. The number of generation is often chosen between [100,500]. For the proposed case, number of generation is equal to 100. The mutation rate is selected to be 0.05.

The GA algorithm aims to find out the optimal value of the unknown parameters in order to minimize the objective function. The GA in 51 generations converges with the optimal solution, [0.762 -4.852 5.003], which on substitution to (5) provide following controller K(s):

$$K(s) = 0.762 - \frac{4.852}{s} + 5.003s \qquad (11)$$

The infinity norm obtained by the designed controller in (11) is 0.5 which is less than 1. Consequently, since this norm is less than 1, then the system is robust according to the concept of mixed-sensitivity robust control.

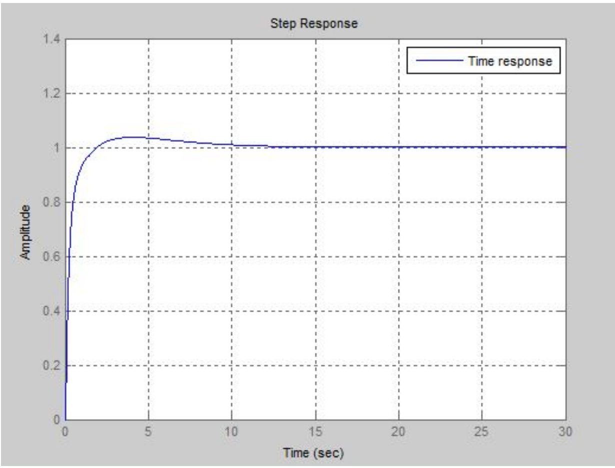

Fig. 4. Closed loop response with PID controller using GA

Fig. 4 shows the step response of the system which shows the following characteristics:
Peak response: 1.05
Overshoot: 9.81%
Rise time: 0.0411 sec
Settling time: 2.83 sec
Steady state error: zero

C. *Polynomial Controller Using GA*

The size of population of GA is often chosen between [20,100]. For the proposed simulation, the size of population is taken as 50. The number of generation is often chosen between [100,500]. For the proposed case, number of generations is equal to 100. The mutation rate is chosen to be 0.05. The GA algorithm aims to find optimal value of unknown parameters to minimize the objective function.

The GA converges with the optimal solution, $X = [2.357, 2.961, 0.009, 2.112, 3.142]$, which on substitution to (6) provide following controller K(s):

$$K(s) = \frac{(s + 2.357) * (s + 2.961)}{(s + 0.009) * (s + 2.112) * (s + 3.142)}$$

The infinity norm obtained by the evaluated controller is 0.21495 which is less than 1. Consequently, since this norm is less than 1, then the system is robust according to the concept of mixed sensitivity robust control and the controller is stable and fulfills the constraint i.e. $\|F_l(P, K)\| < 1$.

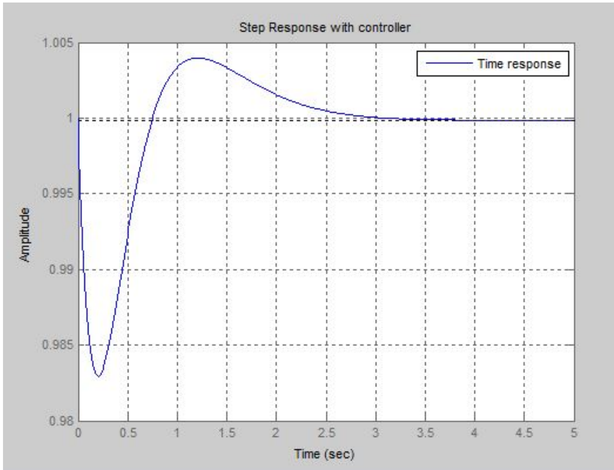

Fig.5. Closed loop response with Polynomial controller of order 3 using GA

Fig. 5 shows the step response which shows the following characteristics:
Peak response: 1
Overshoot: 0.411%
Rise time: 0.000523 sec
Settling time: 2.75 sec
Steady state error: zero

D. *Conventional controller using 'hinfsyn' command*

A conventional mixed sensitivity controller is also designed for comparison using 'hinfsyn' command in Robust Control toolbox of MATLAB. From the conventional technique, the order of the final controller comes out to be 5.

The controller obtained by this method is as follows:

$$K = \frac{4551s^4 + 12870s^3 + 72570s^2 + 98610s + 54290}{0.1s^5 + 36.7s^4 + 2591s^3 + 473s^2 + 1778s + 177}$$

Since the order and complexity of the controller obtained from this method is very high, so other optimization techniques are considered better.

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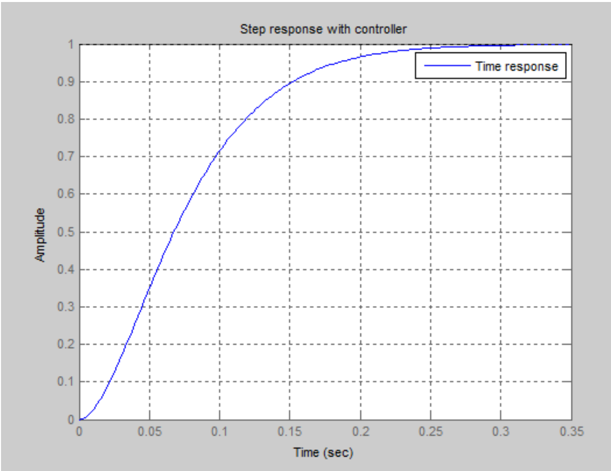

Fig. 6. Closed loop response using *hinfsyn* command

Fig. 6 shows the step response which shows the following characteristics:
Peak response: 1
Overshoot: 0.998%
Rise time: 0.0404 sec
Settling time: 0.224 sec
Steady state error: zero

VI. CONCLUSION

The order of the conventional technique controller i.e. using 'hinfsyn' command is quite high and its structure is also very complicated. Thus, the advantage of simple structure can be obtained by the proposed technique.

Comparative study of PID and Polynomial controllers using GA for speed control of DC motor along with the conventional controller is shown in Table II.

TABLE II
COMPARATIVE STUDY OF PID AND POLYNOMIAL CONTROLLER

Controller	Without controller	PID with GA	'*hinfsyn*' comma	Polynomial with GA
Peak Amplitude	1.17	1.05	0.998	1
Max. Overshoot	19.5 %	9.81%	0.0404	0.411
Rise Time	0.721	0.0411 %	0.13	0.000523
Settling Time	1 3.83	2.83	0.224	2.75
H_∞	--	0.5	0.8153	0.21495

The proposed technique can be applied to control the speed of a DC motor. Based on the incorporation of robust control and the optimization concepts, the proposed technique can achieve robustness and good performance while keeping the structure of the controller as simple. Robustness of the controlled system can be guaranteed via the theory of mixed sensitivity robust control. Since, in general, complex control system design requires many objectives to be met simultaneously along with various constraints; the present design methodology is best suited and also very helpful in practical applications.

It is concluded from the table that Polynomial controller of order 3 using GA gives the best results for speed control of DC motor.

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