

Small-Signal Transfer-Function Extraction of a Lossy Buck Converter Using ASM Technique

A.Skandarnezhad^{1*}, A.Rahmati¹, A.Abrishamifar¹, A.Kalteh

Department of Electrical and Electronic Engineering, Iran University of Science and Technology (IUST), Tehran, Iran¹

Abstract: Nowadays, step-down power converters such as buck scheme are widely employed in a variety of applications such as power supplies, spacecraft power systems, hybrid vehicles and DC motor drives. This paper determines the small signal averaged switch model (ASM) of the buck converter with parasitic elements consideration. Lossy analysis helps one to better understanding of the time-frequency domain behaviour and accurate designing the feedback control circuitry. After determining the control-to-output and audio-successability transfer functions, finally the proposed model results are validated by simulations in PSpice and Matlab software.

Keywords: Buck Converter, Parasitic Elements, Small-Signal Transfer-Function, Averaged Switch Model.

I. INTRODUCTION

PWM converters are highly nonlinear circuits since they contain at least one transistor and one diode. The converters normally require control circuits to regulate the dc output voltage against load and source voltage variations. Typical control aspects of interest are frequency response, transient response, and stability [1].

The former method to modelling the power converter is the state-space averaging (SSA) technique. The state-space equations are weighed according to the fraction of the switching cycles during which the converter remains in a given state. However, the state-space averaging method requires considerable matrix manipulation and is sometimes tiresome when the converter circuit contains a large number of elements or parasitic components. Moreover, it provides little insight into the converter behaviour. In many applications the average values of voltages and currents are of interest, rather than their instantaneous values [2].

On the other hand, the main goal of the averaged switch modelling (ASM) approach is to find an averaged model for the switch network. The resulting averaged switch model can then be inserted into the converter circuit to obtain a complete averaged model of the converter. An important advantage of this approach is that the same model can be used in many different converter schemes. In many cases, the averaged switch model simplifies converter analysis and yields good intuitive understanding of the converter steady-state and dynamic properties [3].

In this work, the small signal averaged switch model of open loop buck converter is obtained. Then, the small signal transfer function is readily derived from this AC small signal model. Simplified equations for control-to-output and audio-successability transfer-functions are derived. Finally the sample buck converter is simulated in PSpice and its results are compared with our complete model simulation results in Matlab. The results are in agreement with each other.

II. AVERAGED SWITCH MODEL (ASM)

A PWM converter has two parts: linear analog part and nonlinear discrete part. The linear part consists of linear components such as capacitors and inductors with their equivalent series resistances. The nonlinear part consists of nonlinear semiconductor devices such as transistor and diode operated as switches that is as discrete components. The nonlinear part may be replaced by an average circuit model which emulates its average low frequency behaviour. The modelling strategy of PWM converters is similar to the transistor modelling and is based on the following principles [4]:

- (i) Replacement of the switching network by an analog continuous circuit model.
- (ii) Leaving the analog part composed of linear components unchanged.

Fig. 1. shows the open-loop buck converter schematic.

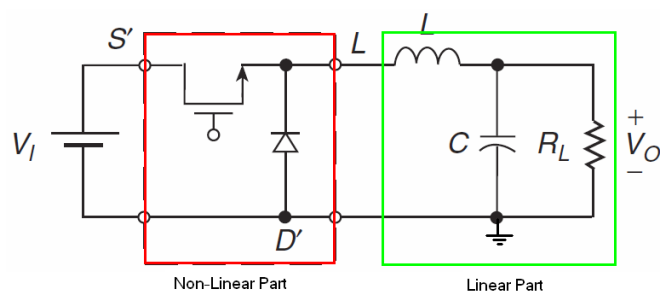


Fig. 1. Open-loop buck converter

The switch sub-circuit is highly nonlinear where Fig. 2(a). shows the switching network of this PWM converter and Fig. 2(b). shows an equivalent circuit of the switching network. The ideal part of the switching network consists of two ideal switches. The transistor is represented by an ideal switch and a linear on-resistance r_{DS} and the diode is

represented by an ideal switch, a linear forward resistance R_F and an offset voltage V_F [2].

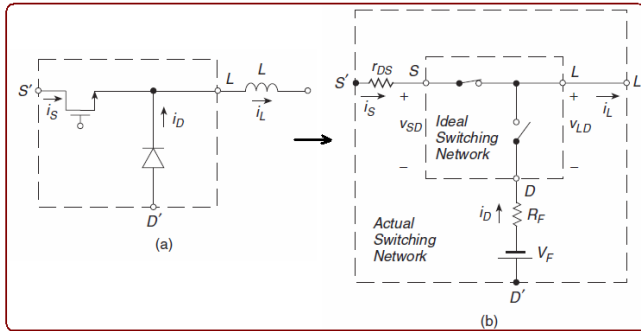


Fig. 2. (a). Switching network (b). Equivalent circuit

When transistor is closed, its current equals with inductor current and when it opens, the diode current equals with i_L where is depicted in Fig. 3.

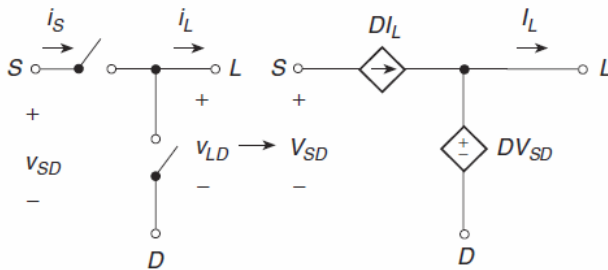


Fig. 3. Averaged large-signal model of the ideal switch network

The law of conservation energy is used to determine the average values of the switched resistances. Modelling of the actual switching network under steady-state conditions for two switch network operating in CCM is seen in Fig. 4.

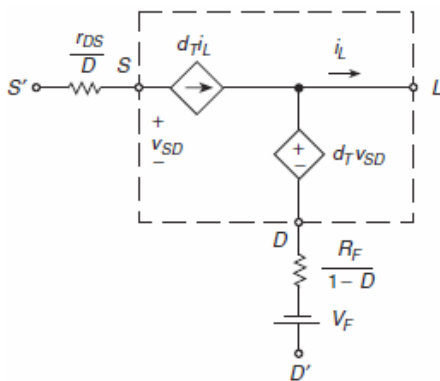


Fig. 4. Averaged actual switch network

By some circuit manipulation, one can move all the parasitic elements to one side, but with considering the conservation law. Simplified large-signal averaged model of the actual switching network with the averaged resistances moved to the inductor branch is depicted in Fig. 5.

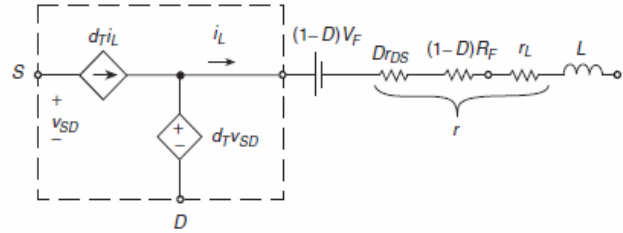


Fig. 5. Averaged actual switch network elements moved to inductor side

Linearization of the large-signal averaged model at a given operating point can be performed by expanding the equations into a Taylor's series about the operating point and neglecting the higher order terms [5]. A linear small-signal averaged model can be obtained by assuming small-signal perturbations, which allows us to take into account only the first order terms, as described in Eqs. (1) and (2).

$$I_S + i_s = (D + d)(I_L + i_l) = DI_L + Di_l + I_L d + i_l d \quad (1)$$

$$V_{LD} + v_{ld} = (D + d)(V_{SD} + v_{sd}) = DV_{SD} + Dv_{sd} + V_{SD}d + v_{sd}d \quad (2)$$

After neglecting the products of the small-signal terms $i_l d$ and $v_{sd}d$, one obtains a set of linear equation where is illustrated in Fig. 6.

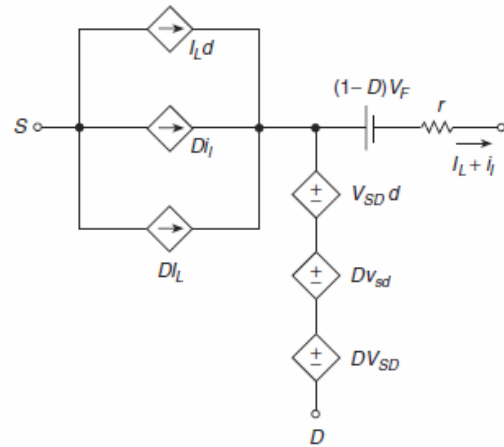


Fig. 6. Large-signal averaged model of the switch network

Small-signal equivalent of the buck converter can be derived after insertion the circuit of Fig. 6. in the buck structure of Fig. 1., where is depicted in Fig. 7.

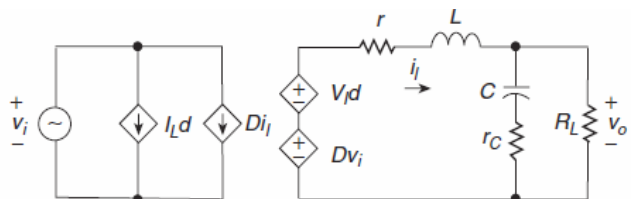


Fig. 7. Small-signal model of the buck converter

Next section presents the converter AC transfer functions.

III. SMALL-SIGNAL TRANSFER-FUNCTION OF THE BUCK CONVERTER

Small-signal model of the buck converter where seen in Fig 7. has two independent input: $v_i(s)$ and $d(s)$.

Setting $d=0$ then doing some kerchiefs law equations, one obtains a small-signal model of the buck converter for determining the audio-successability transfer function that is exhibited in Fig. (8). and presented in Eq. (3).

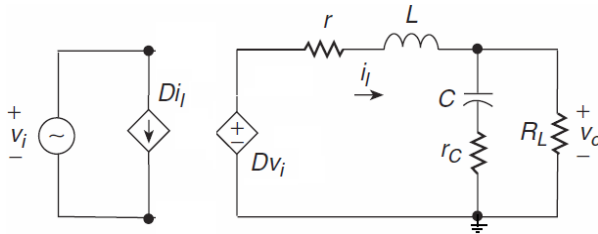


Fig. (8). Small-signal model for determining the audio-successability

$$\begin{cases} (r + sL)j_l(s) + v_o(s) = Dv_i(s) \\ v_o(s) = i_l(s) \cdot (R_L \parallel (r_c + 1/sC)) \end{cases} \Rightarrow$$

$$T_{o-i}(s) = \frac{v_o(s)}{v_i(s)} = D \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{Q\omega_o} + (\frac{s}{\omega_o})^2} \quad (8)$$

Where,

$$r = r_L + Dr_{ds} + (1 - D)R_F \quad (9)$$

$$\omega_z = \frac{1}{r_c \cdot C} \quad (10)$$

$$\omega_o = \frac{1}{\sqrt{LC} \frac{R_L + r_c}{R_L + r}} \quad (11)$$

$$Q = \frac{1}{\frac{\sqrt{L/C}}{r + R_L} + \frac{r_c + (r \parallel R_L)}{\sqrt{L/C}}} ; \xi = \frac{1}{2Q} \quad (12)$$

By setting $v_i=0$, and some equations manipulation, the control-to-output transfer function will be obtained which is illustrated in Fig. (9). and Eq. (13). respectively.

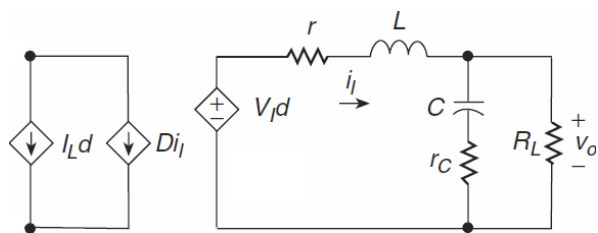


Fig. (9). Small-signal model for determining the control-to-output

$$T_{o-d}(s) = \frac{v_o(s)}{d(s)} = V_I \frac{R_L}{r + R_L} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{Q\omega_o} + (\frac{s}{\omega_o})^2} \quad (13)$$

The delay time t_d , introduced by power transistors driver and pulse width modulator can be described by the function $T_d(s)$ from dc to $fs/2$ that is approximated in Eq. (14).

$$T_d(s) = e^{-st_d} = \frac{1 - \frac{st_d}{2}}{1 + \frac{st_d}{2}} = \frac{s - \frac{2}{t_d}}{s + \frac{2}{t_d}} = \frac{s - \omega_y}{s + \omega_y} \quad (14)$$

$$T_{o-d}^{delay}(s) = T_{o-d}(s) \cdot T_d(s) \quad (15)$$

Let to consider a step change in the input voltage of magnitude ΔV_I , then the transient component of the output voltage of the converter in the time domain will be:

$$v_o(t) = V_O + v_o(t)$$

$$v_o(t) = \ell^{-1} \left\{ \frac{\Delta V_I}{s} \cdot T_{o-i}(s) \right\} \quad (16)$$

Transient change of the output voltage to step change of duty cycle is presented in Eq. (17).

$$v_o(t) = \ell^{-1} \left\{ \frac{\Delta d}{s} \cdot T_{o-d}(s) \right\} \quad (17)$$

Where ℓ^{-1} is the inverse-laplacian operator.

IV. NUMERICAL EXAMPLE AND SIMULATION RESULTS

To validate the proposed model, a prototype buck converter is done with specifications stated in Table 1. This circuit is simulated using PSpice and the model numerical analysis is done by Matlab, then simulation results and experimental results are compared. Note, r_L and r_C are the equivalent serried resistors (ESR) of inductor and capacitor.

TABLE 1. A PROTOTYPE BUCK CONVERTER ELEMENTS

Symbol	Description	Value
E	Source voltage	12V
L	Inductor	100uH
C	Capacitor	10uF
R_L	Load resistance	4.7Ω
D	Duty cycle	0.5
r_C	ESR of capacitor	0.1Ω
r_L	ESR of inductor	0.1 Ω
r_{ds}	Drain-source resistance	0.1 Ω
R_F	Diode forward resistance	0.05 Ω
V_F	Diode forward voltage	0.75V

By considering the Eqs (9)-(12) one can find,

$$r = 0.175\Omega, \omega_z = 1Meg \frac{rad}{sec}$$

$$\omega_o = 31.86K \frac{rad}{sec}, Q = 1.38, \xi = 0.36$$

Then using EQ. 8., audio-successability is depicted in Fig. 10.

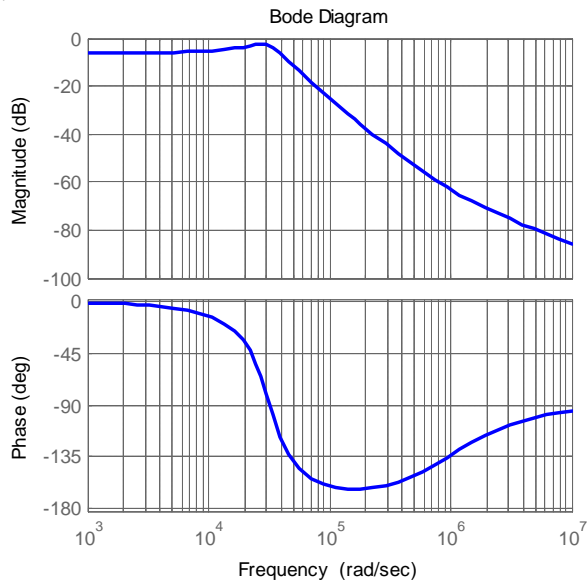


Fig. (10). Audio-successability transfer function of the given converter

And control-to-output transfer function of Eq. (14) is presented in Fig. (11).

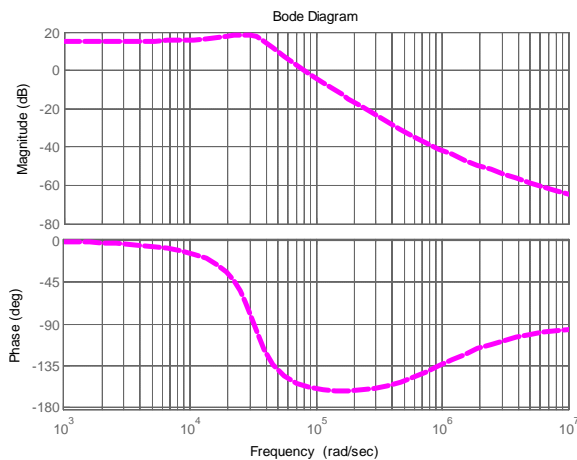


Fig. (11). Control-to-output transfer function of the given converter

When a converter with zero initial values connects to source voltage at $t=0$, indeed, endures to a step change of source voltage with amplitude of $\Delta V_I = E$. Transient behaviour of the output voltage can be figured out using Eq. (16). As seen in Fig. (12). It indicates a good agreement between the proposed model and schematic simulation results.

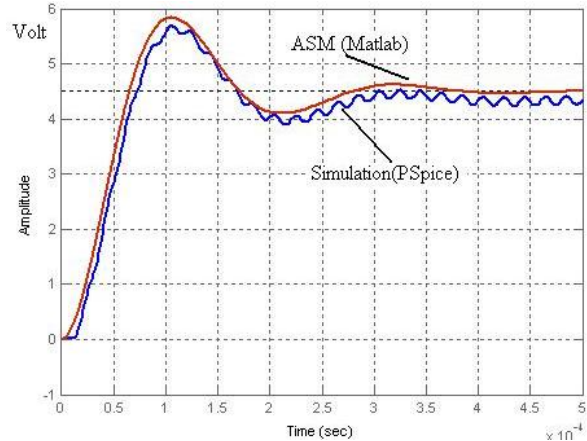


Fig. (12). Control-to-output transfer function of the given converter

Similarly, one can find the converter output voltage along with step change in the duty cycle according to Eq. (17).

V. CONCLUSION

An analytical averaged approach to extract the small-signal model of a buck converter including the consideration of the switches conduction losses is described. Finally, the buck converter Benchmark circuit is simulated in PSpice and its results are compared with our model simulation results in Matlab. Simulation results are in conformity with each other. Further work will include the better impressing the parasitic elements in the modelling approach.

REFERENCES

- [1] T. L. Skvarenina, *The Power Electronics Handbook*, New York, CRC Press, 2002.
- [2] M. K. Kazimierzczuk, *Pulse Width Modulated DC-DC Power Converters*, 3rd, Markono Print Media, WILEY, 2008.
- [3] L. Yanfei, "A Unified Large Signal and Small Signal Model for DC/DC Converters With Average Current Control", Transactions of china electrotechnical society, Vol.22, No.5, pp. 84-91, May 2007.
- [4] M. R. Modabbernia, "An Improved State Space Average Model of Buck DC-DC Converter with all of the System Uncertainties", IJEEF, Vol. 5, No. 1, pp. 81-94, March 2013.
- [5] U. Drofenik and J.W. Kolar, "A general scheme for calculating switching- and conduction losses of power semiconductors in numerical circuit . . .", IPEC, 2005, 7 pages, Niigata, Japan.

BIOGRAPHY



A. Skandarneshad, obtained his B.E. and M.S. degree in Electronics Engineering from Iran University of Science and Technology (IUST) at 2003 and 2005 respectively. He is now pursuing the PhD degree in the power electronic field at the same university. His areas of interest are switching power supplies design and modeling, power factor correction techniques, electro-magnetic compatibility methods in SMPS's.