ISSN 2321 – 2004
ISSN 2321 – 5526

*INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING*
*Vol. 2, Issue 1, January 2014*

# PID controller design for unstable Processes With time delay

**S.Kumar[1], V.K.Singh[2]**

M.Tech, Student, Department of Electrical and Electronics, BTKIT, Dwarahat, Uttarakhand, India [1]

Assistant Professor, Dept. of Electrical and Electronics,BTKIT,Dwarahat,Uttarakhand,India [2]

**Abstract**: **:** the present study compare the PID Lead-lag tuning formulas derived for two poles unstable second order plus dead time (SOPDT) processes based on IMC principle for disturbance estimator controller by Liu.et.al,S.Park et.al,and Lee et al. Respectively by using the matlab simulation..

**Keywords**: IMC, 2DOF, unstable, PID

## 1.INTRODUCTION

. The proportional integral derivative (PID) controller algorithm is undoubtedly the most adopted controllers for industrial plants , mainly due to their simplicity ,and they can assure satisfactory performances for wide range of processes. In general, two types of time delayed unstable process are the first–order delayed unstable processes (FODUP) and the second–order delayed unstable process (SODUP). Recently, tuning of controller for a time-delay unstable process has been an active area of research in the literature.

Huang and Chen [1] suggested a three element structure, which is equivalent to a two-degree-of–freedom (2DOF) control scheme, for controlling the open loop unstable processes. However its method still gives about 100% overshoot to a setpoint change. S.Park et al [2] and Wang and Cai [3] had proposed a 2DOF control methods for several processes to overcome excessive overshoot and large settling time in setpoint response.M.Lee et al [4]. Had proposed a tuning formula which is simple and easy to memorize and also applicable to several classes of unstable process with time delay in a unique manner .

Due to its internal instability, IMC structure cannot be directly applied for the controlling of unstable processes. it is very powerful for controlling stable processes with time delay by the reason of the internal instability Morari and Zafiriou[5].some modified IMC methods of 2DOF for controlling unstable processes with time delay had been developed by Huang and Chen [1],Tan et al.[6],Liu et al.[7]. a 2DOF control method based on Smith-predictor (SP) were proposed by {Kwak et al.[8], Majhi and Atherton [ 9],Zhang et al.[10]} to achieve a smooth nominal setpoint response without overshoot for first–order unstable processes with time delay.

Liu et al.[7] proposed a control structure is shown in fig 1, where $G_{m0}$ is the delay free part of the process model $G_m$ i.e .$G_{m0}e^{-\theta_m s}$, and C is responsible for the setpoint tracking, and F is used for rejecting the load disturbances and therefore is called a disturbance estimator. And a controller $G_c$ is employed for stabilizing the setpoint response. The design of $G_c$ enables it to contributes as a P or PD controller and converts the system an open–loop for setpoint tracking. The analytical design procedure for both C and F is developed based on the $H_2$ optimal performance objective [5], which is equivalent to the integral-squared–error (ISE) performance specification, both of the nominal setpoint response and load disturbance response can be quantitatively regulated to achieve the optimality. Moreover, both of C and F can be monotonously tuned on – line by a single adjustable parameter respectively to cope with the process uncertainty in practice and thus to make the best compromise between the nominal system performance and its robust stability ,which is a dominant virtue of the proposed two-degree-of – freedom control scheme.

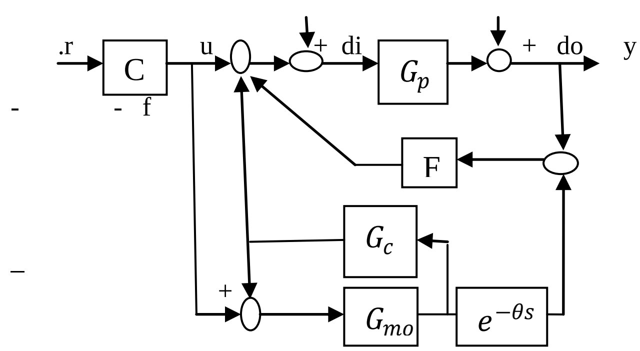


Fig.1 Two–degree–of–freedom control structure

## 2.Controller design procedure by T.Liu et al.

### 2.1 Design of stabilizing controller Gc

The setpoint transfer function is given by

ISSN 2321 – 2004
ISSN 2321 – 5526

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 2, Issue 1, January 2014

$$Hr=\frac{Y}{rf}=\frac{CGp}{1+GcGmo}.\frac{1+FGmo\,e^{-\theta mS}}{1+FGp} \qquad (2.1)$$

In the nominal case (i.e,Gm=Gp) eq. 2.1 is simplified as

$$Hr=\frac{Y}{rf}=\frac{CGp}{1+GcGmo} \qquad (2.2)$$

Since the dead-time is discarded in the above characteristic equation of the nominal setpoint transfer function it certainly contributes to achieving a smooth servo response.

The closed-loop transfer function for disturbance rejection is given by.

$$\text{Hdi}=\frac{y}{di}=\frac{Gp}{1+FGp} \qquad (2.3)$$

Let the transfer functions of the process model be.

$$1.\text{Gp}=\frac{Kp\,e^{-\theta s}}{(\tau_1 s-1)(\tau_2 s-1)} \qquad (2.4)$$

Where Kp is the steady state gain $\tau_1$ and $\tau_2$ are the time constants and $\theta$ is the time delay of the process model.Choosing the stabilizing controller $G_c=K_d$s,($K_d > (\tau_1+\tau_2)/K_p$).Thus the charaterstic equation of the setpoint response transfer function of equation (2.1). becomes

$$[(\tau_1 s-1)(\tau_2 s-1)+sK_dK_p]=0 \qquad (2.5)$$

By employing the Routh-Hurwitz stability criterion, obtaining the tuning constraint for stabilizing the setpoint response.it gives

$$K_d > (\tau_1+\tau_2)/K_p \qquad (2.6)$$

**2.2 Setpoint tracking controller C**

By the ISE performance specification i;e $\min\|e\|_2^2$ is used to design the setpopint tracking controller C.that is ,it should be implemented to achieve the system performance objective $\min\|W(s)(1-Hr(s))\|_2^2$ ,where W is the setpoint weight function and can be chosen as 1/s for the rigorous step change of the setpoint input and load that occurs in industries.As for the unstable process type $\text{Gp}=\frac{Kp\,e^{-\theta s}}{(\tau s-1)}$ ,by using the v/v order all-pass pade approximation for the pure time delay term $e^{-\theta s}$ ,obtain

$$\text{Gp}=\frac{Kp}{(\tau s-1)}\,\frac{Q_{vv}(-\theta s)}{Q_{vv}(\theta s)}$$ , where

$$Q_{vv}(\theta s)=\sum_{j=0}^{v}\frac{(2v-j)!v!}{(2v)!j!(v-j)!}(\theta s)^j \qquad (2.7)$$

And v is chosen to be an integer large enough to guarantee that the introduced approximation error can be neglected in comparison with the process model mismatch in practice .by using

$$\|W(s)(1-Hr(s))\|_2^2 = \left\|\frac{1}{s}\left(1-\frac{kC(s)Q_{vv}(-\theta s)}{(\tau s+k_ck-1)Q_{vv}(\theta s)}\right)\right\|_2^2$$

$$=\left\|\frac{Q_{vv}(\theta s)}{sQ_{vv}(-\theta s)}-\frac{kC(s)}{s(\tau s+k_ck-1)}\right\|_2^2$$

Note that $Q_{vv}(0)$=1 and all zero of $Q_{vv(-\theta s)}$ are located in RHP. Utilizing the orthogonality property of $H_2$ norm yield

$$\|W(s)(1-Hr(s))\|_2^2 =$$

$$\left\|\frac{Q_{vv}(\theta s)-Q_{vv(-\theta s)}}{sQ_{vv(-\theta s)}}\right\|_2^2+\left\|\frac{\tau s+k_ck-1-kC(s)}{s(\tau s+k_ck-1)}\right\|_2^2 \qquad (2.8)$$

Minimizing the right side i;e letting its second term be zero .obtain the optimal controller

$$C_{in}(s)=\frac{\tau s+k_ck-1}{k} \qquad (2.9)$$

However it is not proper and cannot be physically realized in practice . hence a first order low pass filter

$$\text{F}=\frac{1}{\lambda_f s+1} \qquad (2.10)$$

then the practically optimal controller is obtained as

C(s)= $\frac{\tau s+k_ck-1}{k(\lambda_f s+1)}$ , where $\lambda_f$ is the adjustable parameter and when it is tuned to zero .C recovers the optimality.By using the above procedure the setpoint tracking controller for the process

$$\text{Gp}=\frac{Kp\,e^{-\theta s}}{(\tau_1 s-1)(\tau_2 s-1)}$$ is

$$\text{C(s)}=\frac{\tau_1\tau_2 s^2+(k_dk-\tau_1-\tau_2)s+1}{k(\lambda_f s+1)^2} \qquad (2.11)$$

**2.3 Disturbance Estimator Controller(F)**

This section describes disturbance estimator controller design based on on IMC Principle. The proposed modified IMC structure is shown in fig 1 . As per principle of IMC

ISSN 2321 – 2004
ISSN 2321 – 5526

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 2, Issue 1, January 2014

controller the following condition should be satisfy for internal stabilization of closed loop system.

(1) $G_{IMC}$ is stable. (2.12)

(2) $G_{IMC}\ G_p$ is stable. (2.13)

(3) (1- $G_{IMC}\ G_p$ )$G_P$ is stable. (2.14)

The process model $G_m$ is separated into the invertible or delay free $G_1$ (s) and non invertible or delay part $G_{N1(s)}$ .i;e $G_m$(s)= $G_1$(s)×$G_{N1}$ .As per modified structure $G_1$(s)=$P_m$ and similarly $G_{N1}$ =$P_A$

### 2.3.1 IMC Controller Design

**(1) Process Model with two unstable poles.**

To obtain a superior response for unstable processes. if the process model ($G_P$) has unstable poles ,$u_{p1}$.......................$u_{pm}$ , then the IMC controller $G_{imc}$ should have zeros at $u_{p1}$ ...............$u_{pm}$ and also 1-$G_P G_{imc}$ should have zeros at $u_{p1}$.............$u_{pm}$ .

Let the transfer function of the process model be.

$$G_P(s)=\frac{K_p e^{-\Theta s}}{(\tau_1 S-1)(\tau_2 S-1)} \quad (2.15)$$

First step of IMC controller design is to factor the process model into invertible and non invertible part.

$$G_1(s)=\frac{K_p}{(\tau_1 S-1)(\tau_2 S-1)} \qquad G_{NI}=e^{-\theta s} \quad (2.16)$$

The invertible and non invertible part of modified model.are.

$$P_m=\frac{K_p}{(\tau_1 S-1)(\tau_2 S-1)} \qquad P_A=e^{-\Theta s} \quad (2.17)$$

Second step is to define IMC Controller as

$$G_{IMC}=P_m^{-1}(s)\times f(s) \quad (2.18)$$

Where f(s) $=\frac{(a_2 s^2+a_1 s+1)}{(\lambda f s+1)4}$ is the low pass filter with adjustable time constant λ that reduce the effect of process model mismatch and improves the closed loop performance.

$$G_{IMC}=\frac{(\tau_1 s-1)(\tau_2 s-1)(a_2\ _S 2+ a_1 s+1)}{k_p(\lambda_s+1)4} \quad (2.19)$$

The first condition is satisfied automatically because $P_m^{-1}$ is the inverse of the model portion with the unstable poles or poles near zero.The second condition is satisfied because RHP poles of $G_P$ must be cancelled by the zeros of $G_{IMC}$ .then the third condition can be fulfilled by designing the IMC filter f is to make the controller proper to cancel the unstable poles near zeros of $G_P$.The RHP poles of $G_P$ must be cancelled by the zeros of (1-$G_P G_{imc}$ ) .The values of $a_1$ and $a_2$ can be calculated with the help of equation.( 1-$G_P G_{imc}$ ) .this required

$$[1-G_P G_{imc}\ ]|s=1/\tau_1, 1/\tau_2 = 0 \quad (2.20)$$

$$[\ 1-\ \frac{(e^{-\theta s})(a_2\ _S 2+ a_1 s+1)}{k_p(\lambda_s+1)4}\ ]|s=1/\tau_1, 1/\tau_2 = 0 \quad (2.21)$$

The values of $a_1$ and $a_2$ are obtained after simplification are given below.

$$a_1=\{\tau_1^2\ (\frac{\lambda_f}{\tau_1}+1)^4\ e^{\theta/\tau_1}\ -\tau_2^2\ (\frac{\lambda_f}{\tau_2}+1)^4\ e^{\theta/\tau_2}+(\tau_2^2-\tau_1^2)\}/(\tau_1-\tau_2) \quad (2.22)$$

$$a_2=\tau_1^2\{\ (\frac{\lambda_f}{\tau_1}+1)^4\ e^{\theta/\tau_1}-1\}-a_1\tau_1 \quad (2.23)$$

### 2.3.2 Disturbance Estimator Design.

The two unstable poles impose a phase lead and time delay terms impose a phase lag.in the present work, to make the controller realizable a PID controller in series with a lead lag compensator is considered.IMC controller is transform into PID lead-lag filter controller

$$F_{im}\ (s)=\frac{G_{imc}}{1-G_p G_{imc}} \quad (2.24)$$

$$F_{im}\ (s)=\frac{(\tau_1 s-1)(\tau_2 s-1)(a_2 s^2+a_1 s+1\ )}{k[(\lambda_f s+1)^4-(a_2\ s^2+a_1\ s+1)e^{-\theta s}]} \quad (2.25)$$

However it is not difficult to discover that there exists a RHP zero-pole cancelling at s=$1/\tau$ in eq.(2.24) which tends to cause the desired disturbance estimator to work unstably and cannot be removed directly .hence the mathematical maclaurin expansion series is here utilized to copy out the ideally desired disturbance estimator note that s=0 is a connotative pole of the ideally desired disturbance estimator in (2.24) which indicates that it has the property of integrating to eliminate the system output deviation from the setpoint value.therefore letting $F_{im}$(s)=$^{M(s)}/_{S}$ it follows that.

ISSN 2321 – 2004
ISSN 2321 – 5526

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 2, Issue 1, January 2014

$$F_{im}(s)=\frac{1}{s}\left[M(s)+M'(0)s+\frac{M''(0)}{2}s^2+\ldots.\frac{M^i}{i!}s^i+\cdots\right]$$
(2.26)

Obviously the first three terms of the above maclaurin expansion constitute exactly a standard PID controller in form of

$$F=(K_F+\frac{1}{T_1 s}+T_D s) \quad (2.27)$$

Where, $K_F=M'(0)$ ; $T_1=\frac{1}{M'(0)}$; $T_D=\frac{M''(0)}{2}$ ; (2.28)

**2.4 simulation example**

Consider the process with doubly unstable poles studied by Tan et al.[6]

$$G_P=\frac{2e^{-0.3s}}{(3s-1)(s-1)}$$

It is a typical second-order delayed unstable process.by Tan et al[6] method, they have taken $K_d = 3$ and $\lambda = 1.7\theta = 0.51$ just for obtain the set-point tracking controller, that is,

$$C(s)=\frac{1.5s^2+s+0.5}{(0.51s+1)^2}$$

The disturbance estimator F in form of PID is tuned as $K_C = 1.7638$, $\tau_1 = 1.059$, $\tau_2 = 4.0642$ (i.e. $\lambda = 1.7\theta = 0.51$), and also for illustration, they also took $N = 3$ and $\lambda = 1.5\theta = 0.45$ to obtain the third order approximation controller by Using the analytical design formulae

$$F=\frac{3.82s^3+439.41s^2+232.66s-129.79}{0.53s^3+0.8s^2+100s}$$

By adding a unit step change to the setpoint input at t=0 and an inverse unit step change of load disturbance to the process input at t=15, obtain the simulation results shown in fig 2

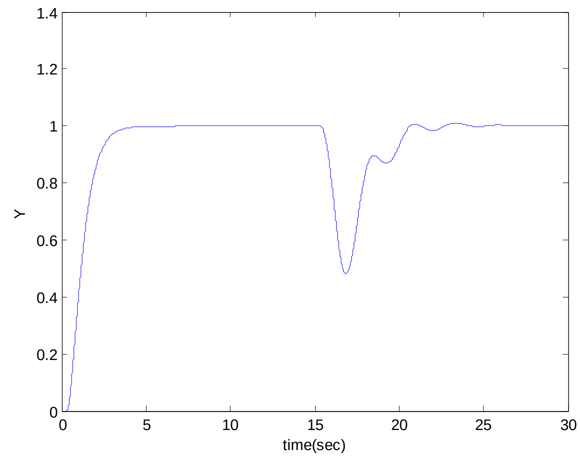


Fig.2.1

## 3 CONTROLLER DESIGN PROCEDURE BY LEE ET AL

### 3.1 Design of stabilizing controller Gc

The setpoint transfer function is given by

$$Hr=\frac{Y}{rf}=\frac{CGp}{1+GcGmo}.\frac{1+FGmo\,e^{-\theta mS}}{1+FGp} \quad (3.1)$$

In the nominal case (i.e,Gm=Gp) eq. 3.1 is simplified as

$$Hr=\frac{Y}{rf}=\frac{CGp}{1+GcGmo} \quad (3.2)$$

Since the dead-time is discarded in the above characteristic equation of the nominal setpoint transfer function,it certainly contributes to achieving a smooth servo response.

The closed-loop transfer function for disturbance rejection is given by.

$$Hdi=\frac{y}{di}=\frac{Gp}{1+FGp} \quad (3.3)$$

Let the transfer functions of the process model be.

$$1.Gp=\frac{Kp\,e^{-\theta s}}{(\tau_1 s-1)(\tau_2 s-1)} \quad (3.4)$$

Where Kp is the steady state gain $\tau_1$ and $\tau_2$ are the time constants and $\theta$ is the time delay of the process model.Choosing the stabilizing controller $G_c=K_d$s,($K_d>(\tau_1+\tau_2)/K_p$).Thus the charaterstic equation of the setpoint response transfer function of equation (3.4). becomes

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 2, Issue 1, January 2014

$$[(\tau_1 s - 1)(\tau_2 s - 1) + sK_d K_p]=0 \quad (3.5)$$

By employing the Routh-Hurwitz stability criterion, obtaining the tuning constraint for stabilizing the setpoint response. It gives

$$K_d > (\tau_1 + \tau_2)/K_p \quad (3.6)$$

### 3.2 Disturbance Estimator Controller(F)

This section describes disturbance estimator controller design based on on IMC Principle. The proposed modified IMC structure is shown in fig 3.1. As per principle of IMC controller the following condition should be satisfy for internal stabilization of closed loop system.

(1) $G_{IMC}$ is stable.

(2) $G_{IMC}$ $G_p$ is stable.

(3) (1- $G_{IMC}$ $G_p$ )$G_P$ is stable.

The process model $G_m$ is separated into the invertible or delay free $G_1$ (s) and non invertible or delay part $G_{N1(s)}$ .i;e $G_m$(s)= $G_1$(s)×$G_{N1}$ .As per modified structure $G_1$(s)=$P_m$ and similarly $G_{N1}$ =$P_A$

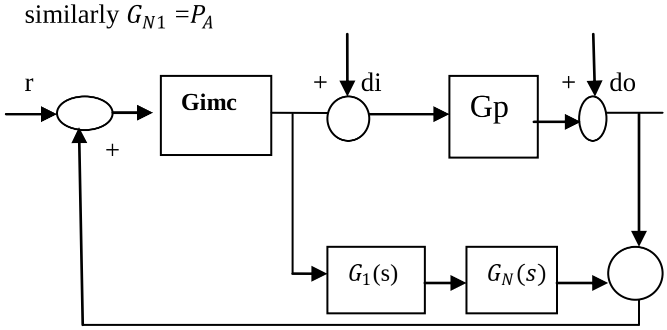


Fig 3.1 Modified IMC structure

### 3.2.1 IMC Controller Design

#### (1) Process Model with two unstable poles.

To obtain a superior response for unstable processes. if the process model ($G_P$) has unstable poles ,$u_{p1}$......................$u_{pm}$ , then the IMC controller $G_{imc}$ should have zeros at $u_{p1}$ ...............$u_{pm}$ and also 1-$G_P G_{imc}$ should have zeros at $u_{p1}$.............$u_{pm}$ .

Let the transfer function of the process model be.

$$G_P(\mathrm{s})=\frac{K_p e^{-\theta s}}{(\tau_1 S-1)(\tau_2\, S-1)} \quad (3.7)$$

First step of IMC controller design is to factor the process model into invertible and non invertible part.

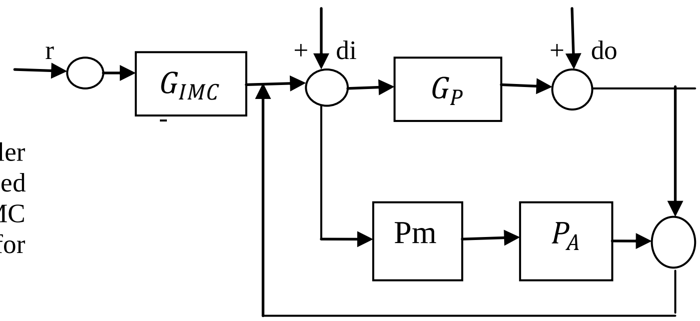


Fig .3.2 Modified IMC structure

$$G_1(\mathrm{s})=\frac{K_p}{(\tau_1 S-1)(\tau_2\, S-1)} \qquad G_{NI}=e^{-\theta s} \quad (3.8)$$

The invertible and non invertible part of modified model.are.

$$P_m=\frac{K_p}{(\tau_1 S-1)(\tau_2\, S-1)} \qquad P_A=e^{-\theta s} \quad (3.9)$$

Second step is to define IMC Controller as

$$G_{IMC}=P_m^{-1}(\mathrm{s})\times \mathrm{f(s)} \quad (3.10)$$

Where f(s) $=\frac{(a_2 s^2+a_1 s+1)}{(\lambda f s+1)4}$ is the low pass filter with adjustable time constant λ that reduce the effect of process model mismatch and improves the closed loop performance.

$$G_{IMC}=\frac{(\tau_1\, s-1)(\tau_2\, s-1\,)(a_2\,{}_S 2+\,a_1\,s+1)}{k_p\,(\lambda_s+1)4} \quad (3.11)$$

The first condition is satisfied automatically because $P_m^{-1}$ is the inverse of the model portion with the unstable poles or poles near zero.The second condition is satisfied because RHP poles of $G_P$ must be cancelled by the zeros of $G_{IMC}$ .then the third condition can be fulfilled by designing the IMC filter f is to make the controller proper to cancel the unstable poles near zeros of $G_P$.The RHP poles of $G_P$ must be cancelled by the zeros of (1-$G_P G_{imc}$ ).

The values of $a_1$ and $a_2$ can be calculated with the help of equation.(3.11) .this required

$$[1\text{-}G_P G_{imc}\ ]|\mathrm{s}=1/\tau_1,1/\tau_2 = 0 \quad (3.12)$$

ISSN 2321 – 2004
ISSN 2321 – 5526

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 2, Issue 1, January 2014

$$[\,1- \frac{(e^{-\theta s})(a_2\, s^2+ a_1\, s+1)}{k_p(\lambda_s+1)^4}\,]|s=1/\tau_1,1/\tau_2 = 0 \qquad (3.13)$$

The values of $a_1$ and $a_2$ are obtained after simplification are given below.

$$a_1=\{\tau_1^2\ (\frac{\lambda_f}{\tau_1}+1)^4\ e^{\theta/\tau_1} - \tau_2^2\ (\frac{\lambda_f}{\tau_2}+1)^4\ e^{\theta/\tau_2} +(\tau_2^2-\tau_1^2)\}/(\tau_1-\tau_2) \qquad (3.14)$$

$$a_2=\tau_1^2\{\ (\frac{\lambda_f}{\tau_1}+1)^4\ \ e^{\theta/\tau_1} - 1\} - a_1\tau_1 \qquad (3.15)$$

### 3.2.2 Disturbance Estimator Design.

The two unstable poles impose a phase lead and time delay terms impose a phase lag.in the present work, to make the controller realizable a PID controller in series with a lead lag compensator is considered . IMC controller is transform into PID lead-lag filter controller

$$F_{im}\ (s)=\frac{G_{imc}}{1-G_pG_{imc}} \qquad (3.16)$$

$$F_{im}\ (s)=\frac{(\tau_1 s-1)(\tau_2 s-1)(a_2 s^2+a_1 s+1)}{k[(\lambda_f s+1)^4-(a_2\ s^2+a_1\ s+1)e^{-\theta s}]} \qquad (3.17)$$

Where $e^{-\theta s}$ is the dead time which is approximated by pade expansion i;e

$$e^{-\theta s}=\frac{(1-\frac{\theta s}{2})}{(1+\frac{\theta s}{2})} \qquad (3.18)$$

$$F_{im}(s)=\frac{(\tau_1 s-1)(\tau_2 s-1)(a_2 s^2+a_1 s+1)\left(1+\frac{\theta s}{2}\right)}{k(\theta+4\lambda_f-a_1)s\left[1+\frac{(a_1\theta/2-a_2+2\lambda_f\theta+6\lambda_f^2)}{(\theta+4\lambda_f-a_1)}s^1+\frac{(a_2\theta^2/2+3\lambda_f^2\theta+4\lambda_f^3)}{(\theta+4\lambda_f-a_1)}s^2+\frac{(2\lambda_f^3\theta+\lambda_f^4)}{(\theta+4\lambda_f-a_1)}s^3+\frac{(\lambda_f^4\,\theta^2/2)}{(\theta+4\lambda_f-a_1)}s^4\right]} \qquad (3.19)$$

However it is not difficult to discover that there exists a RHP zero-pole cancelling at s=$1/\tau$ in eq.(3.12) which tends to cause the desired disturbance estimator to work unstably and cannot be removed directly .hence the mathematical maclaurin expansion series is here utilized to copy out the ideally desired disturbance estimator note that s=0 is a connotative pole of the ideally desired disturbance estimator in eq.( 3.12) which indicates that it has the property of integrating to eliminate the system output deviation from the setpoint value.therefore letting $F_{im}(s)=M(s)/_S$ it follows that.

$$F_{im}(s)=\frac{1}{s}\left[M(s)+M'(0)s+\frac{M''(0)}{2}\ s^2\ +\ \ldots\ldots\ldots\ldots\frac{M^i}{i!}\ s^i\ +\ \ldots\ldots\right] \qquad (3.20)$$

Obviously the first three terms of the above maclaurin expansion constitute exactly a standard PID controller in form of

$$F=K_c\,(1+1/\tau_1 s+\tau_D s)(1+\alpha s)/(1+\beta s) \qquad (3.21)$$

Where

$$K_c=\frac{a_1}{k(\theta+4\lambda_f-a_1)}\ ;\ \tau_1=a_1\ ;\tau_D=\frac{a_2}{a_1}\ ;\ \alpha=0.5\theta\ ;$$

$$\beta=\frac{(a_1\,\theta/2-a_2+2\lambda_f\theta+6\lambda_f^2)}{(\theta+4\lambda_f-a_1)}+(\tau_1+\tau_2) \qquad (3.22)$$

### 3.3 Simulation example

. Consider the process with doubly unstable poles studied by Liu et al. and Lee et al.

$$G_P = \frac{2e^{-0.3s}}{(3s-1)(s-1)}$$

It is a typical second-order delayed unstable process.by Liu et al.[7 ] and Tan et al. [6] method, they have taken $K_d$ = 3 and λ = 1.7θ = 0.51 just for obtain the set-point tracking controller, that is,

$$C(s) = \frac{1.5s^2+s+0.5}{(0.51s+1)^2}$$

The disturbance estimator F in form of PID is tuned as $K_C$ = 1.7638, $\tau_1$ = 1.8679, $\tau_2$= 2.3042 (i.e. λ = 1.7θ = 0.51), and also for illustration, they also took $N$ = 3 and λ = 1.5θ = 0.45 to obtain the third order approximation controller by Using the analytical design formulae

$$F=\frac{3.82s^3+439.41s^2+232.66s-129.79}{0.53s^3+0.8s^2+100s}$$

In our proposed method values of PID with series lead-lag filter parameters for disturbance estimator are

$K_C$= 3.5671 $\tau_1$ = 1.491 $\tau_D$ = 1.3364
$a$ = 0.15 $0.1a$ = 0.01983 $\beta$ = 0.0058

For (λ=0.35) performance comparison, a unit step change to the set-point input at t = 10 is added. The simulation results are shown in fig hethe proposed disturbance estimator has a faster settling time and smaller peak than either the high order or PID controller

ISSN 2321 – 2004
ISSN 2321 – 5526

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 2, Issue 1, January 2014

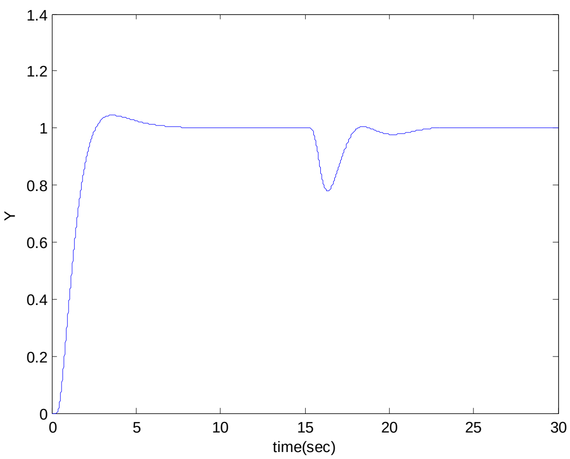


Fig.3.3

**4.controller design procedure by S.Park et al.**

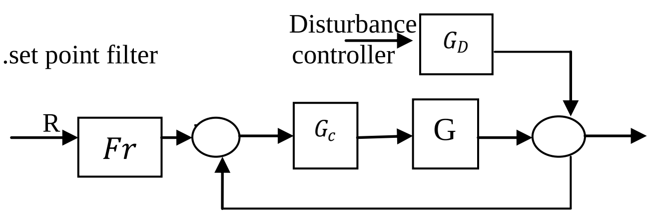


Fig 4.1 model by S.Park

The closed loop transfer functions for designing the feedback controller $G_c$ are

$$\frac{C}{R}=\frac{G_c\,G}{1+G_cG} \qquad (4.1)$$

$$\frac{C}{d}=\frac{G_D}{1+G_cG} \qquad (4.2)$$

The controller $G_c$ should be designed to insure the stability of these two transfer functionsThe above eqs.can be reformulated as the IMC structure (Morari and Zafiriou,1989) by

$$\frac{C}{R}=\frac{Gq}{1+q(G-\hat{G})} \qquad (4.3)$$

$$\frac{C}{d}=\frac{(1-qG)G_D}{1+q(G-\hat{G})}$$
(4.4)

If G=$\hat{G}$, then$\frac{C}{R}$ =Gq, $\frac{C}{d}$=(1-q$\hat{G}$)$G_D$ (4.5)

Where q is the IMC controller

Let us consider an unstable process model

G(s)=$P_M(s)P_A(s)$ (4.5)

Where $P_M(s)$ contains the invertible portion of the model and $P_A(s)$ contains all the noninvertible portion .The invertible portions are the part of the model with stable poles and unstable poles. The noninvertible portions are the p[ortion of model with right–half–plane zeros and time delays .The following two conditions should be satisfied to stabilize the closed–loop response.

1. if the process model G has unstable poles u$p_1$ … … … … … … .. u$p_k$ q should have zeros at u$p_1$.............u$p_k$

2 if the process model $G_D$has unstable poles du$p_{1,}$ … … … … … … .. du$p_m$, (1-q$\hat{G}$) should have zeros at du$p_{1,}$ … … du$p_m$

If these two conditions are satisfied, the closed –loop responses for both a setpoint change and a load change become stable

The IMC controller is set as q=$P_M^{-1}(s)f$. Here, q has zeros at u$p_1$.............u$p_k$ because $P_M^{-1}(s)$ is the inverse of the model portion with the unstable poles . Thus, the first condition is satisfied .Then, through the filter design, the second condition should be satisfied .The filter for the IMC controller can be designed by two part ;$f_s$ is the portion to make the controller proper ,and $f_d$ is the portion to cancel the unstable poles or stable poles near zero of $G_D$

.f=$f_sf_d$ , $f_s=\frac{1}{(\lambda s+1)^n}$ , $f_d=\frac{\sum_{i=1}^{m}\alpha_i s^i+1}{(\lambda s+1)^m}$ (4.6)

Where n is chosen to make the controller realizable ,$\alpha_i$ are determine to cancel the unstable poles of $G_D$ and m is the number of unstable poles .f functions as a filter with adjustable time constant $\lambda$

|1-q$\hat{G}$)|s= du$p_{i,}$ … … … … … … .. du$p_m$, = 0 (4.7)

Where du$p_{i,}$≠0

Thus ,the IMC controller is

$$.q=\frac{P_M^{-1}(s)}{(\lambda s+1)^n}\times\frac{\sum_{i=1}^{m}\alpha_i s^i+1}{(\lambda s+1)^m} \qquad (4.8)$$

Then we get

ISSN 2321 – 2004
ISSN 2321 – 5526

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 2, Issue 1, January 2014

$\frac{C}{R}$=Gq=$\frac{P_A(s)}{(\lambda s+1)^n} \times \frac{\sum_{i=1}^{m} \alpha_i s^i+1}{(\lambda s+1)^m}$ (4.9)

$\frac{C}{d}$=(1-Gq) $G_D$=(1-$\frac{P_A(s)}{(\lambda s+1)^n} \times \frac{\sum_{i=1}^{m} \alpha_i s^i+1}{(\lambda s+1)^m}$) $G_D$ (4.10)

The lead term ($\sum_{i=1}^{m} \alpha_i s^i + 1$) in eqs (4.9) causes an overshoot in the closed-loop response to a setpoint change .This problem can be resolved if we add a setpoint filter

Fr =$\frac{1}{(\sum_{i=1}^{m} \alpha_i s^i+1)}$ (4.11)

The classical feedback controller we need is obtained as

$G_C=\frac{q}{1-Gq}$ (4.12)

Thus ,the controller $G_C$ is

$$G_C=\frac{\frac{P_M^{-1}(s)}{(\lambda s+1)^n} \times \frac{\sum_{i=1}^{m} \alpha_i s^i+1}{(\lambda s+1)^m}}{1-\frac{P_A(s)}{(\lambda s+1)^n} \times \frac{\sum_{i=1}^{m} \alpha_i s^i+1}{(\lambda s+1)^m}} \quad (4.13)$$

The controller $G_C$ can be approximated to a PID controller by first noting that it can be expressed as

$G_C$=f(s)/s (4.14)

Expanding $G_C$(s) in a Maclaurin series in s gives

$G_C$(s)=$\frac{1}{s}$(f(0)+$f'(0)s$+$\frac{f''(0)}{2!}s^2$+.................) (4.15)

The first three terms of the above expansion can be interpreted as the standard PID controller given by

$G_C$(s= K$_c$(1+$\frac{1}{\tau_1 s}$+$\tau_D s$) ,where K$_c$ = $f'(0)$ , $\tau_1$=$f'(0)/(f(0)$ , $\tau_D$=$\frac{f''(0)}{2!}/f'(0)$ (4.16)

$\tau_1 \geq 0$; $\tau_D \geq 0$

### 4.2 PID controller setting

Unstable process model with two unstable poles and time delay

The process model is

$G(s) = G_D$(s)=$\frac{K_p e^{-\Theta s}}{(\tau_1 S-1)(\tau_2 S-1)}$ (4.17)

And from eq.4.6

$f_s$=$\frac{1}{(\lambda s+1)^2}$ , $f_d$=$(\alpha_2 s^2 + \alpha_1 s + 1)/ (\lambda s + 1)^2$.

The IMC controller becomes

.q=$(\tau_1 S - 1)(\tau_2 S - 1)(\alpha_2 s^2 + \alpha_1 s + 1)/ K$ (4.18).

Then we get

$G_C(s)$=$\frac{(\tau_1 S-1)(\tau_2 S-1)\alpha_2 s^2+\alpha_1 s+1)}{K[(\lambda s+1)^4-e^{-\theta s}(\alpha_2 s^2+\alpha_1 s+1)]}$ (4.19)

Expanding $G_C(s)$ in a Maclaurin series in s gives

K$_c$ =$\frac{\tau_1}{K(4\lambda+\theta-\alpha_1)}$ (4.20)

$\tau_1$=-$\tau_1$-$\tau_2$+$\alpha_1$-$\frac{6\lambda^2-\alpha_2+\alpha_1\theta-\theta^2/2}{4\lambda+\theta-\alpha_1}$ (4.21)

$\tau_D$=$\frac{\alpha_2+\tau_1\tau_2-(\tau_1+\tau_2)\alpha_1-\left(4\lambda^3+\theta\alpha_2+\frac{\theta^3}{6}-\alpha_1\theta^2/2\right)/(4\lambda+\theta-\alpha_1)}{\tau_1}$-$\frac{6\lambda^2-\alpha_2+\alpha_1\theta-\theta^2/2}{4\lambda+\theta-\alpha_1}$ (4.22)

Where $\alpha_1$,$\alpha_2$ values are calculated by solving

1-$\frac{(\alpha_2 s^2+\alpha_1 s+1)e^{-\Theta s}}{(\lambda s+1)^4}$ |s=$\frac{1}{\tau_1}$, $\frac{1}{\tau_2}$ = 0 (4.23)

With this controller ,the transfer function of the setpoint change is given by

$\frac{C}{R}$ =$\frac{(\alpha_2 s^2+\alpha_1 s+1)e^{-\Theta s}}{(\lambda s+1)^4}$ (4.24)

Therefore ,if only a PID controller is used ,the closed –loop response is at best ,and the lead term $\alpha_2 s^2 + \alpha_1 s + 1$ causes an overshoot .adding a setpoint filter fr=$1/\alpha_2 s^2 + \alpha_1 s + 1$ results the closed –loop transfer function $C/R'$=)$e^{-\Theta s}/(\lambda s + 1)^4$.

### Simulation example

Consider an unstable process with two unstable poles as follows

$G(s) = G_D$(s)=$\frac{K_p e^{-\Theta s}}{(\tau_1 S-1)(\tau_2 S-1)}$

ISSN 2321 – 2004
ISSN 2321 – 5526

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN ELECTRICAL, ELECTRONICS, INSTRUMENTATION AND CONTROL ENGINEERING
Vol. 2, Issue 1, January 2014

The filter time constant $\lambda$ was chosen as $\lambda$=0.45, the tuning parameters are $K_C$=2.3153, $\tau_1 = 1.7843$,$\tau_D$=1.8859, setpoint filter is Fr=1/($3.252s^2 + 1.7147s + 1$).fig show the closed –loop response of the unstable process given by eq.to a unit step change in setpoint and load.

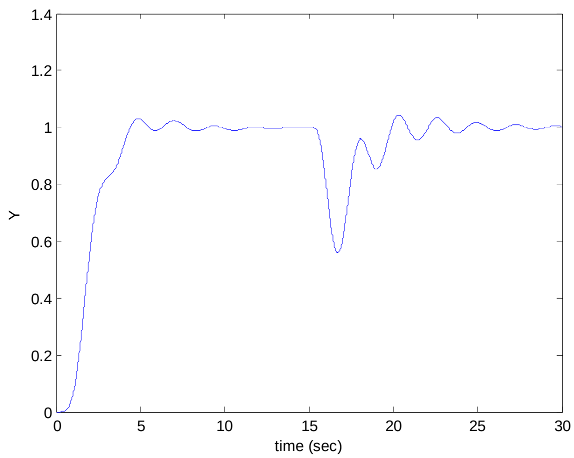


Fig.4.2

## 5.COMPARISION OF SIMULATION MODEL BY LEE ET.AL,LIU ET.AL, S.PARK

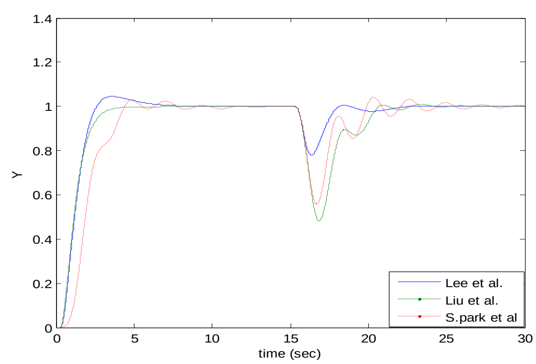


Fig.5

## 6. CONCLUSION

The simulation result shown in Fig.5 .the model proposed by Lee has a faster settling time and smaller peak than either the high order or PID controller by Liu[7]and S.Park[2]

## REFERENCES

1. H.P Huang and C.C Chen " control –system synthesis for open –loop unstable process with time delay " IEE process –control theory and Application, Vol.144, pp.334,1997
2. S.Park,Y.Lee and J.Lee " PID controller tuning for integrating and unstable process with time delay" Chem.Eng.Sci., Vol. 55,pp 3481-3493,2000.
3. Y.G Wang and W.J Cai " Advanced PID tuning for integrating and unstable processes gain and phase margin specifications" Ind Eng.Chem.Res.Vol.41,No.12, pp 2910-2914,2002.
4. M.Shamsuzzoha and Moonyong Lee "Enhanced disturbance rejection for open –loop unstable process with time delay" ISA Transaction 48(2009) 237-244.
5. M.Morari and E Zafiriou "Robust process control", Prentice-Hall: Englewood Cliffs ,NJ,1989.
6. W.Tan,H.J.Marquez and T.Chen , "IMC design for unstable processes with time delays," J.process control,Vol 13,pp.203-213,2003.
7. T.Liu, W.Zhang and D.Gu, " Analytical designof 2DOF control scheme for open loop unstable process with time delay" J.Process Control, Vol 15,pp. 559-572,2005.
8. Kwak HJ, Sung SW, Lee IB, Park JY. Modified Smith predictor with a new structure for unstable processes. Ind Eng Chem Res 1999;
9. Majhi S, Atherton DP.obtaining controller parameter for a new Smith Predictor using autotuning .Automatica 2000;
10. Zhang WD,GU D, Wang W, Xu x.quantitative performance design of a modified SP for unstable processes with time delay. Ind Eng Chem Res 2004;