

# “PID Tuning Rules for First Order plus Time Delay System”

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**Abstract:** This paper demonstrates an efficient method of tuning the PID controller parameters using different PID tuning techniques. The method implies an analytical calculating the gain of the controller ( $K_c$ ), integral time ( $T_i$ ) and the derivative time ( $T_d$ ) for PID controlled system whose process is modelled in first order plus time delay (FOPTD) form. In this Paper a First order time delay system is selected for study. The performance of PID tuning techniques is analysed and compared on basis of time response specifications.

**Keywords:** PID controller, Tuning rules, MATLAB Simulation, Comparison.

## I. INTRODUCTION

Proportional–Integral–Derivative (PID) controllers are still by far the most widely adopted controllers in industry owing to the advantageous cost/benefit ratio they are able to provide. In fact, although they are relatively simple to use, they are able to provide a satisfactory performance in many process control tasks. The PID controller is the most common control algorithm used in process control applications. According to the survey more than 90% of the control loops were of the PID type. The popularity of the PID controller can be attributed to its different characteristic features: it provides feedback; it has the ability to eliminate steady-state offsets through integral action; and it can anticipate the future through derivative action.

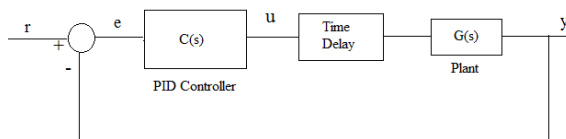


Fig.1 Feedback control system

## II. THE PID STABILIZATION PROBLEM

Consider the feedback control system shown in Figure 1. The plant  $G(s)$  is a first-order model with dead-time given by the following transfer function

$$G(s) = \frac{k}{1 + Ts} e^{-\tau s} \quad \text{----- (1)}$$

Where  $K$  represents the steady-state gain of the plant,  $\tau$  the time delay, and  $T$  the time constant. The controller  $G_c(s)$  is of the PID type, i.e., it has a proportional, an integral and Derivative term

$$G_c(s) = K_c \left( 1 + \frac{1}{T_i s} + T_d s \right) \quad \text{----- (2)}$$

With  $K_c$  = Proportional gain,  $T_i$  = Integral time constant and  $T_d$  = Derivative time constant. Different methods have been therefore proposed in the literature to estimate the three parameters by performing a simple experiment on the plant. They are typically based either on an open-loop step response or on a closed-loop relay feedback system.

## III. CONVENTIONAL PID TUNING TECHNIQUES

### A. ZIEGLER-NICHOLS METHOD

The Ziegler-Nichols design methods are the most popular methods used in process control to determine the parameters of a PID controller [2]. Ziegler Nichols tuning methods (ZN tuning methods) are the principal methods used in PID controller tuning. The two methods are called step response method and ultimate frequency method [6]. The unit step response method is based on the open-loop step response of the system. The unit step response of the process is characterized by two parameters, delay  $L_1$  and time constant  $T$ . These are determined by drawing a tangent line at the inflexion point, where the slope of the step response has its maximum value. The intersections of the tangent and the coordinate axes give the process parameters as shown in Figure 2, and these are used in calculating the controller parameters. The parameters for PID controllers



obtained from the Ziegler-Nichols step response method are shown in Table 1.

Table 1. PID controller parameters for Ziegler-Nichols method

Sr. No.	Type of controller	$K_p$	$T_i$	$T_d$
1	PID	$1.2T/L_1$	$2L_1$	$0.5L_1$

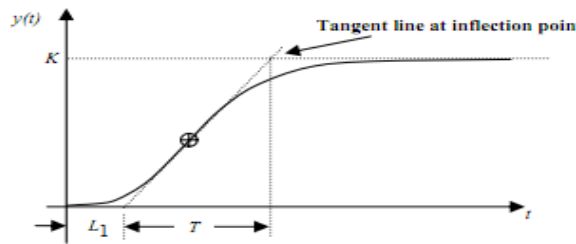


Fig.2. Response curve for Z-N method

From the step response in Fig. 2,  $T$  &  $L_1$  are obtained as  $T = 2.8$ ,  $L_1 = 3$ . As per Table 1,  $K_p = 1.285$ ,  $K_i = 1/T_i = 0.178$  and  $K_d = 1.4$ , with the above values of  $K_p$ ,  $K_i$  and  $K_d$ , step response is shown in Fig. 3.  $M_p = 22.2\%$ ,  $t_s = 21.9$  sec,  $e_{ss} = 0$ .

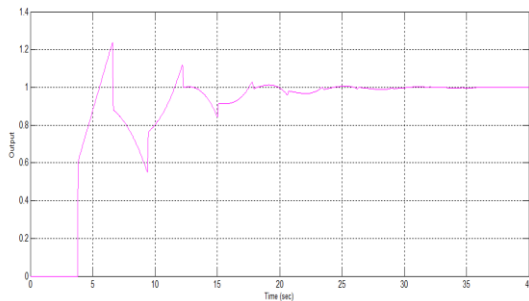


Fig.3. Step response of Ziegler-Nichols method

### B. FINE TUNED METHOD

With Ziegler-Nichols PID tuning formula, the resulting system exhibit a large settling Time in the step response method, which is unacceptable. In such a case, we need a series of fine-tuning until an acceptable result is obtained.

The individual effect of  $K_p$ ,  $K_i$  and  $K_d$  summarized in Table 2 can be very useful in fine tuning of PID controller [7]. Beginning with the values of  $K_p$ ,  $K_i$  and  $K_d$  obtained from Z-N step response method, unit step response for different combination of  $K_p$ ,  $K_i$  and  $K_d$  were observed. After fine tuning, PID controller parameters obtained are  $K_p = 1.285$ ,  $K_i = 0.27$  and  $K_d = 0.9$ . The unit step response for  $K_p =$

$1.285$ ,  $K_i = 0.27$  and  $K_d = 0.9$  is shown in Fig. 4, which gives  $M_p = 26.3\%$ ,  $t_s = 13.6$  sec and  $e_{ss} = 0$ .

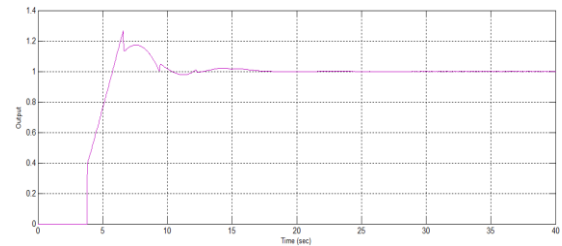


Fig.4 Step Response of Fine Tuned PID controller

Table.2. Effect of  $K_p$ ,  $K_i$  and  $K_d$

Close loop Response	Overshoot	Settling Time	Steady – state Error
Increasing $K_p$	Increase	Small Increase	Decrease
Increasing $K_i$	Increase	Increase	Large Decrease
Increasing $K_d$	Increase	Decrease	Minor change

### C. CHIEN, HORNES AND RESWICK METHOD

CHR in an abbreviation from the authors names – Chien, Hrones and Ryswick. CHR was developed from the Ziegler-Nichols's method for implementation of certain quality requirements of open systems. Using the a periodic step response, the conditional parameters of the process will be determined [6].

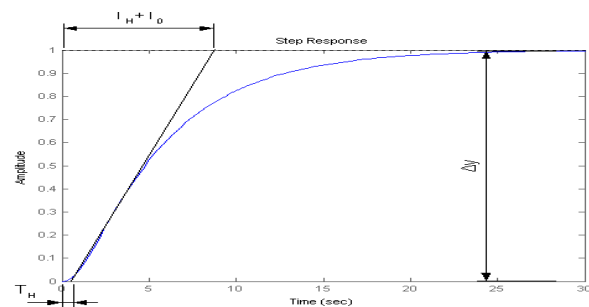


Fig.5. Open loop response of CHR method

$$K_C = \frac{0.6 T_D}{K_p T_H} \quad \text{----- (3)}$$

$$K_i = \frac{1}{T_i} = T_D \quad \text{----- (4)}$$

$$K_d = 0.5 T_H \quad \text{----- (5)}$$

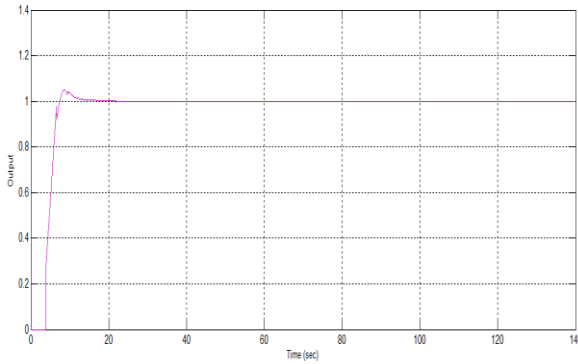


Fig. 6 .Step Response of Chien, Hrones and Reswick method

From the step response in Fig. 5,  $T_H$  &  $T_0$  are obtained as  $T_H = 1.92$ ,  $T_0 = 1.8$ . With  $K_p = 1, K_c = 1.013, K_i = 1/T_i = 0.2654, K_d = 0.756$ . With the above values of  $K_p, K_i$  and  $K_d$ , step response is shown in Fig 6.  $M_p = 4.91 \%$ ,  $t_s = 10.1$  sec,  $e_{ss} = 0$ .

#### D. IMC PID TUNING METHOD

While IMC controller implementations are becoming more popular, the standard industrial controllers remain the proportional plus integral and derivative (PID) controllers [1]. Based on Rivera *et al.* [1986], the goal of control system design is to achieve a fast and accurate set-point tracking.

$$y \cong y_s \quad \forall t, \quad \forall d \quad \text{----- (6)}$$

This implies that the effect of external disturbances should be corrected as efficiently as possible and also being assured of insensitivity to modelling error.

$$y' \cong y_s - d \quad \forall t, \quad \forall d \quad \text{----- (7)}$$

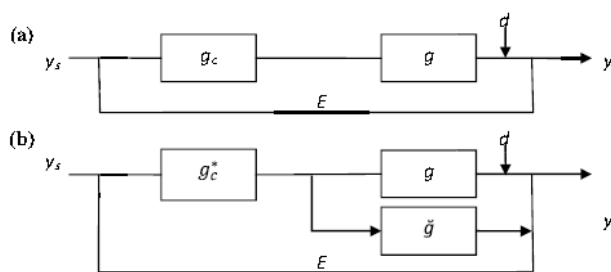


Fig.7: (a) conventional configuration and (b) Internal Model Control configuration

The PID parameter tuning law based on the relationship of the IMC and the PID controller has been proposed by Rivera *et al.* [1986]. PID control structure is shown in Fig.7 (a), where  $g_c$  and  $g$  are the PID controller and the controlled process, respectively. They are given by

$$g_c = K_c \left( 1 + \frac{1}{T_I s} + T_D s \right) \quad \text{----- (8)}$$

$$g = \frac{K}{1 + T_s s} e^{-\theta s} \quad \text{----- (9)}$$

Where  $k_c, T_i$  and  $T_D$  are the proportional gain, the reset time and the derivative time, respectively. Meanwhile, the structures of the IMC is shown in Fig. 7(b), where  $g_c^*$  are the IMC controller and the internal model respectively. The IMC controller is given by

$$g_c^* = \frac{1 + T_s s}{K} \times \frac{1}{1 + \lambda s} \quad \text{----- (10)}$$

The IMC-PID setting for FOPTD process is shown in Table -3.

Table 3:- PID Controller settings for IMC-PID

PID Parameters	IMC –PID
Proportional Gain ( $K_c$ )	$k_c = \frac{T + 0.5\theta}{K(\lambda + 0.5\theta)}$
Integral Time( $T_i$ )	$T_i = T + 0.5\theta$
Derivative Time( $T_d$ )	$T_d = \frac{T\theta}{2T + \theta}$

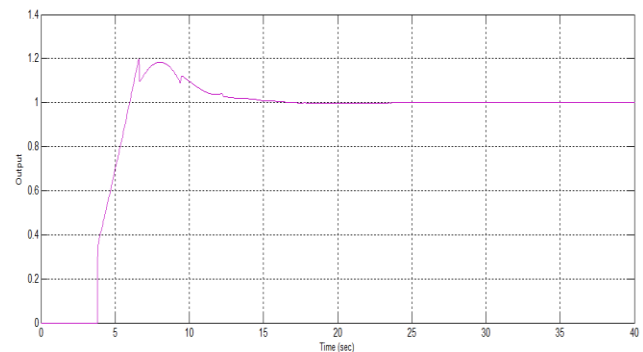


Fig 8. Step Response of IMC PID controller

From equation 9,  $\theta, T, \lambda$  and  $K$  are obtained as  $\theta = 2.8, T = 1.92, \lambda = 1.404$  and  $K = 1$ . With  $K_c = 1.1837, K_i = 1/T_i = 0.301, K_d = 0.8736$ . With the above values of  $K_c, K_i$  and  $K_d$  step response is shown in Fig 8.  $M_p = 18.3 \%$ ,  $t_s = 12.2$  sec,  $e_{ss} = 0$ .

### E. DIRECT SYNTHESIS TUNING METHOD

The direct synthesis methods for PID controllers are typically based on a time-domain or frequency-domain performance criterion [8]. The controller design is based on a desired closed-loop transfer function. Then, the controller is calculated analytically so that the closed-loop set-point response matches the desired response. The obvious advantage of the direct synthesis approach is that performance requirements are incorporated directly through specification of the closed-loop transfer function. One way to specify the closed-loop transfer function is to choose the closed-loop poles.

$$K_c = \frac{0.18 + 0.35(\frac{\tau_m}{T_m})^{-1.00}}{K_m(0.53 - 0.36v^{0.71})} \quad \text{----- (11)}$$

$$K_i = \frac{1}{T_i} = T_m + 0.5\tau_m \quad \text{----- (12)}$$

$$K_d = \frac{T_m \tau_m}{2T_m + \tau_m} \quad \text{----- (13)}$$

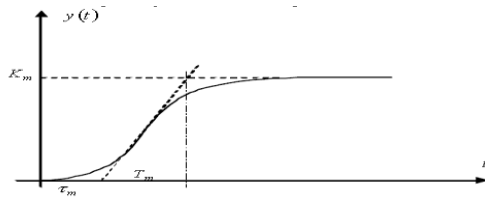


Fig. 9. Response curve for direct synthesis tuning method

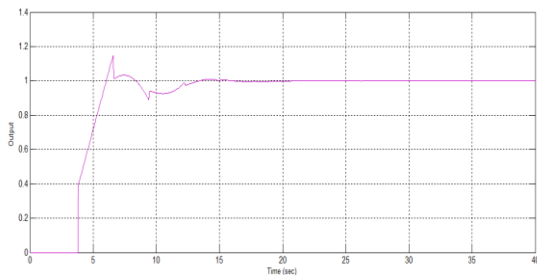


Fig 10. Step Response of direct synthesis method

According to fig 9,  $T_m$ ,  $\tau_m$  and  $v$  are obtained as  $T_m=3$ ,  $\tau_m=2.8$ ,  $v = 0.1$ . With  $K_c = 1.20703$ ,  $K_i = 1/T_i = 0.227272$ ,  $K_d=0.954545$ . With the above values of  $K_c$ ,  $K_i$  and  $K_d$  step response is shown in Fig 10.  $M_p = 13.9\%$ ,  $t_s = 11.6$  sec,  $e_{ss} = 0$ .

### F. LAMBDA TUNING TECHNIQUE

Lambda Tuning originated with Dahlin [3] in 1968. Lambda tuning method is principally a pole placement method. The process model is assumed to be the first order transfer function (5). The closed-loop transfer function of the process is desired to be of first order

$$G(s) = \frac{K}{\lambda s + 1} e^{-sL1} \quad \text{----- (14)}$$

Where,  $\lambda$  is the tuning parameter that determines the pole location. Smaller  $\lambda$  values increase the controller Performance but takes it closer to the instability region we obtain tuning parameters for a PID controller as.

$$K_p = \frac{T + L1/2}{K(\lambda + \frac{L1}{2})} \quad \text{----- (15)}$$

$$K_i = \frac{1}{T_i} = T + L1/2 \quad \text{----- (16)}$$

$$K_d = \frac{TL1}{L1 + 2T} \quad \text{----- (17)}$$

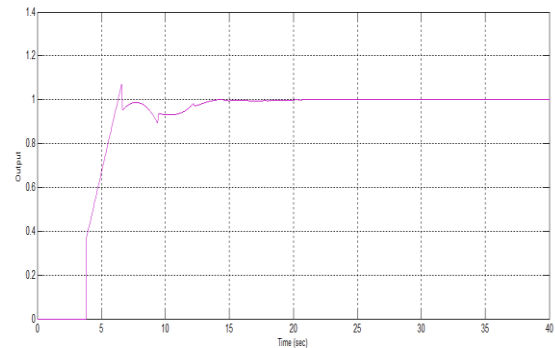


Fig 11. Step Response of lambda tuning method

$T$ ,  $L_1$  and  $\lambda$  are obtained as  $T=3$ ,  $L_1=2.8$  and  $\lambda=2.511$ . With  $K_p=1.125$ ,  $K_i=1/T_i=0.2272$ ,  $K_d=0.9545$ . With the above values of  $K_p$ ,  $K_i$  and  $K_d$  step response is shown in Fig 11.  $M_p = 5.88\%$ ,  $t_s = 11.8$  sec,  $e_{ss} = 0$ .

### IV. MATLAB SIMULATION

The simulation result of proceeding plant is obtained by using MATLAB model. The model is constructed as shown in figure 10. The calculated value is used for  $k_p$ ,  $k_i$  and  $k_d$ .

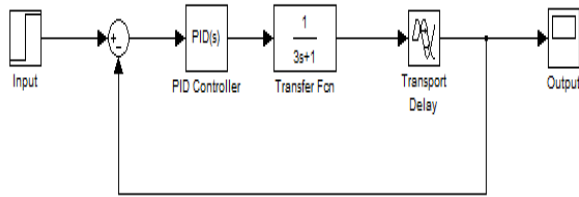


Fig 12. MATLAB/ Simulink Model

### V. PERFORMANCE ANALYSIS

Consider the following stable process

$$G(s) = \frac{K}{Ts + 1} e^{-\theta s} \quad \text{----- (18)}$$

Where K = 1, T = 3, and  $\theta = 2.8$

Simulation results using MATLAB for different PID tuning techniques are summarized in Table 4.

Table 4: Time response parameters

Algorithm	Maximum Overshoot ( $M_p$ )	Settling time ( $T_s$ )	Steady state error ( $e_{ss}$ )
Z-N Method	22.2	21.9	0
Fine Tune	26.3	13.6	0
Chien, Hrones and Reswick method	4.91	10.1	0
IMC Tuning Method	18.3	12.2	0
Direct Synthesis	13.9	11.6	0
Lambda Tuning	5.88	11.8	0

### VI. CONCLUSION

The paper describes design of PID controller for a First order system with time delay. Total Six PID tuning techniques were implemented and their performances analyzed. Ziegler Nicholas Tuning technique exhibit largest settling time and Fine Tuning gives largest maximum overshoot. Chaien ,Hrones and Reswick Method gives smallest maximum overshoot and settling time. Among the Six PID tuning techniques, the Chaien ,Hrones and Reswick Method PID controller gives the best results for a first order time delay system.

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