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Design of Wideband Digital Differentiator Using FIR Approximation

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Abstract: An ideal differentiator has a frequency response that is linearly proportional to frequency. Full Wideband differentiator has an anti-symmetric unit sample response. FIR approximations are preferred to IIR because of the stability and linear phase response offered. In this paper we consider the design of an FIR differentiator based on frequency sampling technique.

Keywords: Differentiator, FIR, Frequency Sampling, Linear phase, Magnitude Response.

I. INTRODUCTION

Differentiation of a signal gives a measure of instantaneous rate of change of signal with frequency. The differentiators have applications in signal processing systems such as, control systems instrumentation, biomedical and communication systems. For example, in radars, the acceleration can be computed from the position measurements using second order differentiator [1]. Linear phase FIR filter approaches mentioned in literatures [2-3] use maximally flat criteria. This paper proposes an alternate simple technique to design wide band digital differentiator.

II. DESIGN

An ideal digital differentiator is defined as the one that has the frequency response

$H_d(\omega)=j\omega, \quad -\pi\leq \omega \leq\pi.$ (1)

and the corresponding magnitude response

$H_r(\omega)= |\omega| , \quad -\pi\leq\omega\leq\pi.$ (2)

The unit sample response corresponding to $H_r(\omega)$ is determined for antisymmetric case from [4]

$$\frac{1}{\pi}\int_0^{\pi} H_r(\omega)\sin k\omega \; d\omega = \frac{1}{\pi}\int_0^{\pi}\omega\sin k\omega \; d\omega = h(n) \quad (3)$$

where $k = (\frac{N-1}{2} - n)$, n=0,1, ... , ((N/2)-1).

In order to have a linear phase antisymmetric FIR differentiator, the impulse response must satisfy the condition [5],

$h(n)= -h(N-1-n), \; n=0,1\ldots,N-1.$ (4)

where the tap length N is even.

The pseudo-magnitude function of a linear-phase antisymmetric FIR filter for a tap length ‘N’ is given by [5],

$$H_r(\omega) = 2\sum_{n=0}^{(N/2-1)} h(n)\sin k\omega. \quad (5)$$

Coefficients of equation (5) are evaluated using(3) as

$$\frac{1}{\pi}\int_0^{\pi} H_r(\omega)\sin p\omega \, d\omega = \frac{1}{\pi}\int_0^{\pi} 2\sum_{n=0}^{(N/2-1)} h(n)\sin k\omega \sin p\omega \, d\omega .$$

where, $p = (\frac{N-1}{2} - n)$,n=0,1, ... , ((N/2)-1).

The filter coefficients h (n) are used in (5) to plot the frequency response of differentiator. The frequency response approximates equation (2) reasonably well as shown in Fig.1

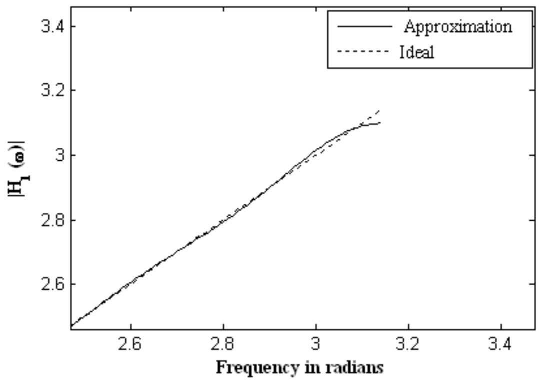

Fig1. Frequency response for N=30.

III. ERROR CRITERIA FOR PERFORMANCE EVALUATION

A. Absolute error

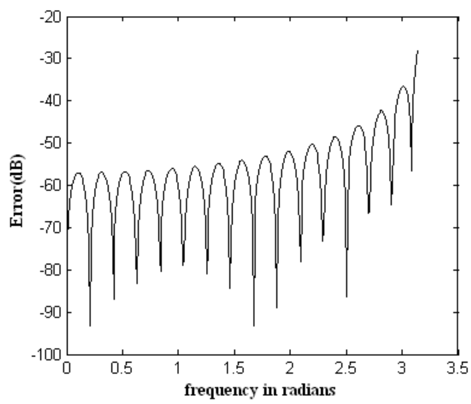

Fig2. Approximation error for N=30

The Error response of our approximation is similar to [4].

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B. New error criteria for differentiators

Differentiating (2) twice we get,

$$\frac{d^2H_1(\omega)}{d\omega^2} = 0 \qquad (7)$$

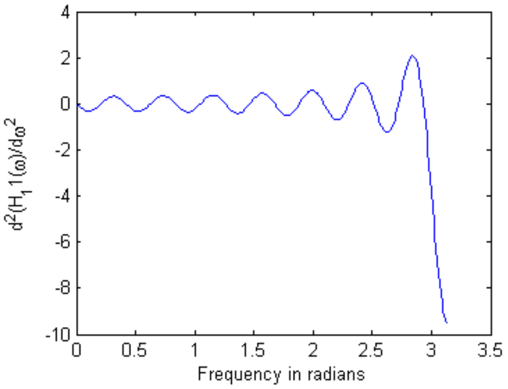

Fig.3. $\frac{d^2H_1(\omega)}{d\omega^2}$ plot for N=30.

C. Comparison with frequency sampling

Although this method is similar to frequency sampling technique, computation of filter coefficient is greatly simplified for a given value of tap length. Since, the integration (3) and (6) are easily evaluated. Whereas frequency sampling [4], coefficients are calculated using summation,

$$h(n) = \frac{1}{N}\left\{\tilde{H}(0) + 2\sum_{k=1}^{N/2-1} \mathrm{Re}[\tilde{H}(k)e^{j2\pi kn/N}\right\} \qquad (8)$$

which requires evaluation of ((N/2)-1)) terms for N even. As N becomes large our method becomes less complex compared to (8).Bandwidth and error in frequency response obtained in our design and frequency sampling technique used in [4] are comparable.

IV. CONCLUSION

The design of FIR approximation of a differentiator has been proposed. The proposed approximation yields impulse response coefficients h(n) giving a close 'fit' to the ideal differentiator response.

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